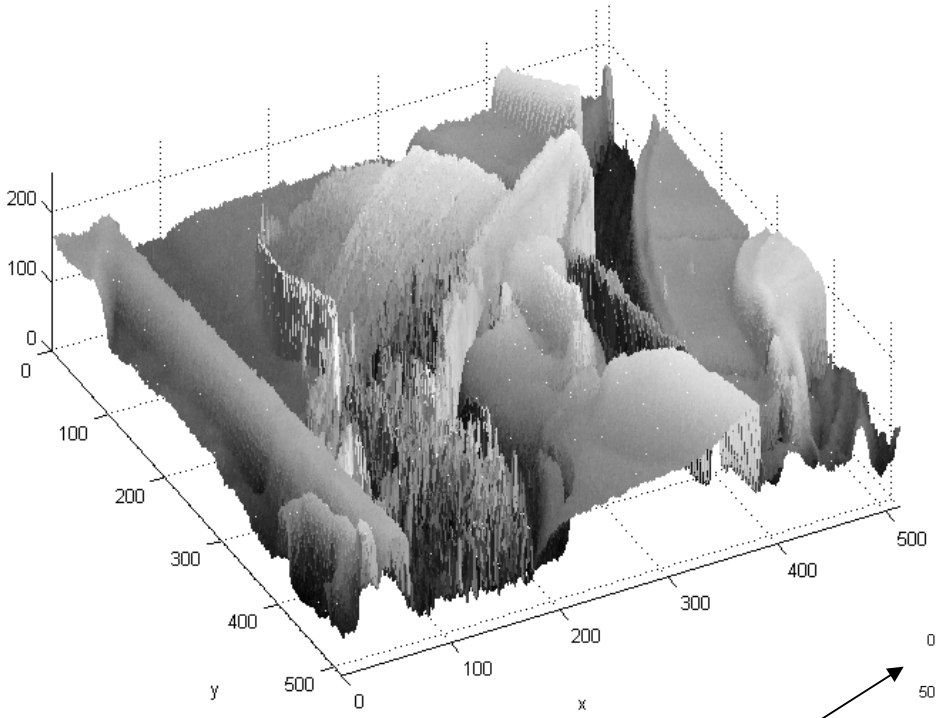
Image Processing

- An image can be represented by functions of two *spatial* variables *f(x,y),* where *f(x,y)* is the *brightness* of the gray level of the image at a spatial coordinate *(x,y)*
- A multispectral image is a **f** is a vector-valued function with components *(f_1, f_2, …, f_n);* a special case is a color image in which the components measure the brightness values of each of three wavelengths, that is:

$$\mathbf{f}(\mathbf{x}) = \left\{ f_{red}(\mathbf{x}),\ f_{green}(\mathbf{x}),\ f_{blue}(\mathbf{x}) \right\}$$

$$\mathbf{x} = (x, y)$$



(0,0) is (usually) the top-left corner.
Other standards exist…

RGB planes decomposed…

$$\mathbf{f}(\mathbf{x}) = \left\{ f_{red}(\mathbf{x}),\ f_{green}(\mathbf{x}),\ f_{blue}(\mathbf{x}) \right\}$$

red

green

blue

Point Operations

- In a point operation each pixel in the output image is a function of the grey-level (or color value) of the pixel at the corresponding position in the input image

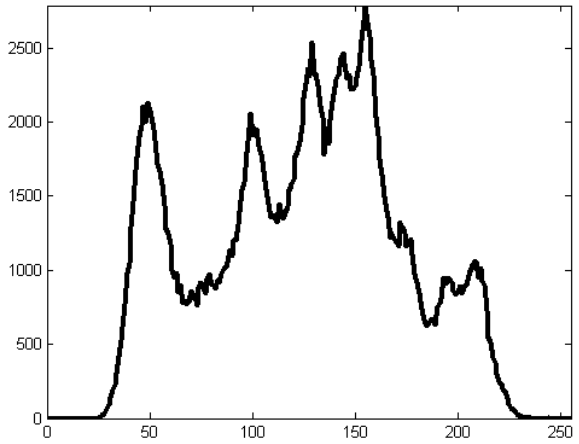
$$out(x,y) = f(in(x,y))$$

- For example: photometric decalibration, contrast stretching, thresholding, background subtraction…

Histogram

- A grey level histogram is a function that gives the frequency of occurrence of each gray level in the image
- If the gray levels are quantized in *n* values (usually 256), the value of the histogram at a particular gray level *p, h(p)*, is the number of pixels in the image with that gray level
- Often it is expressed in terms of *fraction* of pixels

