Hough Transform

- It is a technique used to isolate curves of a given shape in an image
- In its classical formulation it requires the curve to be specified in some parametric form (usually lines, circles or ellipses)
- also… it can be generalized to arbitrary curved shapes
- Advantage: robust to “gaps” in the object
- Disadvantages:
  - parametric description of the shape
  - depending on the number of parameters might become slow

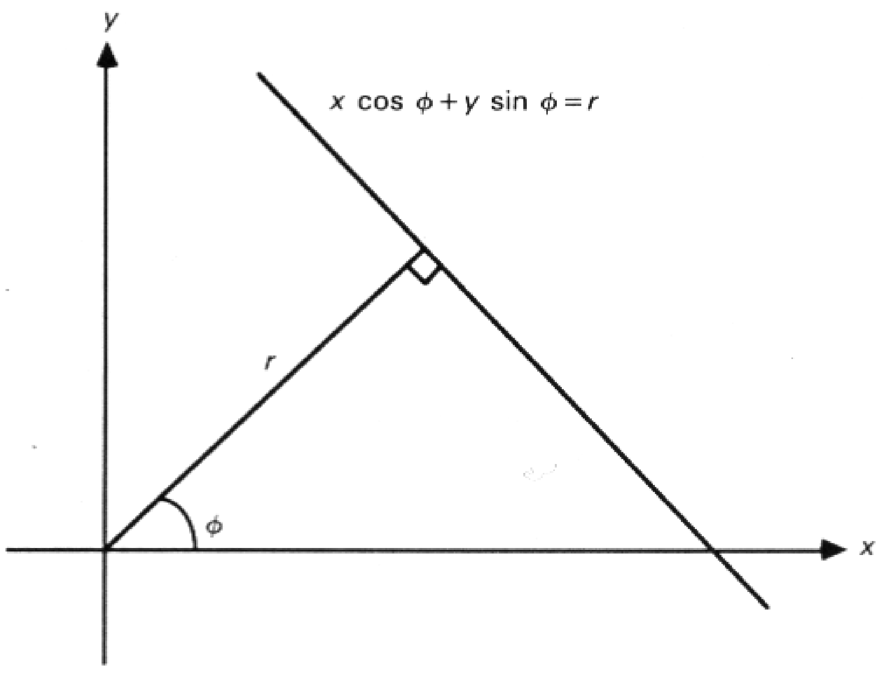
Hough Transform for lines

- We want to detect points lying on a straight line
- Start with the equation of a straight line in parametric form:

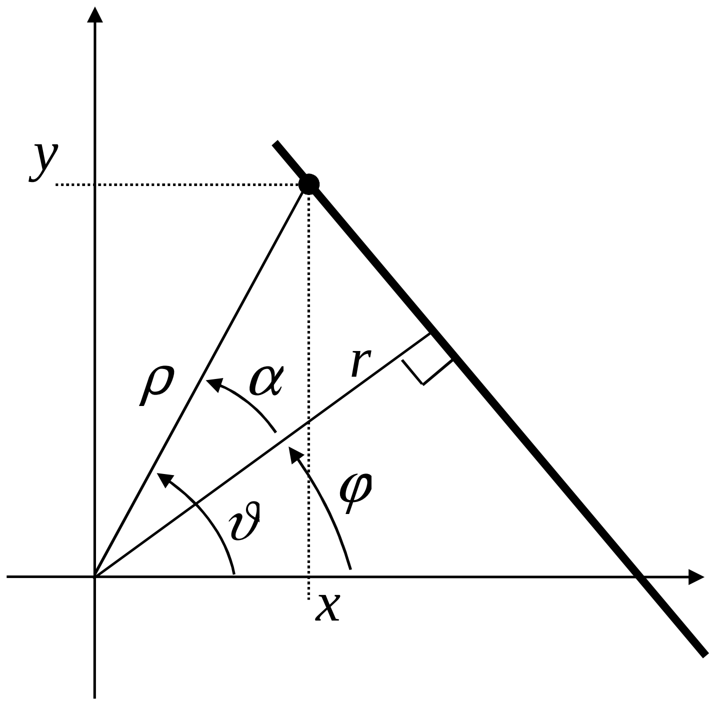
$$x\cos\varphi + y\sin\varphi = r$$

where *r* is the length of a normal to the line from the origin and φ is its angle with the x-axis

The Hough transform



In case you don't trust the equation…



$$r = \rho\cos(\alpha) = \rho\cos(\vartheta - \varphi)$$

$$r = \rho\cos\vartheta\cos\varphi + \rho\sin\vartheta\sin\varphi$$

$$\boxed{r = x\cos\varphi + y\sin\varphi}$$

- Given a point $x_i\, y_i$ on this line we have:

$$x_i \cos\varphi + y_i \sin\varphi = r$$

where r and φ are constant

- Consider now r and φ variable (r_i, φ_i) and $x_i\, y_i$ constant (x,y), the equation describes all possible lines passing through the point, these lines are described by:

$$r_i = x\cos\varphi_i + y\sin\varphi_i \qquad \varphi \in [0, 2\pi]$$

- The *Hough Transform* of the point is the plot of this equation on the (r_i, φ_i) space (*Hough Space*)

we get a sinusoidal curve

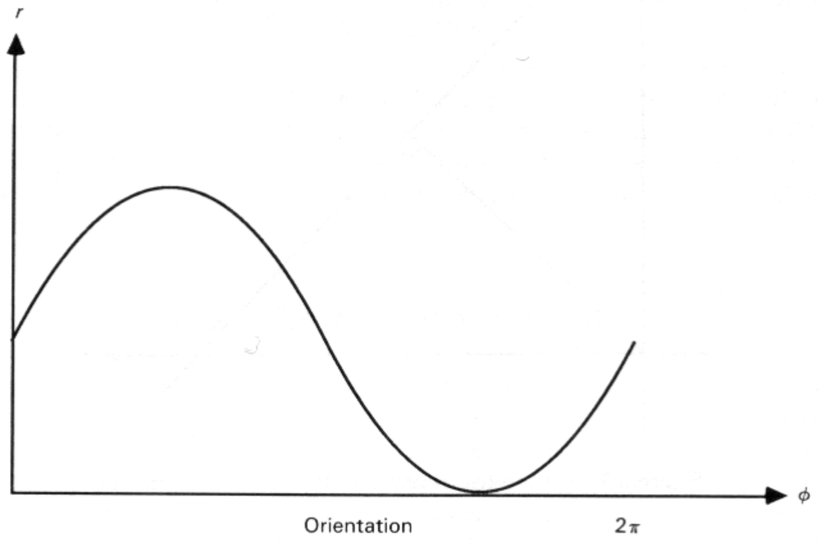
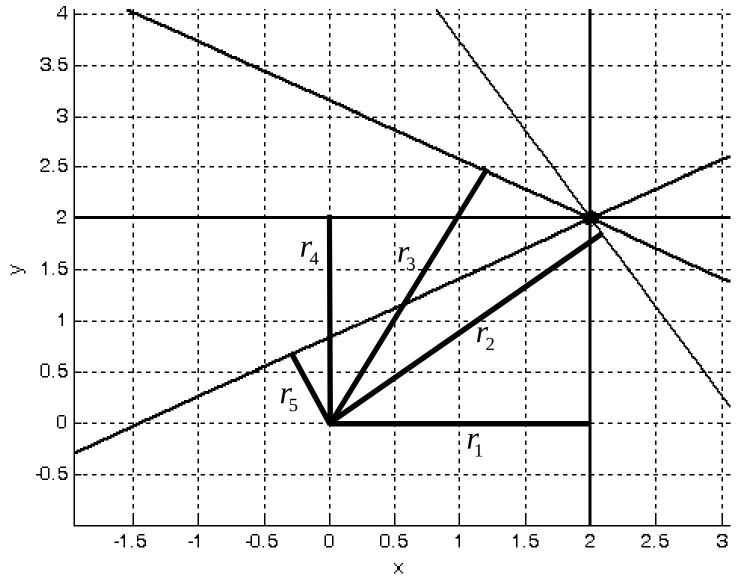
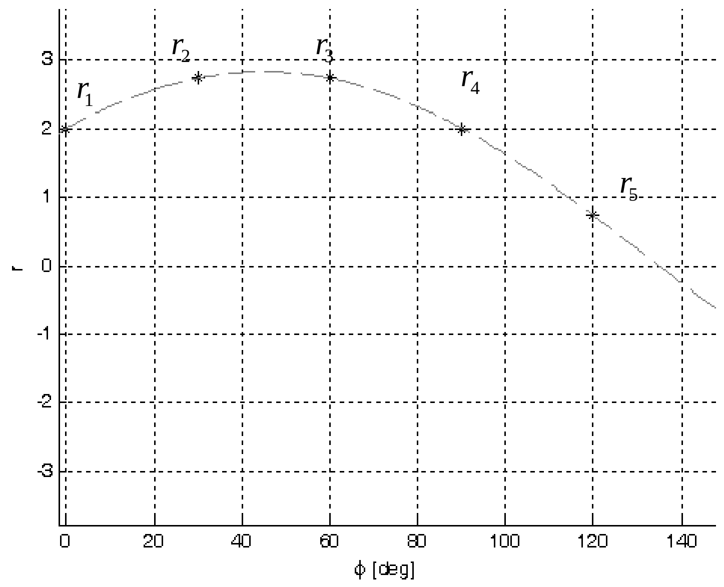


Figure 6.8 Hough transform of a point (x_i, y_i).

