# WALKING PATTERN GENERATION FOR A BIPED ROBOT USING OPTIMIZED POLYNOMIAL APPROXIMATION 

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#### Abstract

In this research, a stable walking pattern generation method for a biped robot is presented. The proposed method is to control the trunk motion along the moving direction to minimize the deviation from the desired ZMP. The trunk motion is approximated by the fifth order polynomial. Small deviation of the desired ZMP is allowed to improve the walking speed. An optimization method for a walking pattern generation is formalized as minimax optimization problem. To solve kinematic problem systematically, constrained multibody dynamics solution methods are used. Foot motions can be designed according to the ground condition and walking speed. Walking simulation for a virtual biped robot is performed to demonstrate the effectiveness and validity of the proposed method. A numerical results show that the proposed method can be applied to the generation of stable walking pattern effectively.

Keywords: Optimized polynomial approximation; ZMP(zero moment point); Biped robot; Stable walking pattern; Constrained multibody system


## 1. Introduction

Many studies have been focused on walking pattern synthesis. Zerrugh et al. researched the walking pattern for a biped robot by recording human kinematic data. ${ }^{1}$ McGeer described a natural walking pattern generated by passive interaction of gravity and inertia on downhill slopes. ${ }^{2}$ To extend the minimum-energy walking method for level
ground and uphill slopes, Channon et al., Rostami et al., and Roussel et al. proposed gait generation method to minimize energy consumption. ${ }^{3-5}$ Silva et al. investigated the required actuator power and energy by adjusting walking parameters. ${ }^{6}$ Since a biped robot tends to fall easily, it is necessary to consider walking stability during the walking pattern generation. Many researchers have been working on the schemes to stabilize the walking of biped robots. Kajita et al. proposed the inverted pendulum mode to generate walking pattern. ${ }^{7}$ Zheng et al. proposed a method of gait synthesis for static stability. ${ }^{8}$ Chevallereau et al. discussed dynamic stability when specifying a low energy reference trajectory. ${ }^{9}$ To ensure the dynamic stability of a biped robot, some researchers proposed methods of a walking pattern synthesis based on ZMP. ${ }^{10-14}$ Huang et al. proposed a method to generate a walking pattern with a high stability margin. ${ }^{15}$ Although the proposed method considered only two variable to obtain the stability they could express complicated hip motion as third order spline interpolation, because one step cycle was divided into one function of double support phase and one function of single support phase. Kang et al. proposed DAE(differential and algebraic equations) solution method. ${ }^{16}$ The method can satisfy the desired ZMP exactly but cannot overcome the limitation of walking speed. Therefore, this method is not suitable for generating high speed walking pattern.

In this paper, a walking pattern generation method to improve walking speed is presented. An optimization method for a walking pattern generation is formalized as minimax optimization problem. ${ }^{17}$ Small deviation of the desired ZMP is allowed to improve walking speed. A trunk motion is approximated to the fifth order polynomial. The coefficients of the polynomial are assigned as the design variables for an optimization problem. To solve kinematic problem systematically, constrained multibody dynamics solution methods are used. To verify the effectiveness and validity of the proposed method, stable walking patterns for the biped robot are generated and walking simulations are performed for the generated walking patterns.

## 2. ZMP Equation

The ZMP is defined as the point on the ground about which the sum of all the moment of active forces is equal to zero. If the ZMP is located within the contact polygon between feet and the ground, the biped robot can walk stably. This contact polygon is called the stable region. Figure 1 shows the reference frames and ZMP. In the figure, $X Y Z$ is global reference frame and $x_{i}^{\prime} y_{i}^{\prime} z_{i}^{\prime}$ is body fixed reference frame on the i-th body. $\mathbf{r}_{i}$ is a position vector to the origin of $x_{i}^{\prime} y_{i}^{\prime} z_{i}^{\prime}$ and $\mathbf{r}_{Z M P}$ is a position vector to the $p_{Z M P}$, which is called the ZMP.

