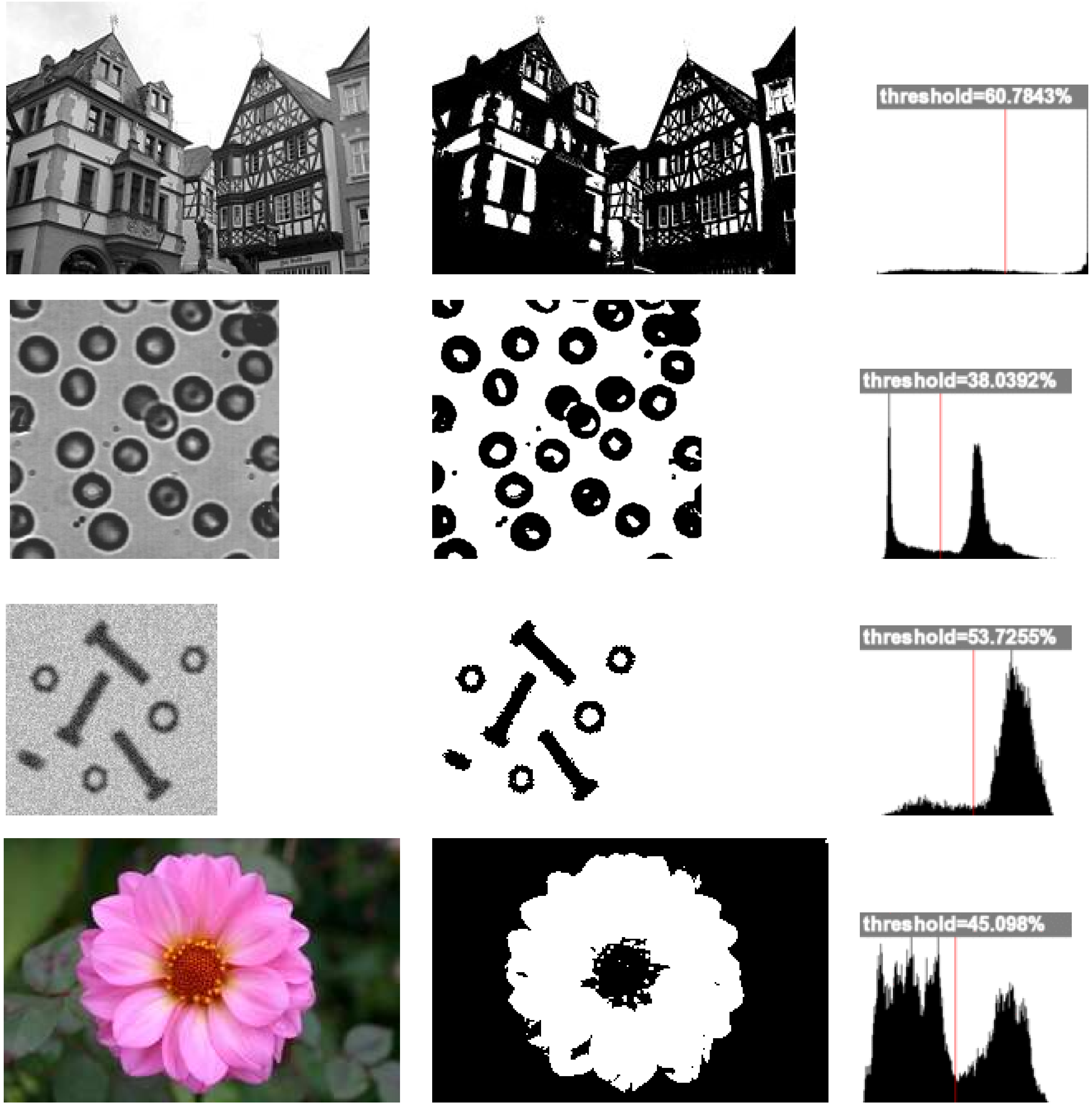
Automatic thresholding

- Automatically perform histogram shape-based image thresholding
- Assume the image contains two classes of pixels, foreground versus background pixels


threshold=60.7843%
threshold=38.0392%
threshold=53.7255%
threshold=45.098%

- Idea: minimize within-group variance of the two groups of pixels separated by the thresholding operator
- histogram is a probability function P, P(0),…,P(I) probabilities of the observed gray values 0,…,I
- P(i)=num (r,c) Image(r,c)=i/(RxC)
- goal: determine threshold *t*

- The *weighted within-class variance* is:

$$\sigma_w^2(t) = q_1(t)\sigma_1^2(t) + q_2(t)\sigma_2^2(t)$$

- Where the class probabilities are estimated as:

$$q_1(t) = \sum_{i=1}^{t} P(i) \qquad q_2(t) = \sum_{i=t+1}^{I} P(i)$$

- And the class means are given by:

$$\mu_1(t) = \sum_{i=1}^{t} \frac{iP(i)}{q_1(t)} \qquad \mu_2(t) = \sum_{i=t+1}^{I} \frac{iP(i)}{q_2(t)}$$

- Finally, the individual class variances are:

$$\sigma_1^2(t) = \sum_{i=1}^{t} [i - \mu_1(t)]^2 \frac{P(i)}{q_1(t)}$$

$$\sigma_2^2(t) = \sum_{i=t+1}^{I} [i - \mu_2(t)]^2 \frac{P(i)}{q_2(t)}$$

- Now, we could actually stop here. All we need to do is just run through the full range of *t* values [1,256] and pick the value that minimizes

$$\sigma_w^2(t)$$

- But the relationship between the within-class and between-class variances can be exploited to generate a recursion relation that permits a much faster calculation.

- After some algebra, we can express the total variance as...

$$\sigma^2 = \underbrace{\sigma_w^2(t)}_{\text{Within-class}} + \underbrace{q_1(t)[1 - q_1(t)][\mu_1(t) - \mu_2(t)]^2}_{\text{Between-class, } \sigma_B^2(t)}$$

- The total variance is independent of t, we can determine t by maximizing $\sigma_B^2(t)$

- So, minimizing the within-class variance is the same as maximizing the between-class variance

- The nice fact is that there is a relationship between the value computed for t and that computed for t+1

Finally...

- *Initialization..* $q_1(1) = P(1)$ •; $\mu_1(0) = 0$

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- *Recursion...*

$$q_1(t+1) = q_1(t) + P(t+1)$$

$$\mu_1(t+1) = \frac{q_1(t)\mu_1(t) + (t+1)P(t+1)}{q_1(t+1)}$$

$$\mu_2(t+1) = \frac{\mu - q_1(t+1)\mu_1(t+1)}{1 - q_1(t+1)}$$