Hough Transform of one point





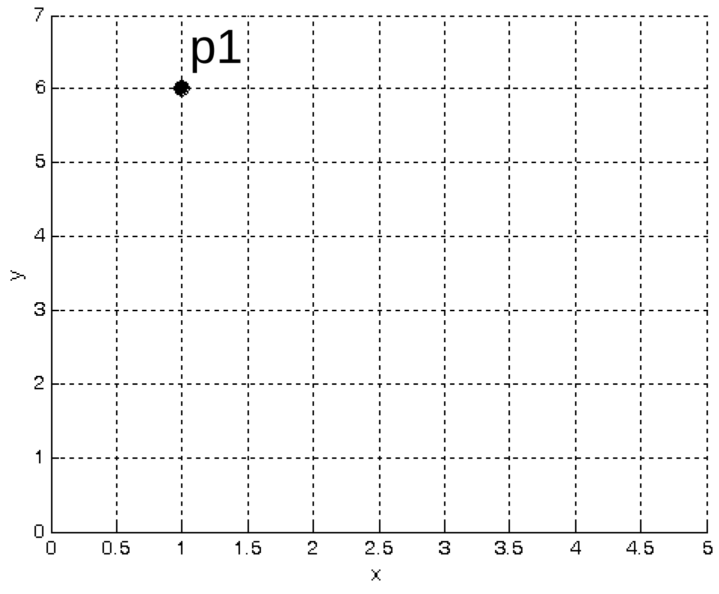
Hough Transform

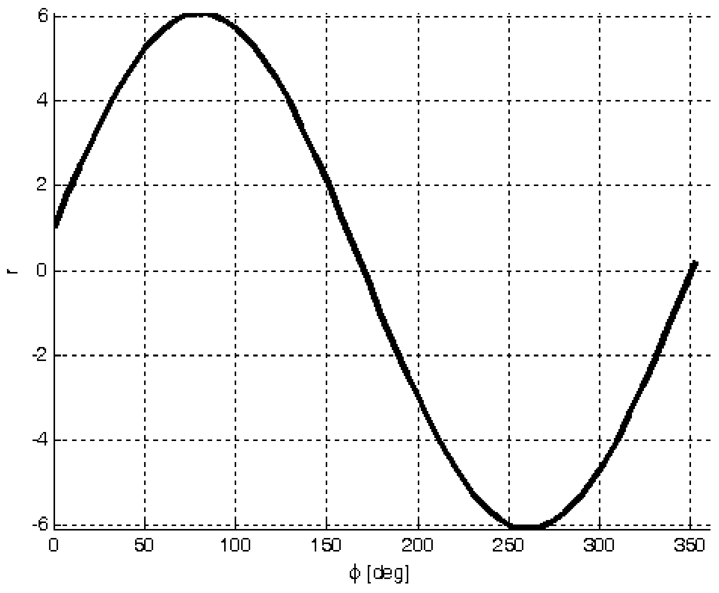
- Define the Hough Space: range for *r* (example: 0-255) and the angular resolution of the sampling of φ (example: 6 degrees)
- This gives a Hough Space (HS) of *256x60* points
- Each point in the image “vote” for a set of lines passing through it; all these votes are accumulated in the HS
- For each *(x,y)* increment all accumulator cells (*r*,φ) which satisfy the equation:

$$r = x\cos\varphi + y\sin\varphi$$

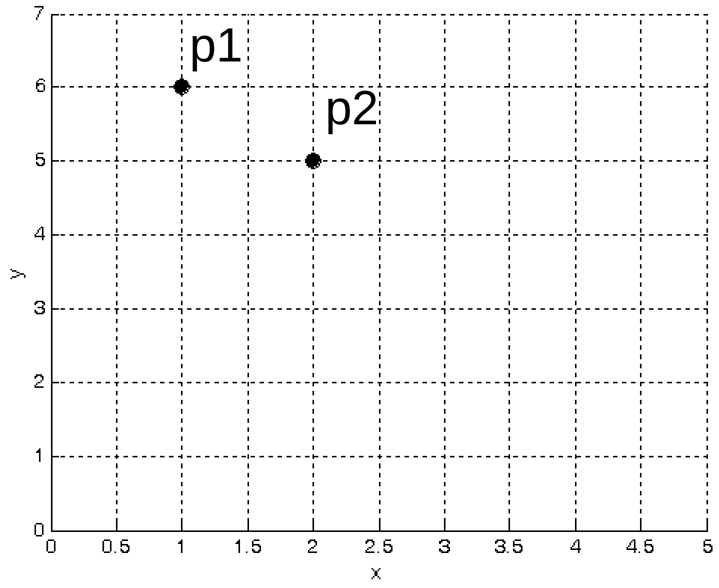
- At the end we scan the accumulator searching for cells which have high count: they correspond to lines for which there are many points in the image plane

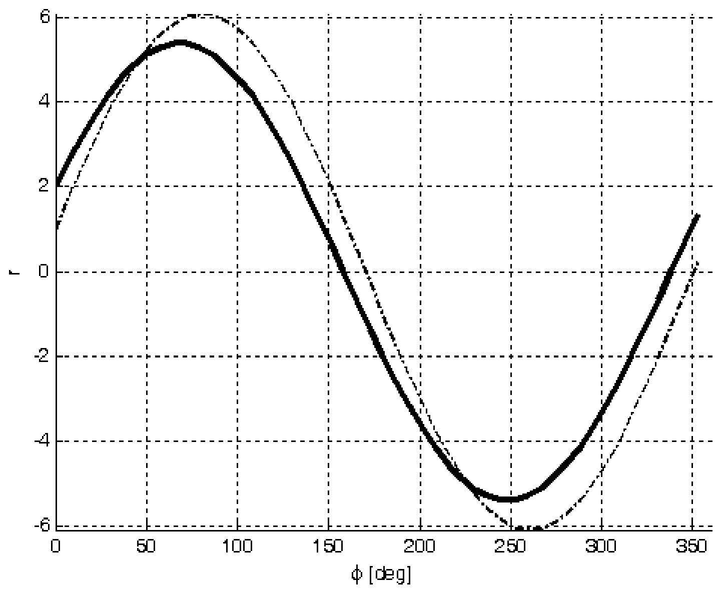
Example: start with one point (pixel)