(image 512x512)

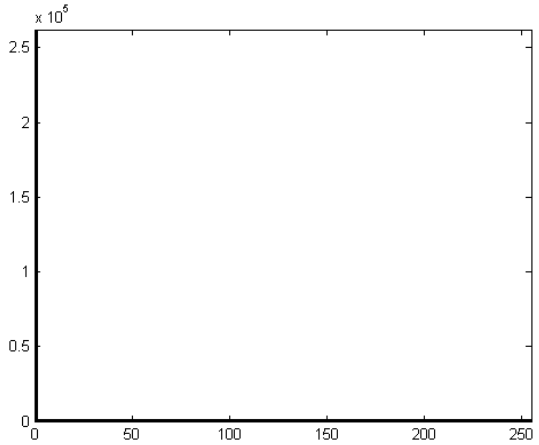


How do we compute the histogram

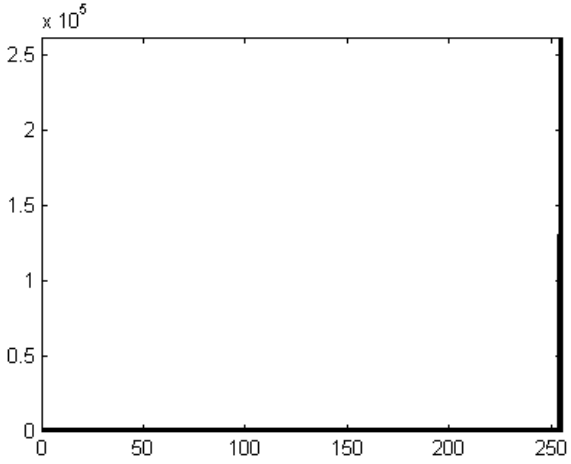
```
function histo=computeHisto(A)

histo=zeros(1,256);

R=size(A,1);
C=size(A,2);

for r=1:R
    for c=1:C
        index=A(r,c);
        histo(index+1)=histo(index+1)+1;
    end
end
```