Suppose that the ground applies a pure force to the biped robot at a location $p_{Z M P}$. Then the dynamics of the biped robot becomes

$$
\begin{equation*}
\sum_{i=1}^{n} m_{i}\left(\tilde{\mathbf{r}}_{i}-\tilde{\mathbf{r}}_{\text {ZMP }}\right)\left(\dot{\mathbf{r}}_{i}+\mathbf{g}\right)=\mathbf{0} \tag{1}
\end{equation*}
$$



Fig. 1 The reference frame and the ZMP.
where

$$
\mathbf{r}_{i}=\left[\begin{array}{lll}
x_{i} & y_{i} & z_{i} \tag{2}
\end{array}\right]^{T}
$$

In the above equation, $n$ is the number of bodies and the tilde over a vector denotes a skew symmetric matrix of the vector. Considering the dynamics in the Zdirection only under the assumption of no external moments, Eq. (1) can be expressed as

$$
\begin{equation*}
X_{\text {ZMP }}=\frac{\sum_{i=1}^{n} m_{i}\left(\ddot{y}_{i}+g_{y}\right) x_{i}-\sum_{i=1}^{n} m_{i}\left(\ddot{x}_{i}+g_{x}\right) y_{i}}{\sum_{i=1}^{n} m_{i}\left(\ddot{y}_{i}+g_{y}\right)} \tag{3}
\end{equation*}
$$

## 3. Kinematic Analysis

Figure 2 shows two-dimensional biped robot to be considered here. The bodies are connected by several kinematic joints as shown in the figure. Constraints which kinematic joints impose can be expressed as ${ }^{18}$

$$
\begin{equation*}
\boldsymbol{\Phi}^{K}(\mathbf{Z})=\mathbf{0} \tag{4}
\end{equation*}
$$



Fig. 2. Model of 6-DOFs biped robot.
where

$$
\begin{align*}
\mathbf{Z} \equiv\left[\begin{array}{llll}
\mathbf{Z}_{1}^{T} & \mathbf{Z}_{2}^{T} & \cdots & \mathbf{Z}_{n}^{T}
\end{array}\right]^{T}  \tag{5}\\
\mathbf{Z}_{i}=\left[\begin{array}{lll}
x_{i} & y_{i} & \theta_{i}
\end{array}\right]^{T} \tag{6}
\end{align*}
$$

A prescribed motion for one or more generalized coordinates can be defined to make the biped robot move in desired pattern. Hereafter, this prescribed motion is called as motion constraint. Motion constraints for the trunk and feet can be expressed as

$$
\boldsymbol{\Phi}^{M}=\left[\begin{array}{l}
\mathbf{Z}_{1}-\mathbf{f}(t)  \tag{7}\\
\mathbf{Z}_{4}-\mathbf{g}(t) \\
\mathbf{Z}_{7}-\mathbf{h}(t)
\end{array}\right]
$$

where $\mathbf{f}(t), \mathbf{g}(t), \mathbf{h}(t)$ are the prescribed motions which are the function of time.
If the motion and kinematic joint constraints are given for the forward and inverse kinematic analysis, the configuration of the biped robot can be determined. For a constrained multibody system, constraints, their time derivatives, and second derivatives are expressed as follows

$$
\begin{gather*}
\boldsymbol{\Phi}=\left[\begin{array}{l}
\boldsymbol{\Phi}^{K} \\
\boldsymbol{\Phi}^{M}
\end{array}\right]=\mathbf{0}  \tag{8}\\
\dot{\boldsymbol{\Phi}}=\boldsymbol{\Phi}_{\mathbf{Z}} \mathbf{Y}+\boldsymbol{\Phi}_{t}=\mathbf{0}  \tag{9}\\
\ddot{\boldsymbol{\Phi}}=\boldsymbol{\Phi}_{\mathbf{Z}} \dot{\mathbf{Y}}-\boldsymbol{\gamma}=\mathbf{0} \tag{10}
\end{gather*}
$$