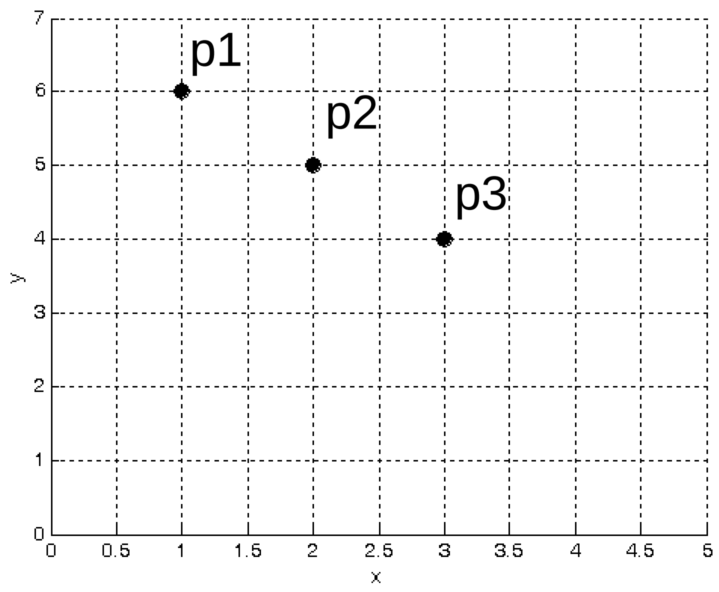


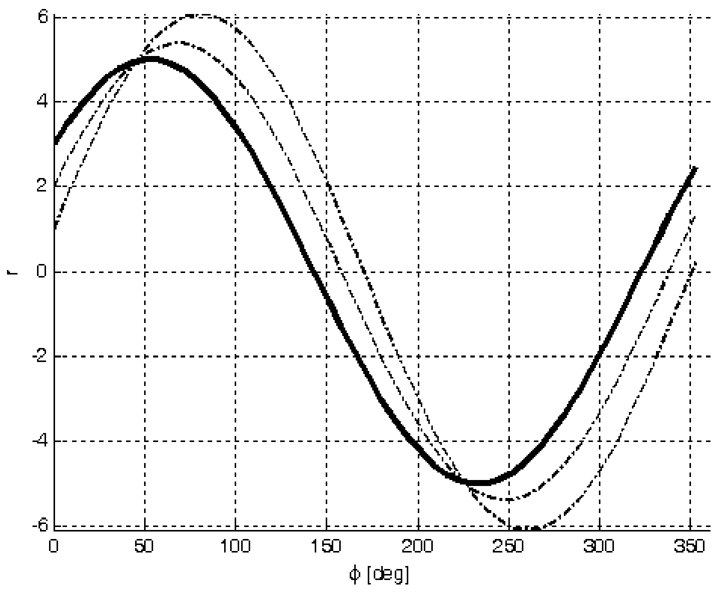
Example: add another pixel



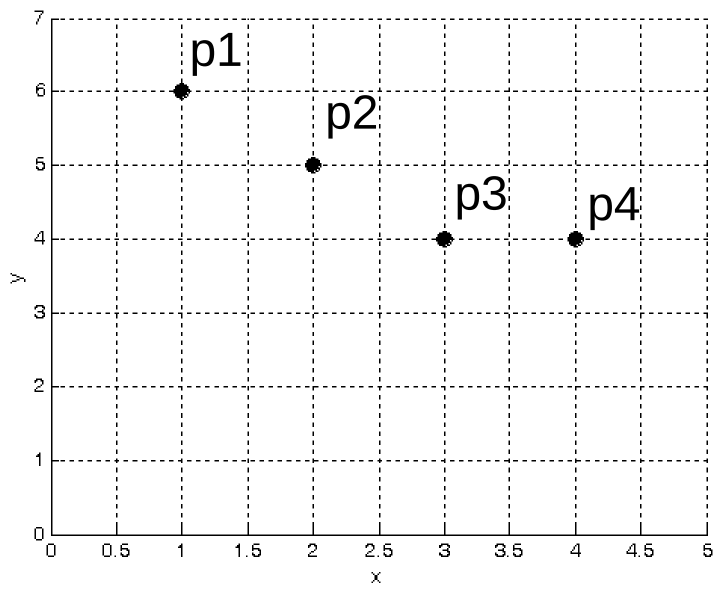


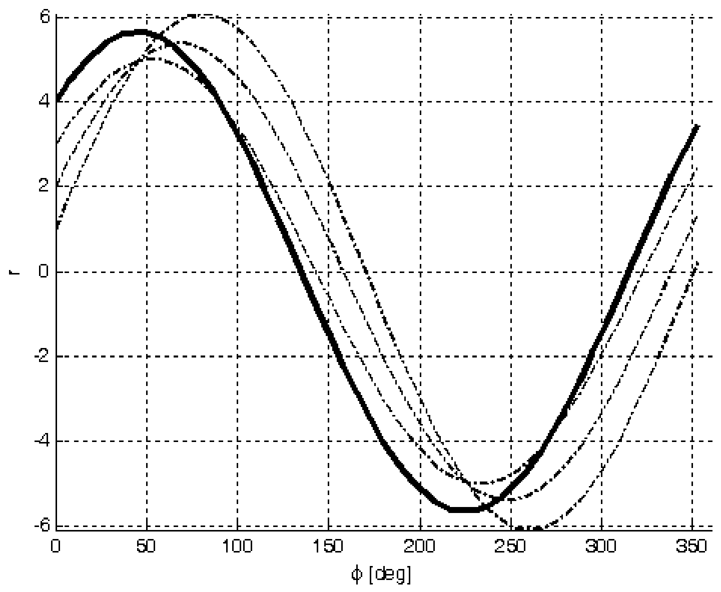
Example: now we have three pixels



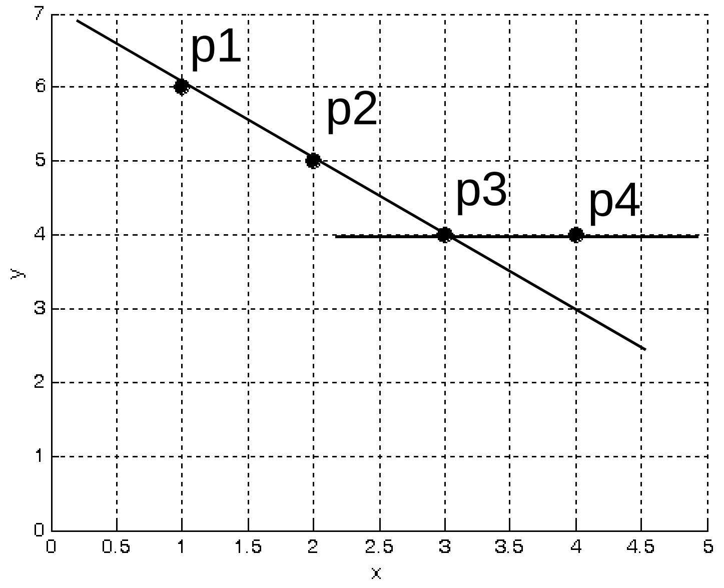


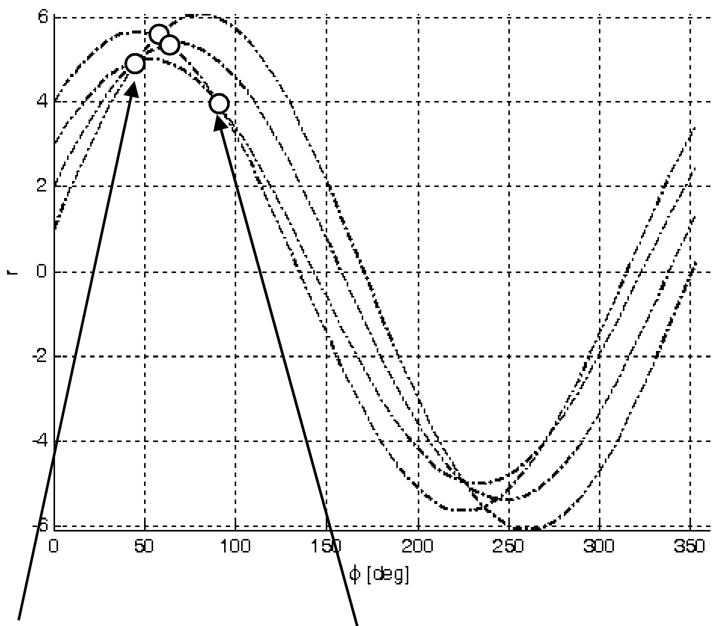
Example: the last one…





Finally: search intersections in the HT





p1,p2 and p3 intersect at the orientation of 45°

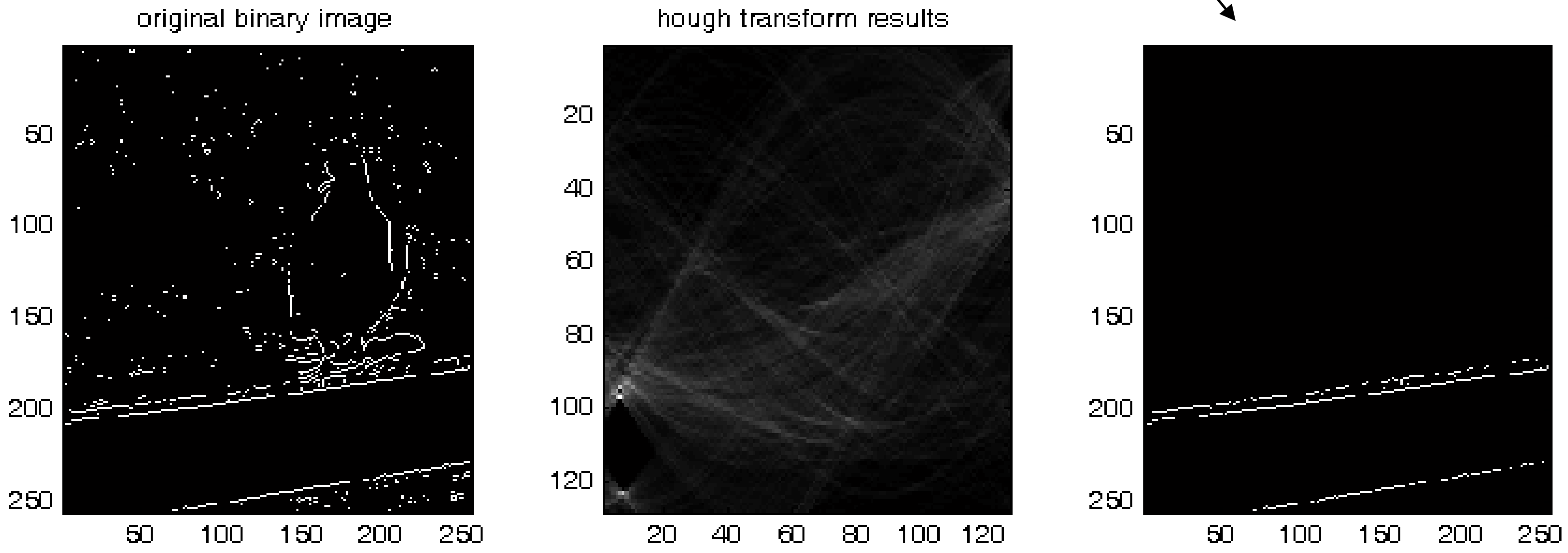
p3 and p4 intersects at 90°, p4 intersects the othe r points around 60°

Improvements:

- Usually start with the output of an edge detector, to reduce the number of points in the image
- Search along directions perpendicular to the image gradient

Hough Example

extract 5 strongest lines



houghdemo.m from http://homepages.cae.wisc.edu/~ece533/matlab/index.html

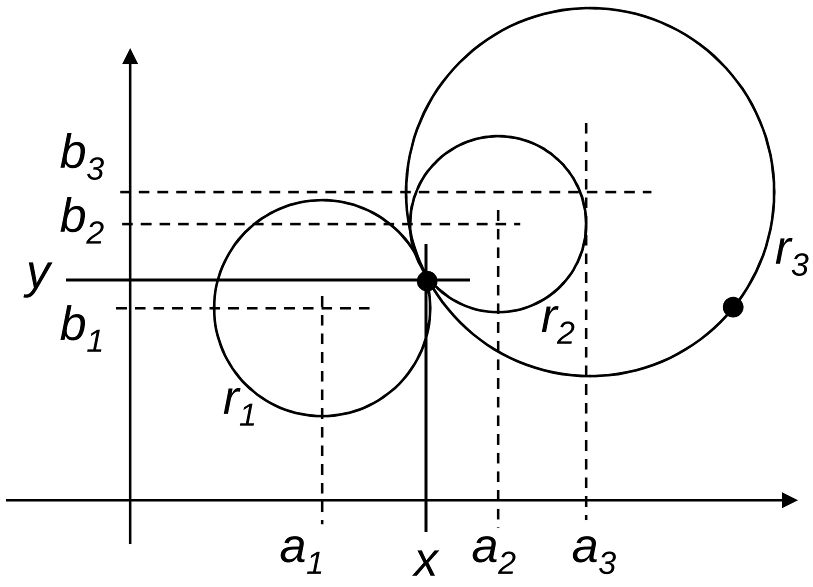
Circular Hough Transform

- Detect circles in the image
- Parametric equation of a circle:

$$(x-a)^2+(y-b)^2=r^2$$

where *(a,b)* are the coordinates of the center of the circle and *r* its radius

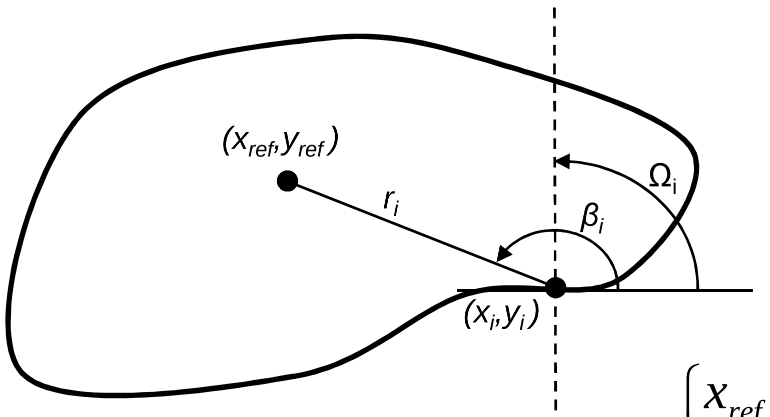
- *a,b* and *r* define the parameter space, the accumulator is three-dimensional



a_1,b_1,r_1
a_2,b_2,r_2
a_3,b_3,r_3
…

Generalized Hough Transform

- Suppose we don't have a simple analytical equation for the object
- Instead we use a LUT defining the relationship between the coordinates of the point, its orientation and the Hough parameters

