Automatic thresholding
• Automatically perform histogram shape-based image thresholding
• Assume the image contains two classes of pixels, foreground versus background pixels
• Idea: minimize within-group variance of the two groups of pixels separated by the thresholding operator
• histogram is a probability function \( P, P(0), \ldots, P(I) \) probabilities of the observed gray values 0,\ldots,I
• \( P(i)=\text{num } (r,c) \text{ Image}(r,c)=i/(RxC) \)
• goal: determine threshold \( t \)
• The weighted within-class variance is:

\[ \sigma_w^2(t) = q_1(t) \sigma_1^2(t) + q_2(t) \sigma_2^2(t) \]

• Where the class probabilities are estimated as:

\[ q_1(t) = \sum_{i=1}^{t} P(i) \quad q_2(t) = \sum_{i=t+1}^{I} P(i) \]

• And the class means are given by:

\[ \mu_1(t) = \sum_{i=1}^{t} \frac{iP(i)}{q_1(t)} \quad \mu_2(t) = \sum_{i=t+1}^{I} \frac{iP(i)}{q_2(t)} \]
• Finally, the individual class variances are:

\[
\sigma_1^2(t) = \sum_{i=1}^{t} [i - \mu_1(t)]^2 \frac{P(i)}{q_1(t)}
\]

\[
\sigma_2^2(t) = \sum_{i=t+1}^{l} [i - \mu_2(t)]^2 \frac{P(i)}{q_2(t)}
\]

• Now, we could actually stop here. All we need to do is just run through the full range of \(t\) values [1,256] and pick the value that minimizes

\[\sigma_w^2(t)\]

• But the relationship between the within-class and between-class variances can be exploited to generate a recursion relation that permits a much faster calculation.
• After some algebra, we can express the total variance as...

\[
\sigma^2 = \sigma_w^2(t) + q_1(t)[1 - q_1(t)][\mu_1(t) - \mu_2(t)]^2
\]

Within-class

Between-class, \(\sigma_B^2(t)\)

• The total variance is independent of \(t\), we can determine \(t\) by maximizing \(\sigma_B^2(t)\)

• So, minimizing the within-class variance is the same as maximizing the between-class variance

• The nice fact is that there is a relationship between the value computed for \(t\) and that computed for \(t+1\)
Finally...

- **Initialization.**  \( q_1(1) = P(1) \cdot ; \mu_1(0) = 0 \)

- **Recursion...**

  \[
  q_1(t + 1) = q_1(t) + P(t + 1)
  \]

  \[
  \mu_1(t + 1) = \frac{q_1(t) \mu_1(t) + (t + 1)P(t + 1)}{q_1(t + 1)}
  \]

  \[
  \mu_2(t + 1) = \frac{\mu - q_1(t + 1)\mu_1(t + 1)}{1 - q_1(t + 1)}
  \]