where

$$
\begin{gather*}
\mathbf{Y} \equiv\left[\begin{array}{llll}
\mathbf{Y}_{1}^{T} & \mathbf{Y}_{2}^{T} & \cdots & \mathbf{Y}_{n}^{T}
\end{array}\right]^{T}  \tag{11}\\
\mathbf{Y}_{i}=\left[\begin{array}{lll}
\dot{x}_{i} & \dot{y}_{i} & \dot{\theta}_{i}
\end{array}\right]^{T}  \tag{12}\\
\boldsymbol{\gamma}=-\left(\boldsymbol{\Phi}_{\mathbf{Z}} \mathbf{Y}\right)_{\mathbf{Z}} \mathbf{Y}-2 \boldsymbol{\Phi}_{\mathbf{Z}} \mathbf{Y}-\boldsymbol{\Phi}_{t t} \tag{13}
\end{gather*}
$$

$\mathbf{Z}$ can be obtained by the iterative procedure that employs the following equation.

$$
\begin{equation*}
\boldsymbol{\Phi}_{\mathrm{z}} \Delta \mathbf{Z}=-\boldsymbol{\Phi} \tag{14}
\end{equation*}
$$

Where $\Phi_{\mathbf{Z}}$ is the Jacobian matrix of $\Phi$ with respect to $\mathbf{Z}$. Using the above equation, the improved solution for the next iteration can be obtained as

$$
\begin{equation*}
\mathbf{Z}=\mathbf{Z}+\Delta \mathbf{Z} \tag{15}
\end{equation*}
$$

Using Eq. (14) and Eq. (15), the iteration continues until the solution variance remains within specified allowable error tolerance. The procedure so far mentioned is usually called the position analysis. With the position analysis solution found, the velocity analysis solution can be obtained by solving the following velocity constraint equations.

$$
\begin{equation*}
\boldsymbol{\Phi}_{\mathrm{Z}} \mathbf{Y}=-\boldsymbol{\Phi}_{t} \tag{16}
\end{equation*}
$$

The solution of Eq. (16) can be obtained without the iterative procedure used in the position analysis. With the position and velocity analysis solutions, the acceleration analysis solution can be obtained by solving the following acceleration constraint equations.

$$
\begin{equation*}
\Phi_{\mathrm{Z}} \dot{\mathbf{Y}}=\gamma \tag{17}
\end{equation*}
$$

With the position, velocity and acceleration analysis solutions, we can calculate the ZMP equation.

## 4. Optimized Polynomial Approximation

Figure 3 shows walking parameters. In the figure, $L_{s}$ is a step length, $H$ is a step height. Foot motions can be determined according to the ground condition and walking speed. It is necessary to determine walking parameters such as a step length, a step height. Trunk motion in X direction is approximated to the fifth order polynomial. The fifth order polynomial can be expressed as

$$
\begin{equation*}
x_{1}(t)=\sum_{k=0}^{5} c_{k} t^{k}, \quad n \tau \leq t \leq(n+1) \tau \tag{18}
\end{equation*}
$$

where $x_{1}(t)$ is the trunk motion in X direction; $c_{i}$ is the coefficient of the fifth order polynomial; $n$ is a step number; $\tau$ denotes a period of step.