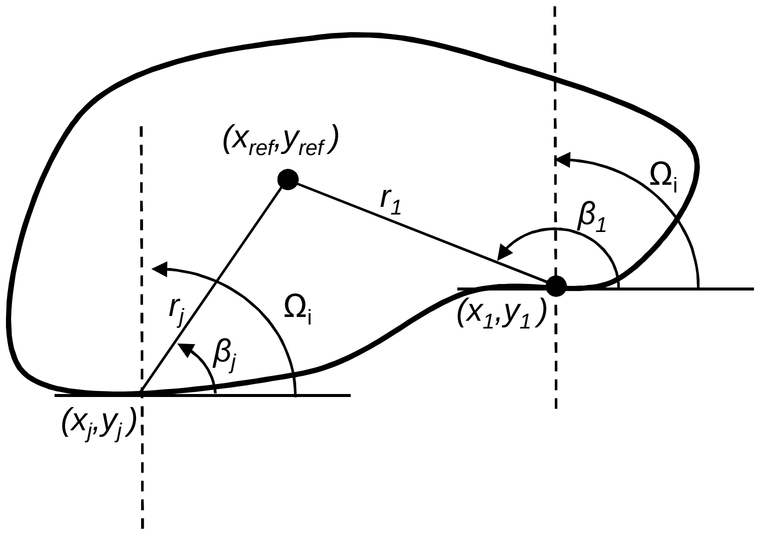


$$\begin{cases} x_{ref} = x_i + r_i \cos\beta_i \\ y_{ref} = y_i + r_i \sin\beta_i \end{cases}$$

Build the LUT:

1. Select an arbitrary reference point (x_{ref},y_{ref})
2. For all points of the boundary (x_i,y_i)
    - draw a line from (x_i,y_i) to (x_{ref}, y_{ref})
    - Measure (β_i,r_i)
    - Compute the orientation of the boundary Ω_i
    - Add (β_i,r_i) to a table indexed by Ω_i

- Probably there will be more than one occurrence of a particular orientation…



$$\begin{cases} x_{ref} = x_i + r_i \cos\beta_i \\ y_{ref} = y_i + r_i \sin\beta_i \end{cases}$$

Ω_0	…
Ω_1	…
…	…
Ω_i	(r_1,β_1), (r_2,β_2) …
…	…

R-table

- Once we have the R-table for the object, we can perform the Hough Transform of the image
- For each point in the image (x_i,y_i), we compute the point (x_{ref},y_{ref}) from:

$$\begin{cases} x_{ref} = x_i + r_i \cos\beta_i \\ y_{ref} = y_i + r_i \sin\beta_i \end{cases}$$

- where (r_i,β_i) are derived from the R-table, starting from the orientation of the point Ω_i
- We accumulate the Hough Space in (x_{ref},y_{ref}):

$$A(x_{ref}, y_{ref})++$$

- Finally search for local maxima in A to identify the center(s) of the object(s)

- It is easy to extend the search for different object orientations φ and scales S:

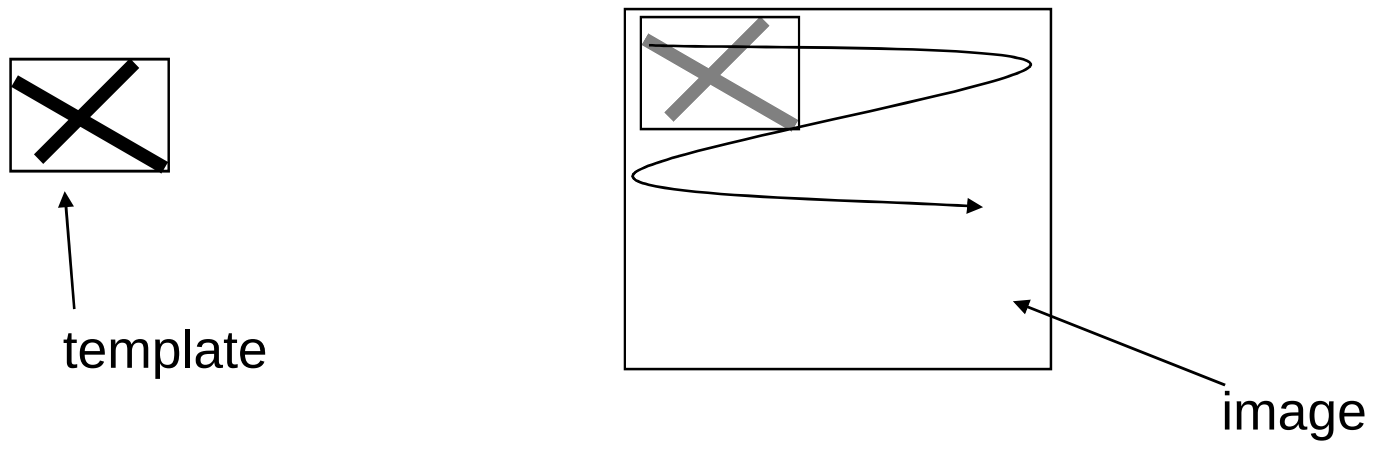
$$\begin{cases} x_{ref} = x_i + S \cdot r_i \cos(\beta_i + \varphi) \\ y_{ref} = y_i + S \cdot r_i \sin(\beta_i + \varphi) \end{cases}$$

- In this case we explore and accumulate a four-dimensional space:

$$A(x_{ref}, y_{ref}, \varphi, S){+}{+}$$

Measures of similarity

- Another approach to detect some previously defined object within a given image is to perform *template matching*
- A *template* is a sub-image representing the “ideal”pattern that is sought in the image
- Involves the translation of the *template* to every possible position of the image, and the evaluation of a the level of *match* between the template and the image in that position
- *Global* versus *local* template



- For each position of the template in the image we evaluate a *similarity measure* between the template and the image
- Possible measures are *summation difference* or *cross-correlation*
- These measures are not only used for template matching but also to estimate the level of similarity between two signals

1. Euclidean distance

- Estimate the similarity between *g* and *t* in *m,n*:

$$E(m,n)=\sqrt{\sum_i \sum_j \left[g(i,j)-t(i-m,j-n)\right]^2}$$

The sum is computed for all points for which (i-*m,j-n)* is a valid coordinate of *t*, in other words for all points in the template image

- To find the *best match* we look for the smallest value in *E(m,n)*

- From an intuitive point of view we are computing the distance between two *HxW* dimensional points, where *H,W* is the size of the template
- Since we are not interested in the exact value of the distance (we search for the minimum in *E*) we can avoid computing the square root (Sum of Squared Differences):

$$SSD(m,n) = E^2(m,n) = \sum_i \sum_j \left[g(i,j) - t(i-m, j-n) \right]^2$$

- Or:

$$S(m,n) = \sum_i \sum_j \left| g(i,j) - t(i-m, j-n) \right|$$

$$E^2(m,n) = \sum_i \sum_j \left[\underbrace{g(i,j)^2}_{\text{assume this constant}} + \underbrace{t(i-m, j-n)^2}_{\text{this is constant}} - 2g(i,j)t(i-m,j-n) \right]$$

2. Cross-correlation

$$R(m,n) = \sum_i \sum_j \left[g(i,j)t(i-m,j-n) \right]$$

t and *g* are similar when *R(i,j)* is **large**

Problems with cross-correlation:

-False matches if the image energy $\sum\sum g(i,j)^2$ varies with the position

-Values of R depends on the size of the template

-Not invariant to illumination change

Consider the correlation with a constant pattern, of gray value *v*

a	b	c
d	e	f
g	h	i

v	v	v
v	v	v
v	v	v

$R = v * (a + b + c + \ldots + i)$

Now consider the correlation with a constant image twice as bright

a	b	c
d	e	f
g	h	i

2v	2v	2v
2v	2v	2v
2v	2v	2v

R=2*v*(a+b+c+…+i)>v*(a+b+c+…i)

We get higher correlation, with the same template…

- Solution: normalize the intensity
  - subtract the mean of both signals
  - divide by std. deviation

$$\hat{g} = \frac{g - \bar{g}}{\sqrt{\sum \left(g - \bar{g}\right)^2}}, \qquad \hat{t} = \frac{t - \bar{t}}{\sqrt{\sum \left(t - \bar{t}\right)^2}}$$

The *normalized cross-correlation* is defined as:

$$NCC(m,n) = \frac{\sum_{i,j}\left[g(i,j) - \bar{g}_{m,n}\right]\left[t(i-m, j-n) - \bar{t}\right]}{\sqrt{\sum_{i,j}\left[g - \bar{g}_{m,n}\right]^2 \sum_{i,j}\left[t(i-m, j-n) - \bar{t}\right]^2}} \in [-1,1]$$

