Non-uniform imaging

SINA 2010/11
Non-uniform sensors

...vision is an information processing task...

One of the major concerns of evolution has been to shape the sensory systems so that information is reduced as soon as possible (ideally even before it enters the processing device)

...may be less information is better

One example is the structure of the human retina
isodensity maps of cone densities (X1000) in the human retina

Curcio et al. 1987
Distribution of the receptors
If the retina was designed as a camera

Visual Field: about 160 deg
Maximum Resolution: about 1/60 deg

According to K. Nakayama and E. Schwartz the saving is from 5,000 to 30,000 times.

Optic nerve: diameter 4 cm
Brain weight: from about 3 to 20 tons
Amount of food: ?
Processing time: ?
The mammalian eye is a remarkable optical device, but its design is not perfect. The blood vessels that supply the inner retina are located in front of the photoreceptor layer, blocking access to light. Their shadows create a pattern of blindness in the field of vision that corresponds precisely to the location of the largest vessels in the eye. We show here that in squirrel monkeys, focal deprivation by blood vessels leads to rewiring of the eye’s geniculocortical projections, imprinting an image of the retinal vascular tree onto the primary visual cortex. This process illustrates vividly that local imbalances in neuronal activity can influence column formation during normal development.
Ocular dominance columns
From macaque data
A growing body of experimental data supports the notion that both patterned spontaneous retinal activity and axon guidance cues together contribute to the formation of ocular dominance columns. Once the initial circuit is formed, studies demonstrate a critical period of time during which ocular dominance columns can be modified in response to visual experience.
Fig. 2. CO (cytochrome oxidase) staining of angioscotoma representations.

(A) At birth, CO staining is uniform. Angioscotomas are absent, but the right eye already monopolizes the left eye’s blind spot representation.

(B) After light exposure, vessels begin to cast shadows, generating angioscotoma representations. Their pattern becomes immutable at the end of the critical period.

(C) Removal of one eye induces an undulating pattern of CO activity, corresponding to the ocular dominance columns. The angioscotomas appear silhouetted against this textured background, because they stain solidly. Note the tendency for the other eye to “frost” the vessel and blind spot representations.
V1 (primary visual cortex)
Cortical magnification factor

\[ M_a = 508(2.26+E)^{-2.74} \]

(Macaque, Van Essen et al, 1984)
Various numbers

• In the human eye we have a very high density of cones (the color sensitive photoreceptors) in the central part of the retina and a decreasing density when we move towards the periphery. The second kind of receptors (the rods, sensitive to luminance), are absent in fovea, but they have a similar spatial distribution.

• In fact the cone density in the foveola (the central part of the fovea) is estimated at about $150–180,000$ cones/mm$^2$. Towards the retinal periphery, cone density decreases from $6000$ cones/mm$^2$ at a distance of $1.5$ mm from the fovea to $2500$ cells/mm$^2$ close to the ora serrata (the extremity of the optic part of the retina, marking the limits of the percipient portion of the membrane).

• Rod density peaks at $150,000$ rods/mm$^2$ at a distance of about $3–5$ mm from the foveola.

• Cone diameter increases from the center ($3.3$ μm at a distance of $40$ μm from the foveola) towards the periphery (about $10$ μm). Rod diameter increases from $3$ μm at the area with the highest rod density to $5.5$ μm in the periphery.
Logpolar mapping

\[ w = f(z) = \log_a(z), \quad w, z \in \mathbb{C} \]
\[ z = x + iy = r(\cos \varphi + i \sin \varphi) \]
\[ w = \rho(z) + i \vartheta(z) \]
\[ z = re^{i\varphi} \]
\[
\begin{cases}
\rho = \log_a r \\
\vartheta = h\varphi
\end{cases}
\]
Geometrical interpretation

\[
\begin{align*}
\eta &= q \vartheta \\
\xi &= \log_a \frac{\rho}{\rho_0}
\end{align*}
\]

\[
\begin{align*}
x &= \rho \cos \vartheta \\
y &= \rho \sin \vartheta
\end{align*}
\]

\(\rho\) and \(\vartheta\) are the standard polar coordinates
Log-polar images
Properties

- Conformal mapping, proximity
  - Use of local operators
- Scale change
  - Translation along the real axis
- Rotation change
  - Translation along the imaginary axis
- Translations
- Fourier-Mellin Transform
Angle preservation

Figure 4.1. Angle Preservation: The angles are locally preserved after a log-polar Transformation. (a) Cartesian domain. (b) log-polar domain. Please note that both images a and b are particulars, so the origin of the mapping falls outside of the cartesian image.
Scale

Figure 4.2. Scale Change: A pure scale change referred to the origin of the mapping, with no translational components, becomes a pure translation after the log-polar transform. (a) Cartesian domain. (b) log-polar domain. (c), (d) Enlargement of the shaded areas respectively in (a) and (b).
Rotation invariance

Figure 4.3. Rotation: A pure rotation referred to the origin of the mapping, with no translational components, becomes a pure translation after the log-polar transform. (a) Cartesian domain. (b) log-polar domain. (c), (d) Enlargement of the shaded areas respectively in (a) and (b).
Examples of rot-scale invariance
Translations

Figure 4.4. Translation: A pure translation implies a deformation of the object after the log-polar transform. (a) Cartesian domain. (b) log-polar domain. (c), (d) Enlargement of the shaded areas respectively in (a) and (b).
Figure 4.5. The Fourier-Mellin Transform.
Logpolar optical flow
1996 – CMOS

Monochromatic
8000 Pixel (128x56)
Diameter 8.1 mm
Min Pix Size = 14 μm
Max/Min = 14.
Q = 600

“polar” fovea
1998 – CMOS Color

Same Layout

Giotto
How to compare Giotto-2 with a constant resolution sensor array?

Suppose we start with 5000 pixels at constant resolution

The same 5000 pixels plus 27000 retina-like pixels
33,000 pixels with same field of view
(less resolution)

Enlarging the constant resolution image shows the loss in resolution

The two images are shown so that the size of the smallest pixel is the same (same maximum resolution)
Another example

6600 Pixels

x 2 Pixels
SubQ-CIF

x 3 Pixels

x 4 Pixels
Q-CIF

...advantages increase
Table 1. A comparison between three generations of log-polar sensors.

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<th>Sensor Version</th>
<th>Total Number of Pixels</th>
<th>Pixels in Fovea</th>
<th>Pixels in Periphery</th>
<th>Total Number of Rings</th>
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<th>Rings in Periphery</th>
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<th>Size of the Smallest Pixel</th>
<th>R</th>
<th>Q</th>
<th>Technology Used</th>
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Figure 5.2. Mirror design: (a) Profile of the mirror. (b) The mirror is designed so that vertical resolution of a cylindrical surface is mapped into constant radial resolution in the image plane.
Figure 5.1. Panoramic View: (a) Image acquired by an OMNIVIEWS log-polar camera (software simulation). (b) Image acquired by a conventional omnidirectional camera. Note that the image from OMNIVIEWS camera is immediately understandable while the image from a conventional camera requires more that 1.5 million operations to be transformed into something similar with no added advantage.