After determining the foot motions, trunk motion in Y direction, and trunk rotational motion, $c_{i}$ can be obtained through an optimization. The goal of an optimization is to find $c_{i}$ which can ensure walking stability. Thus, the objective function can be formulated as maximum absolute difference between $Z M P_{D}$ (desired ZMP) and $Z M P_{A}$ (actual ZMP). When $\max \left|Z M P_{A}-Z M P_{D}\right|$ is used as the objective function, gradient of the objective function cannot be defined at a zero objective. To avoid this numerical difficulty, square of $\max \left(Z M P_{A}-Z M P_{D}\right)$ is assigned as the objective function. Thus, objective function can be expressed


Fig. 3. Walking parameters.

$$
\begin{equation*}
\max _{n \leq \leq \leq(n+1) \tau}\left(Z M P_{A}-Z M P_{D}\right)^{2} \tag{19}
\end{equation*}
$$

According to the prescribed foot motions, the trunk motion is kinematically restricted. Figure 4 shows the kinematical limitations. In the figure, $X_{0}^{\min }$ and $X_{0}^{\max }$ are kinematical limitations to avoid kinematic constraint violation at $t=n \tau . X_{m}^{\min }$ and $X_{m}^{\max }$ are the kinematical limitations at a single support phase. $X_{f}^{\min }$ and $X_{f}^{\text {max }}$ are the kinematical limitations at $t=(n+1) \tau$. In the figure, it is known that the trunk motion cannot exceed some limitations. To consider these limitations, the following inequality constraints are introduced.

$$
\begin{gather*}
X_{0}^{\min }+\alpha_{0} \leq x_{1}(n \tau) \leq X_{0}^{\max }-\alpha_{0}  \tag{20}\\
X_{m}^{\min }+\alpha_{m} \leq x_{1}(n \tau+0.5 \tau) \leq X^{\max }-\alpha_{m}  \tag{21}\\
X_{f}^{\min }+\alpha_{f} \leq x_{1}((n+1) \tau) \leq X_{f}^{\max }-\alpha_{f} \tag{22}
\end{gather*}
$$

where $\alpha_{0}, \alpha_{m}$, and $\alpha_{f}$ are push off factors to avoid singularity during the inverse kinematic analysis.

For the smooth trunk motion at all time, equality constraints of the continuity are required at break points $(t=n \tau$ and $t=(n+1) \tau)$. To meet the continuity condition of position, the following equality constraint should be satisfied.

$$
\begin{equation*}
x_{1}((n+1) \tau)-x_{1}(n \tau)=L_{s} \tag{23}
\end{equation*}
$$



Fig. 4. Kinematic limitations of the biped robot.

Velocity and acceleration discontinuities can deteriorate the walking stability. For the improvement of the walking stability, velocity and acceleration of the trunk motion should satisfy the continuity conditions at break points. The continuity conditions are expressed as

$$
\begin{align*}
& \dot{x}_{1}((n+1) \tau)-\dot{x}_{1}(n \tau)=0  \tag{24}\\
& \ddot{x}_{1}((n+1) \tau)-\ddot{x}_{1}(n \tau)=0 \tag{25}
\end{align*}
$$

The trunk motion at all time can be obtained if the trunk motion satisfying all constraints is given for one period. That is, the trunk motion at all steps can be expressed as repetitive trunk motion of a period.

From Eq. (19) ~ Eq. (25), the minimax optimization problem can be formalized as

$$
\begin{align*}
& \min _{\mathrm{C}} \max _{n \leq \leq \leq \leq(n+1) \tau}\left(Z M P_{A}-Z M P_{D}\right)^{2} \\
& \text { subject to } \\
& \quad g_{j}(\mathbf{c}) \leq 0, \quad j=1,2, \ldots, m  \tag{26}\\
& h_{k}(\mathbf{c})=0, \quad k=1,2, \ldots, p \\
& c_{i}^{\min } \leq c_{i} \leq c_{i}^{\min }
\end{align*}
$$

where $\mathbf{c}=\left[\begin{array}{llllll}c_{0} & c_{1} & c_{2} & c_{3} & c_{4} & c_{5}\end{array}\right]^{T}$ is a design variable vector. $g_{j}(\mathbf{c})$ and $h_{k}(\mathbf{c})$ can be expressed as