$\bar{t}$ mean value of t, $\bar{g}_{m,n}$ mean value of g in the region under t

Problems with template match

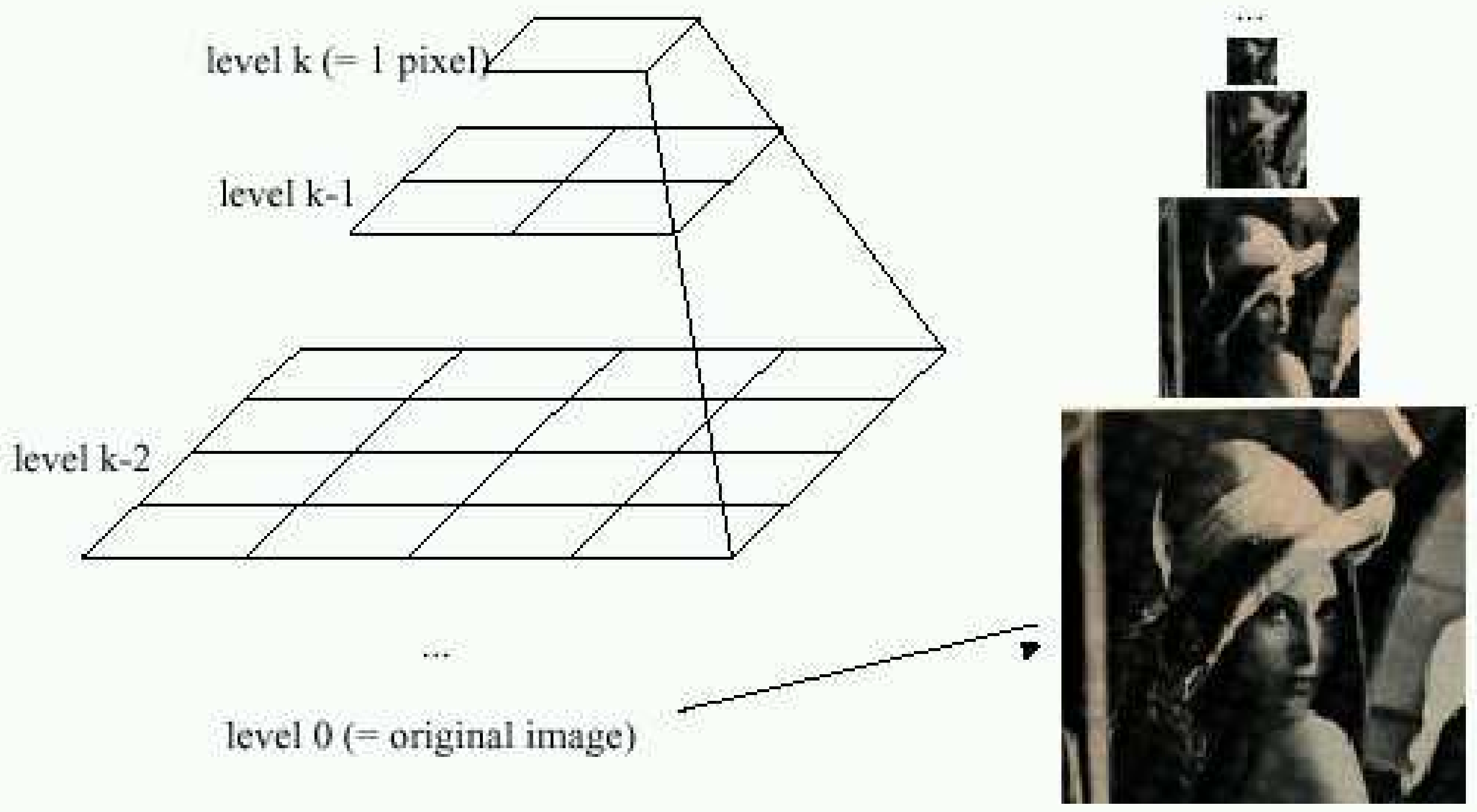
- The template represents the object as we expect to find it in the image
- The object can indeed be scaled or rotated
- This technique requires a separate template for each scale and orientation
- Template matching become thus too expensive, especially for large templates
- A possible solution is to reduce the size of the templates, and detect *salient* features in the image that characterize the object we are interested in
- We then analyze the spatial relationships between those features

$\longrightarrow$ local template matching

Gaussian Pyramid (Burt and Adelson 1983)

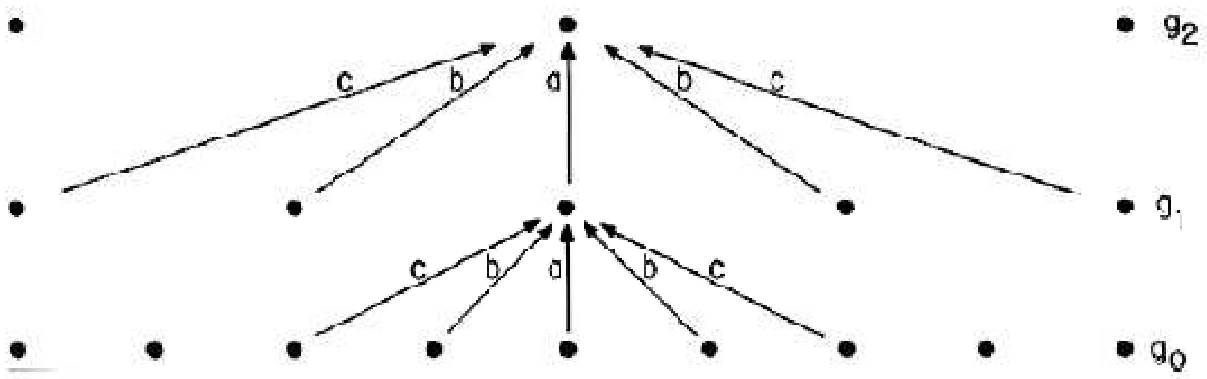
- Image pyramid is a collection of representations of an image
- Typically each layer of the pyramid is half the width and half the height of the previous layer
- If we stack the layers on top of each other, we get a pyramid

Idea: Represent NxN image as a “pyramid” of 1x1, 2x2, 4x4,…, $2^k x 2^k$ images (assuming $N=2^k$)



- In a Gaussian pyramid, each layer is smoothed by a symmetric Gaussian kernel, and resampled to get the next layer
- The smallest image is the most heavily smoothed

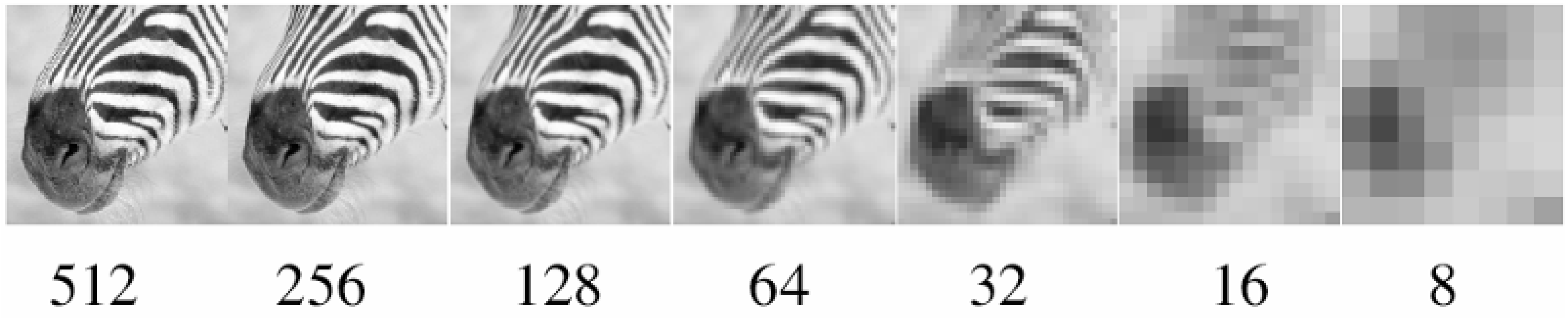
GAUSSIAN PYRAMID

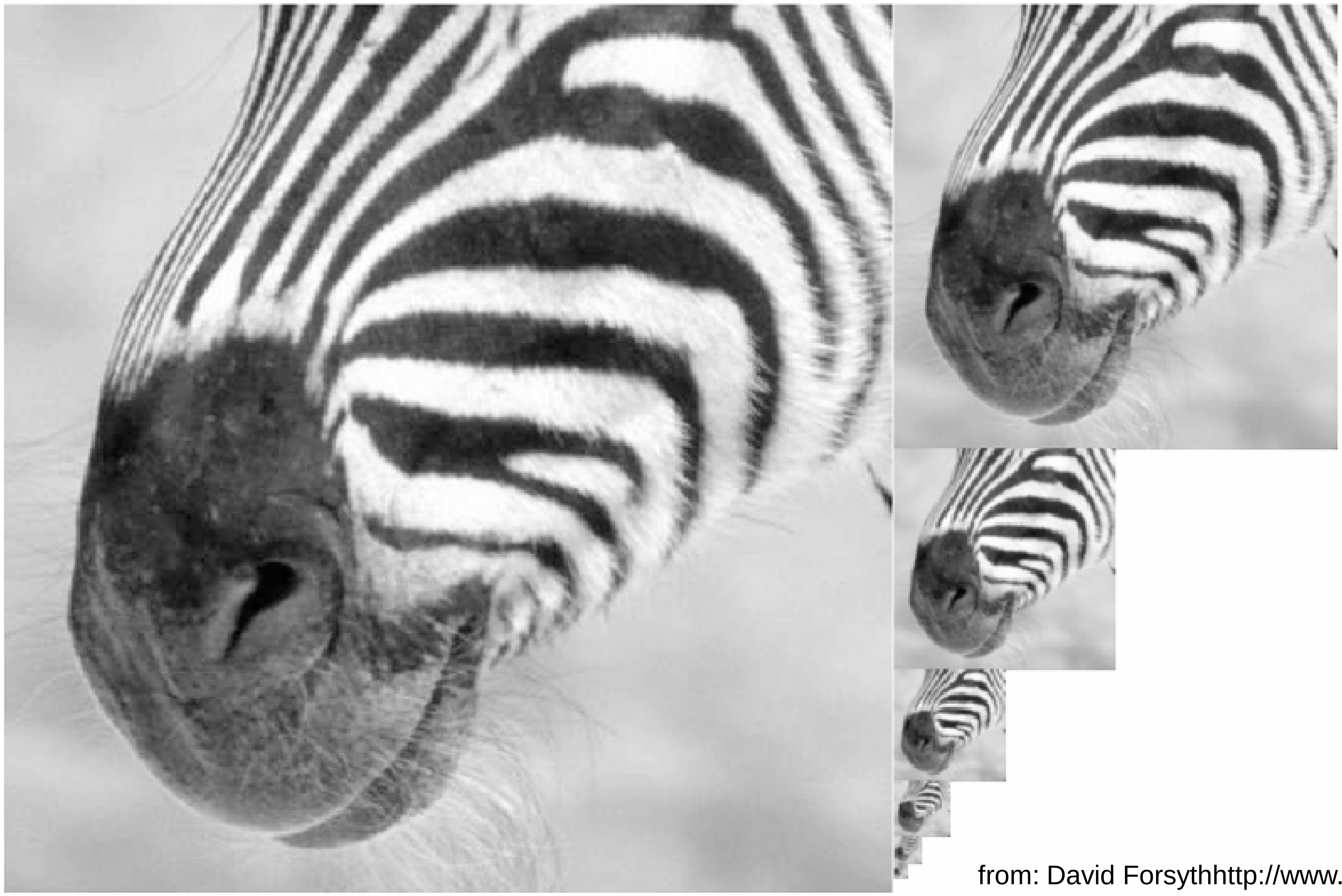


g_0 = IMAGE

g_L = REDUCE [g_{L-1}]

$$g_l(i, j) = \sum_{m=-2}^{2} \sum_{n=-2}^{2} w(m, n) g_{l-1}(2i + m, 2j + n).$$





from: David Forsythhttp://www.cs.berkeley.edu/~daf/

Applications (1)

- Search over scale – objects can be represented as small image patterns; if we want to search across different *scales* we can search across the layers of the gaussian pyramid; bigger objects will be found in the coarser scale layers, smaller objects will be found in the finer scales

Applications (2)

- Spatial search: often we have a point in one image and want to find the same point in another image (example: stereo vision). This can be achieved more efficiently if we first start to search the object in the coarser layers, and then refine the match by searching in the finer layers (*coarse-to-fine matching)*

Applications (3)

- Feature tracking: features (e.g. edges) found at coarse levels are associated with high-contrast image events (low contrast patches are easily lost during consequent smoothing); at fine scales there are probably many more features with lower contrast. A common strategy for improving a set of features obtained at a fine scale is to track features to coarser scales and accept only the fine scale features that are identifiable at coarser scales (*feature tracking)*

Gabor filters

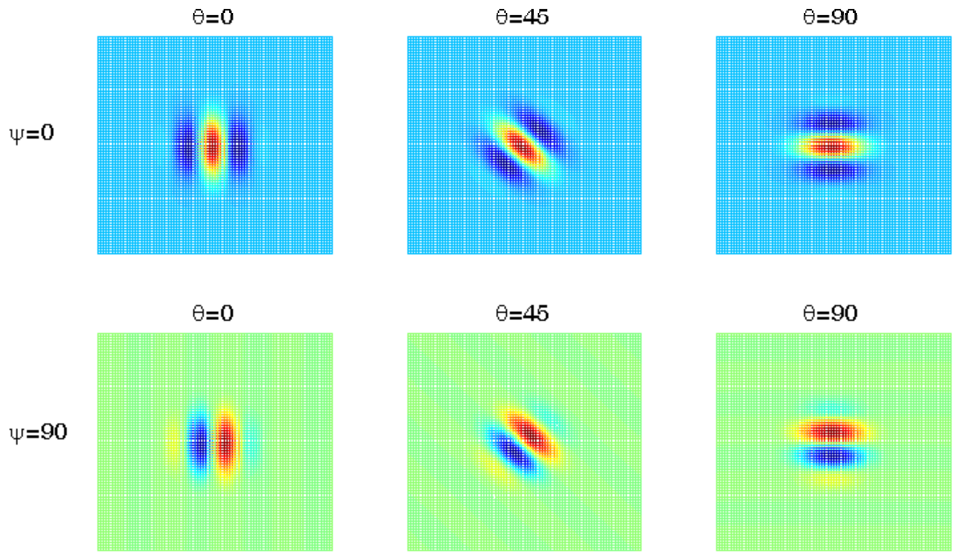
- Suppose we want to analyze the spatial frequency content of an image
- Fourier transform is a way to do this, the problem with this approach is that Fourier coefficients depend on the entire image
- In this way we loose spatial information
- Gabor filters allow to do this; they have stronger response at points in an image where there are components that *locally* have a particular spatial frequency and orientation

- Gabor filters have impulse response defined by a harmonic function multiplied by a Gaussian function
- Also used as models of the receptive fields of simple cells

$$g(x,y) = e^{-\frac{x'^2+\gamma^2 y'^2}{2\sigma^2}} \cos(2\pi\frac{x'}{\lambda}+\psi)$$

$$x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$



σ – width of the Gaussian

γ – shape of the Gaussian

θ – orientation

λ – wavelength

ψ – phase

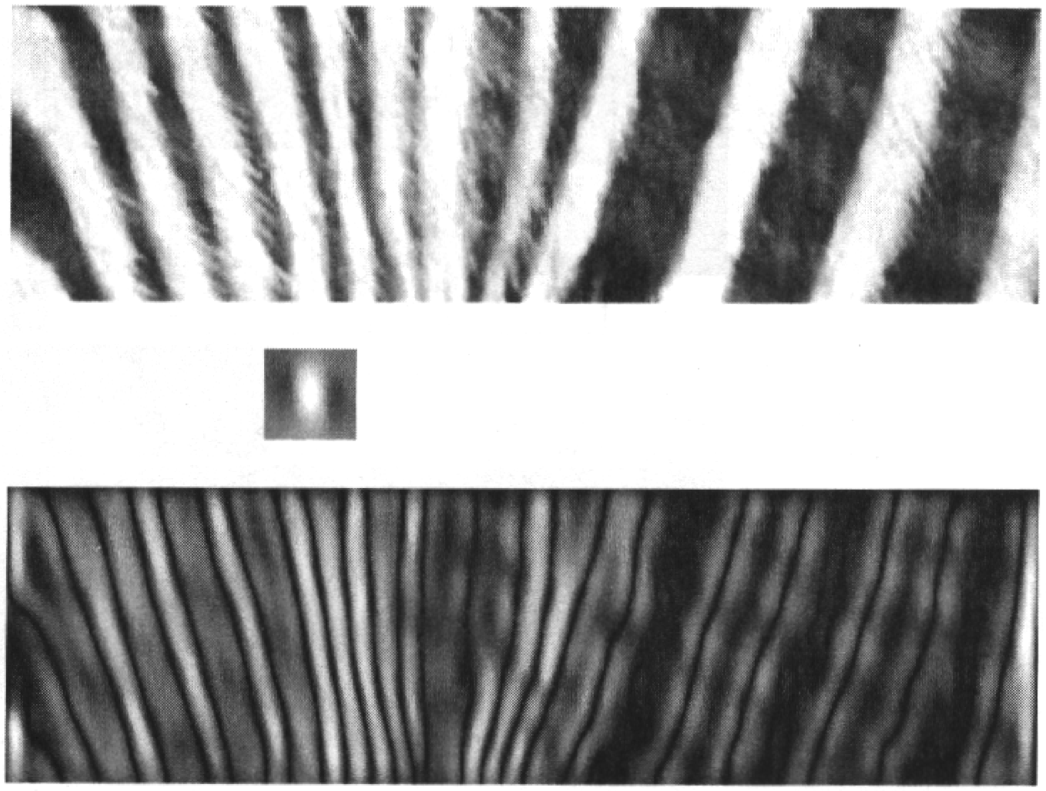
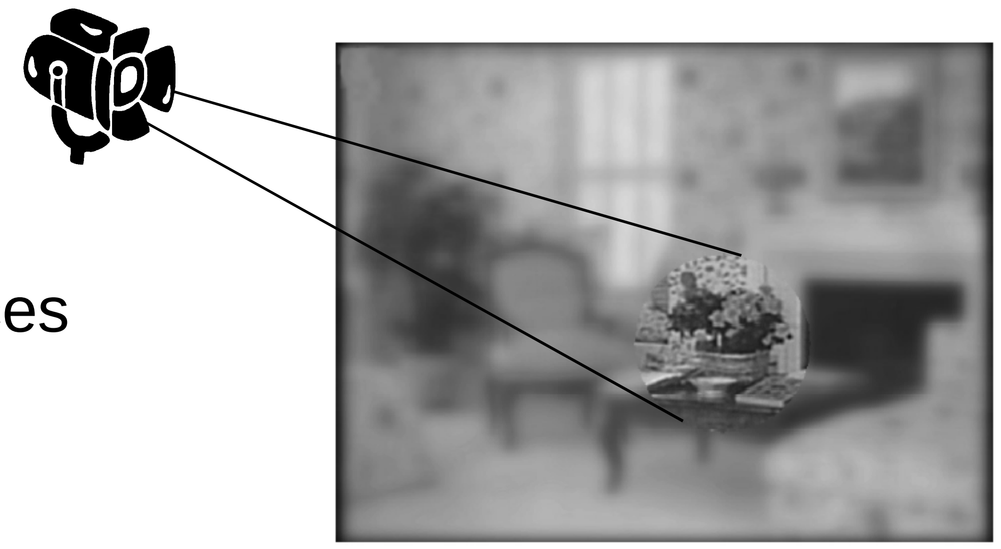


Figure 9.12 The image on the **top** shows a detail from an image of a zebra, chosen because it has a stripes at somewhat different scales and orientations. This has been convolved with the kernel in the center, which is a Gabor filter kernel. The image at the **bottom** shows the absolute value of the result; notice that the response is large when the spatial frequency of the bars roughly matches that windowed by the Gaussian in the Gabor filter kernel (i.e., the stripes in the kernel are about as wide as, and at about the same orientation as, the three stripes in the kernel). When the stripes are larger or smaller, the response falls off; thus, the filter is performing a kind of local spatial frequency analysis. This filter is one of a quadrature pair (it is the symmetric component). The response of the anti-symmetric component is similarly frequency selective. The two responses can be seen as the two components of the (complex valued) local Fourier transform, so that magnitude and phase information can be extracted from them.