$$
\begin{align*}
& g_{1}(\mathbf{c})=-x_{1}(n \tau)+X_{0}^{\min }+\alpha_{0}  \tag{27}\\
& g_{2}(\mathbf{c})=x_{1}(n \tau)-X_{0}^{\max }-\alpha_{0}  \tag{28}\\
& g_{3}(\mathbf{c})=-x_{1}(n \tau+0.5 \tau)+X_{m}^{\min }+\alpha_{m}  \tag{29}\\
& g_{4}(\mathbf{c})=x_{1}(n \tau+0.5 \tau)-X_{m}^{\max }-\alpha_{m}  \tag{30}\\
& g_{5}(\mathbf{c})=-x_{1}((n+1) \tau)+X_{f}^{\min }+\alpha_{f}  \tag{31}\\
& g_{6}(\mathbf{c})=x_{1}((n+1) \tau)-X_{f}^{\max }-\alpha_{f}  \tag{32}\\
& h_{1}(\mathbf{c})=x_{1}((n+1) \tau)-x_{1}(n \tau)-L_{s}  \tag{33}\\
& h_{2}(\mathbf{c})=\dot{x}_{1}((n+1) \tau)-\dot{x}_{1}(n \tau)  \tag{34}\\
& h_{3}(\mathbf{c})=\ddot{x}_{1}((n+1) \tau)-\ddot{x}_{1}(n \tau)  \tag{35}\\
& h_{4}(\mathbf{c})=\mathbf{\Phi}_{\mathbf{Z}} \mathbf{\Delta Z}+\boldsymbol{\Phi}  \tag{36}\\
& h_{5}(\mathbf{c})=\boldsymbol{\Phi}_{\mathbf{Z}} \mathbf{Y}+\boldsymbol{\Phi}_{t}  \tag{37}\\
& h_{6}(\mathbf{c})=\boldsymbol{\Phi}_{\mathbf{Z}} \dot{\mathbf{Y}}-\gamma \tag{38}
\end{align*}
$$

Figure 5 shows the flowchart for stable walking pattern generation.


Fig. 5. Flowchart for walking pattern generation.

## 5. Numerical Example

Numerical simulations are performed for the biped robot shown in Fig. 2. The biped robot has 1.04 m height and 60 kg weight. Inertia and geometric properties are presented in Table 1. Trunk motion $y_{1}(t)$ has constant value of 0.843 m . Pitch angles of the trunk and feet have constant value of 0.0 rad . To obtain the optimized trunk motion, the minimax optimization using SQP method in MATLAB is used. ${ }^{19}$ Simulations are performed for different walking speeds.

Table 2 presents walking parameters for different walking speeds. In order to verify the proposed method, the simulation is done for relatively low walking speed,

Table 1. Length and inertia properties of the biped robot.

| Body number | Length $[\mathrm{m}]$ | Mass $[\mathrm{kg}]$ | Inertia $\left[\mathrm{kg} \cdot \mathrm{m}^{2}\right]$ |
| :---: | :---: | :---: | :---: |
| Body 1 | 0.2 | 34 | $10^{-1}$ |
| Body 2 | 0.4 | 5 | $10^{-2}$ |
| Body 3 | 0.4 | 5 | $10^{-2}$ |
| Body 4 | 0.05 | 3 | $10^{-2}$ |
| Body 5 | 0.4 | 5 | $10^{-2}$ |
| Body 6 | 0.4 | 5 | $10^{-2}$ |
| Body 7 | 0.05 | 3 | $10^{-2}$ |