from: Forsyth and Ponce

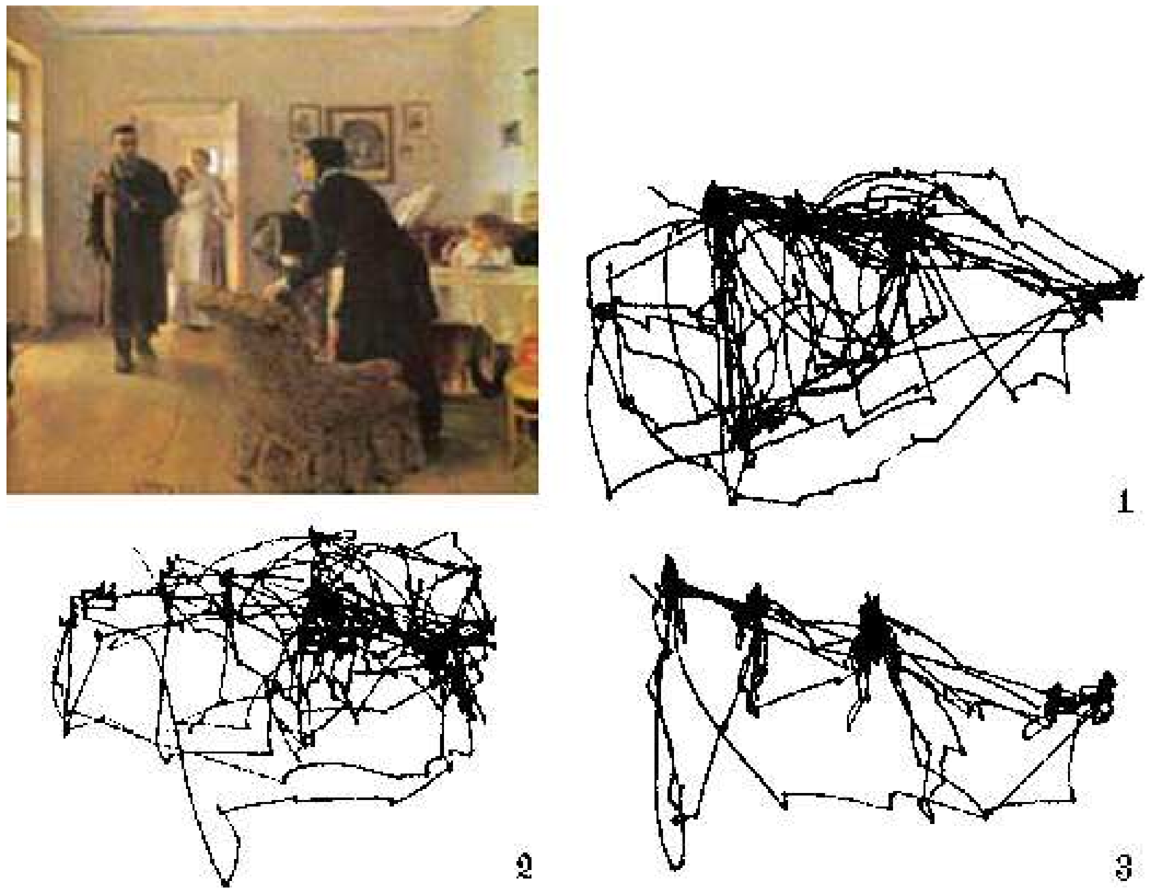
Visual attention

- Only a small fraction of the information registered by the visual system at any given time reaches levels of processing that directly influence behavior
- Visual attention controls access to this privileged level and ensures that the selected information is relevant to behavioral priorities and objectives



- A *spotlight* that enhances important information
- Can diverge from the direction of the gaze

Bottom-Up & Top-Down cues

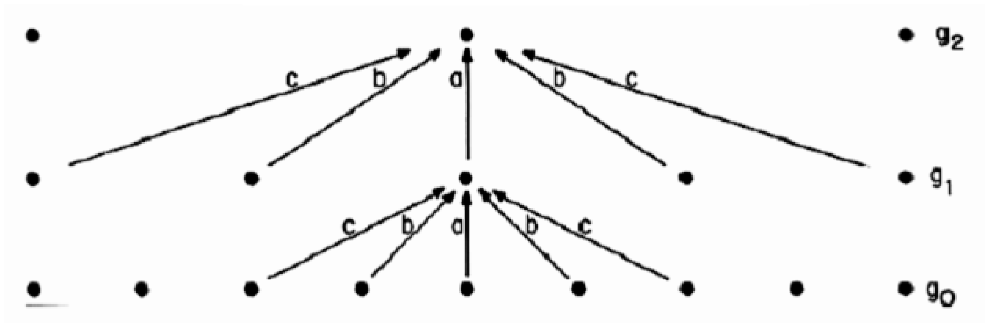


Example: Itti’s model (1998)

- Visual attention model, inspired by the behavior and neuronal architecture of the early primate visual system
- “Feature integration theory” to explain human visual search strategies

- Visual input is decomposed in a set of feature maps
- Different spatial locations compete for saliency within each map
- All maps converge into a “master saliency map” which codes local saliency over the entire visual scene
- This map has internal dynamics which generate attentional shifts

1. Create nine spatial scales using Gaussian pyramids (low-pass and subsample), scales from 0 to 8 (1:1… 1:256)
2. Features are computed by a set of linear “center-surround” operations, implemented as the difference between fine and coarser scales:



this produces multi-scale feature extraction, including different ratios between the center and surround regions (in the paper 6 different ratios are used)

- Compute:

$$I = (r+g+b)/3$$
$$R = r-(g+b)/2; G = g-(r+b)/2$$
$$B = b-(r+g)/2; Y = (r+g)/2-\left|r-g\right|/2-b$$

- And center-surround differences at different scales (s,c):

$$I(c,s) = \left|I(c)-I(s)\right|$$
$$RG(c,s) = \left|\left(R(c)-G(c)\right)-\left(G(s)-R(s)\right)\right|$$
$$BY(c,s) = \left|\left(B(c)-Y(c)\right)-\left(Y(s)-B(s)\right)\right|$$

- Local orientation information is obtained from *I* using Gabor filters at different scale and orientation (0, 45, 90, 135) :

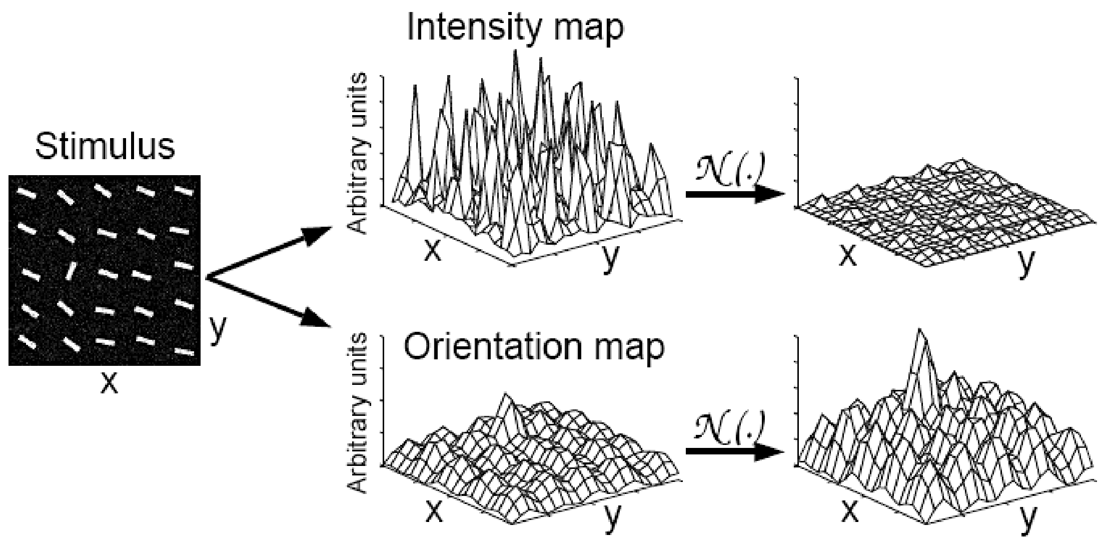
$$O(c,s,\theta) = \left|O(c,\theta)-O(s,\theta)\right|$$

- Difference are taken between fine and coarse scales, if the center is a pixel at scale *c,* the surround is the corresponding pixel at scale $s=c+\delta$, where:

$$c \in \{2,3,4\}$$
$$\delta \in \{3,4\}$$
$$\Rightarrow \text{six combinations}$$