Table 2. Walking parameters for different walking speeds

| Walking speed $[\mathrm{km} / \mathrm{h}]$ | $L_{s}[\mathrm{~m}]$ | $H[\mathrm{~m}]$ | $\tau[\mathrm{sec}]$ |
| :---: | :---: | :---: | :---: |
| 0.9 | 0.4 | 0.06 | 0.8 |
| 1.8 | 0.8 | 0.10 | 0.8 |
| 2.4 | 0.8 | 0.14 | 0.6 |

about $0.9 \mathrm{~km} / \mathrm{h}$. Figure 6 shows the desired ZMP and the prescribed foot motions. The trunk motion, $x_{1}$ is optimized to guarantee the walking stability. Figure 7 shows $\Delta \mathrm{ZMP}\left(=Z M P_{A}-Z M P_{D}\right)$ during the one step movement. In the figure, maximum $\Delta$ ZMP has a negligible value of 0.006 m . To verify the walking stability of the generated walking pattern, a virtual biped robot is constructed on commercial program. ${ }^{20}$ In this model, contact between the ground and feet are considered. Trajectories of relative joint angles, which are obtained by the proposed method, are imposed to the relative joint motions on ADAMS. Figure 8 shows the trunk motion $x_{1}(t)$ to minimize $\Delta$ ZMP. Figure 9 shows superposed stick diagram for five walking steps. From the analysis result, the robot can walk stably without tipping over. This result shows that the proposed method can be effectively applied to the stable walking pattern generation.

Another walking pattern generation is done for the biped robot with relatively high speed, about $1.8 \mathrm{~km} / \mathrm{h}$. Figure 10 shows the desired ZMP and the prescribed foot motions used in the simulation. Figure 11 shows the convergence history. In the figure, it is known that optimum is obtained after 12 iterations. Figure 12 shows that the maximum of $\triangle$ ZMP has an allowable value of 0.0156 m at one step. The initial design variables and optimum are presented in Table 3. Figure 13 shows the desired ZMP and actual ZMP. In the figure, it is known that $\triangle$ ZMP becomes smaller at the optimum compared to the initial design variables. Figure 14 shows the optimized trunk motion in X direction. Figure 15 shows superposed stick diagram for five walking steps. Finally, walking pattern with walking speed $2.4 \mathrm{~km} / \mathrm{h}$ is generated. Figure 16 shows superposed stick diagram for five walking steps.

(a) Prescribed foot motions.

(b) The desired ZMP

Fig. 6. Prescribed foot motions and the desired ZMP for the case $0.9[\mathrm{~km} / \mathrm{h}]$.


Fig. 7. $\Delta$ ZMP for the optimized trunk motion, $x_{1}(t)$.


Fig. 8. Optimized Trunk motion, $x_{1}(t)$.


Fig. 9. Stick diagram of the biped walking (spaced 0.04 sec$)$.

(a) Prescribed foot motions.

(b) The desired ZMP

Fig. 10. Prescribed foot motions and the desired ZMP for the case $1.8[\mathrm{~km} / \mathrm{h}]$.


Fig. 11. Convergence history of $\triangle$ ZMP


Fig. 12. $\Delta$ ZMP for the optimized trunk motion, $x_{1}(t)$.

Table 3. The initial coefficients and optimum of the proposed $5^{\text {th }}$ polynomial.

|  | $c_{0}$ | $c_{1}$ | $c_{2}$ | $c_{3}$ | $c_{4}$ | $c_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Initial design variables | 0 | 0.25 | 0 | 0 | 0 | 0 |
| Optimum | -2.6399 | 16.457 | -39.292 | 44.558 | -23.268 | 4.4499 |



Fig. 13. The optimized ZMP compares with the initial ZMP.


Fig. 14. Optimized trunk motion, $x_{1}(t)$


Fig. 15. Stick diagram of the biped walking (spaced 0.04 sec ).


Fig. 16. Stick diagram of the biped walking (spaced 0.03 sec ).

## 6. Conclusion

This paper presents a new optimized polynomial approximation method which guarantees both the walking stability and high speed. To obtain the high speed walking pattern, small deviation of the desired ZMP is allowed to improve the walking speed without sacrificing the stability. The minimax optimization method is used to solve the optimization problem. Constrained multibody dynamics solution methods are used to solve the inverse kinematic problem systematically. To verify the effectiveness and validity of the proposed method, stable walking pattern for the biped robot is generated and walking simulation is performed for the given walking pattern.

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