- Total of 42 maps: 6 for intensity, 12 for color and 24 for orientation

Saliency maps: example

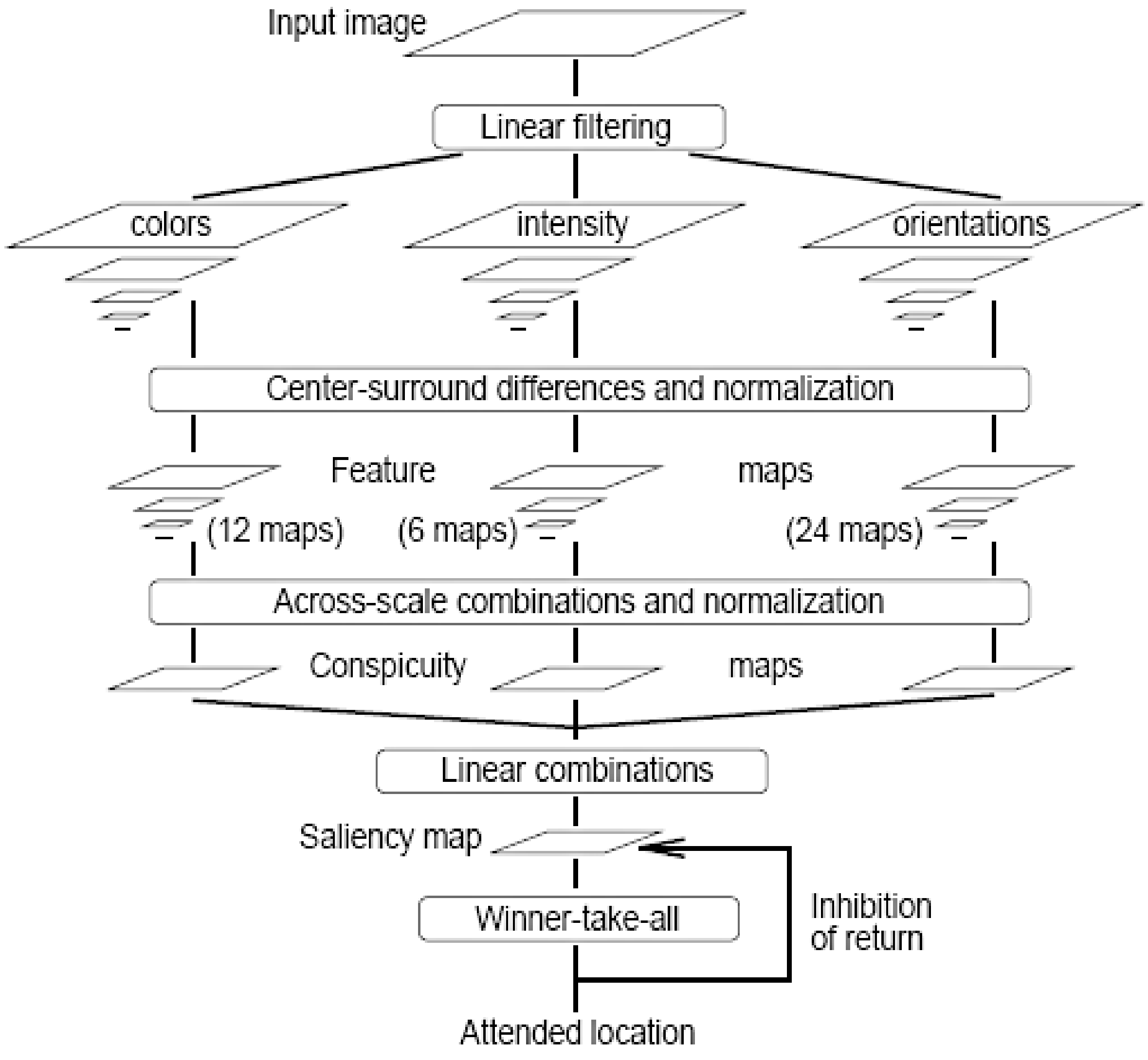


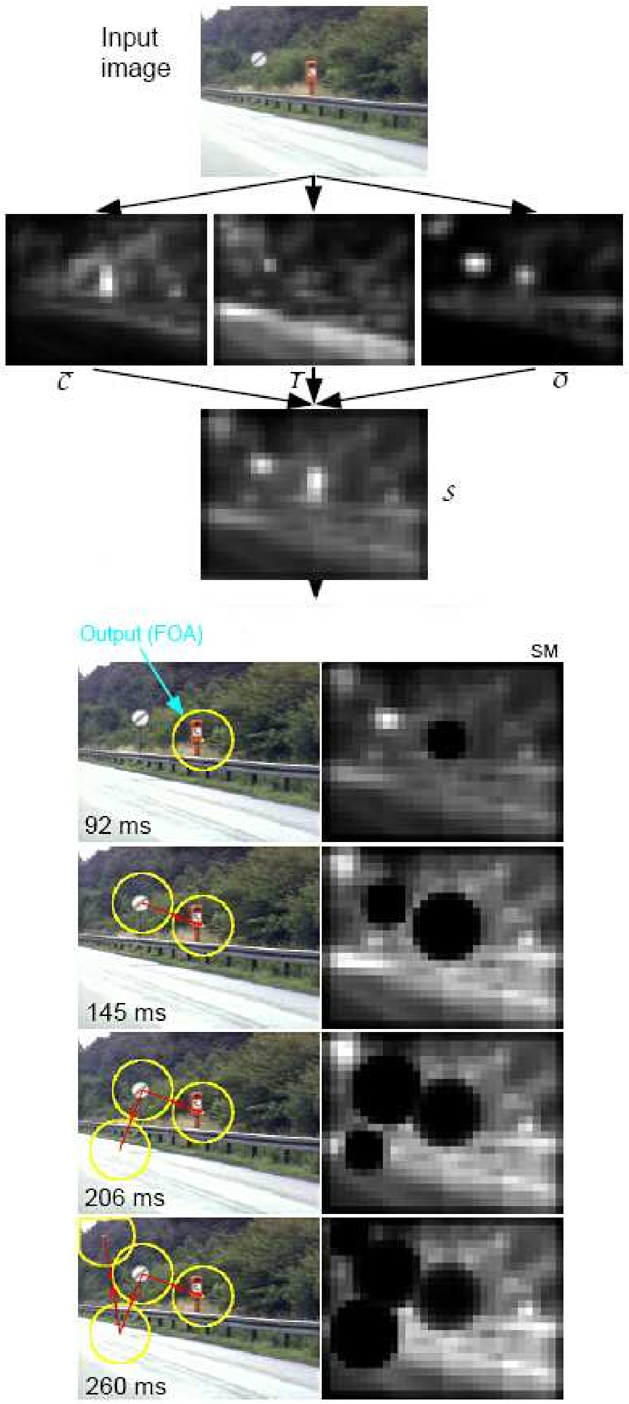
N() operator, promote maps in which a small number of strong peaks of activity is present, suppress maps with a large number of comparable responses:

-normalize the map to a fixed range [0..M]

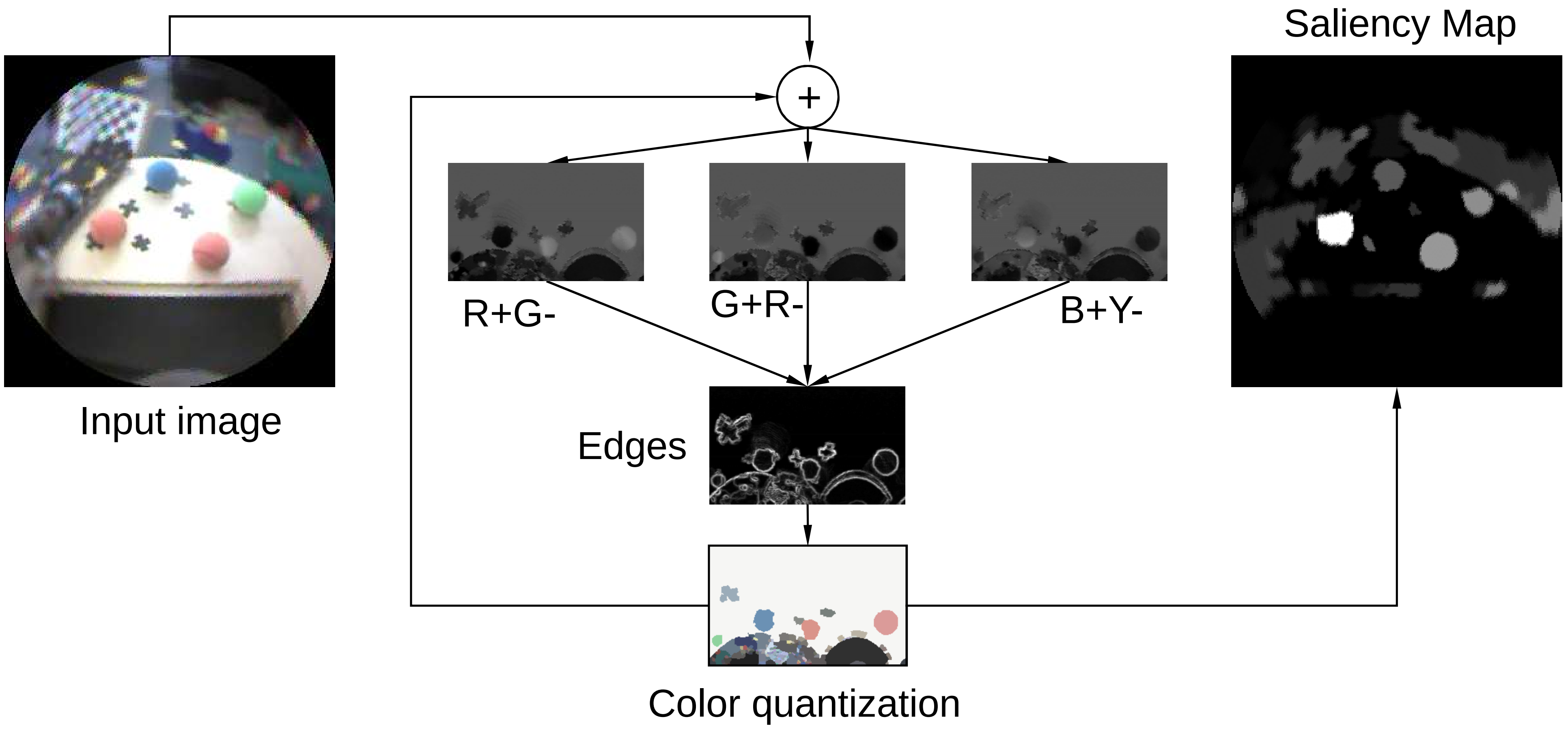
-compute the average of *m* of all other local maxima

-multiply the map by $(M-m)^2$, boost maps with small number of strong peaks


Input image
Linear filtering
colors
intensity
orientations
Center-surround differences and normalization
Feature maps
(12 maps)
(6 maps)
(24 maps)
Across-scale combinations and normalization
Conspicuity maps
Linear combinations
Saliency map
Winner-take-all
Inhibition of return
Attended location


Input image
C
I
O
S
Output (FOA)
SM
92 ms
145 ms
206 ms
260 ms

Example 2 (Orabona et al. 2005)


Saliency Map
+
R+G-
G+R-
B+Y-
Input image
Edges
Color quantization