Some characteristic histograms…

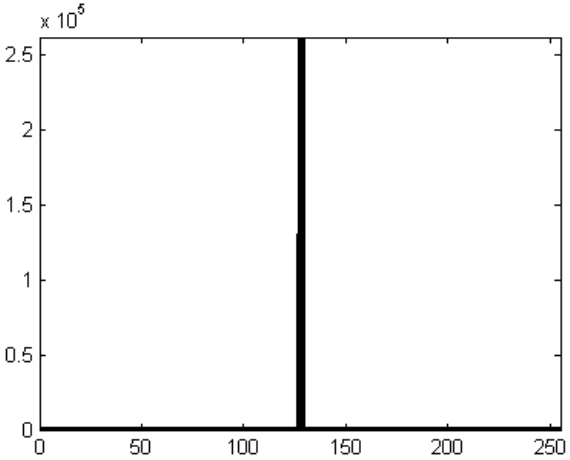
A black image

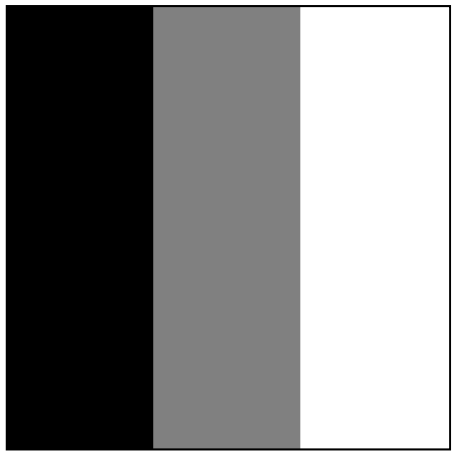


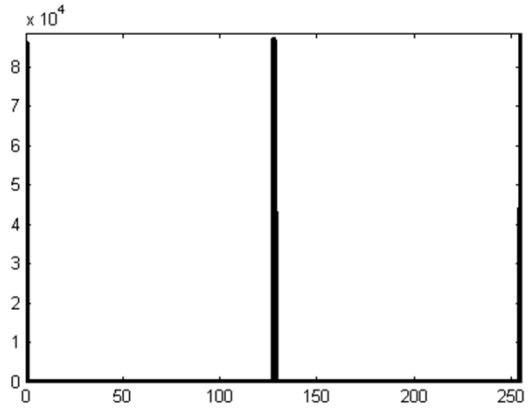
A white image

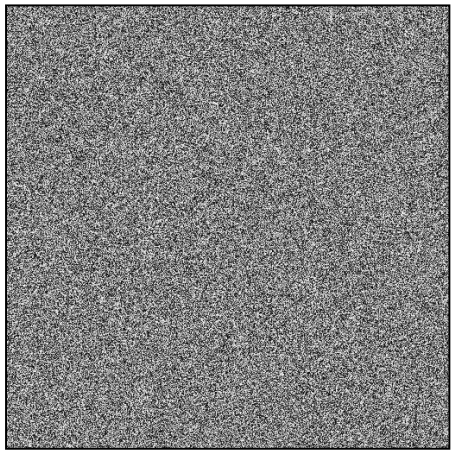


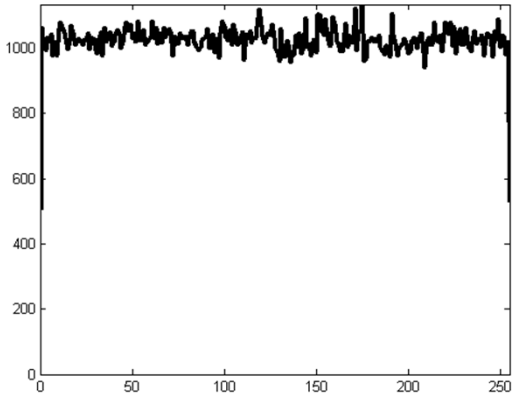
A gray image)






x 10^4
8
7
6
5
4
3
2
1
0
0
50
100
150
200
250

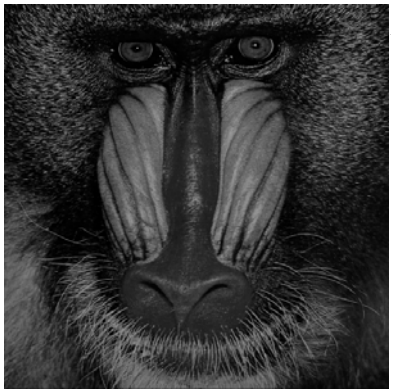


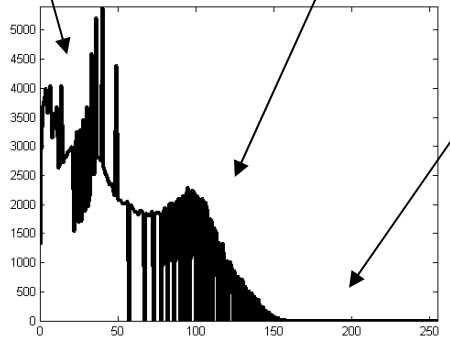

1000
800
600
400
200
0
0
50
100
150
200
250

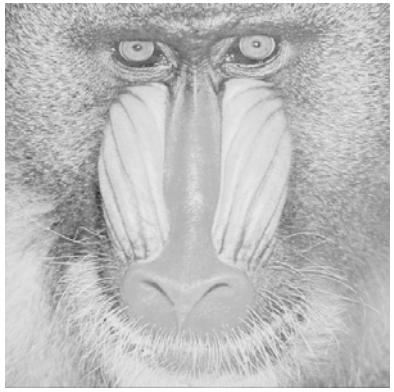
shadows

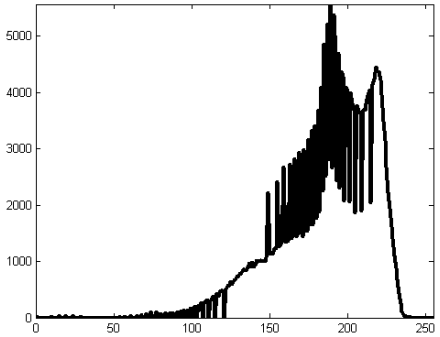
mitdtones

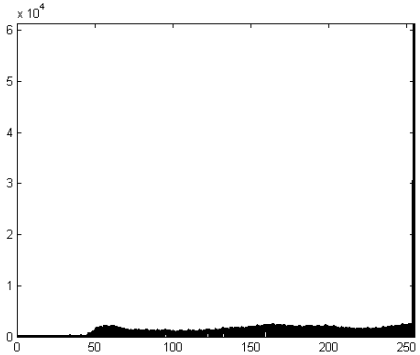
highlights











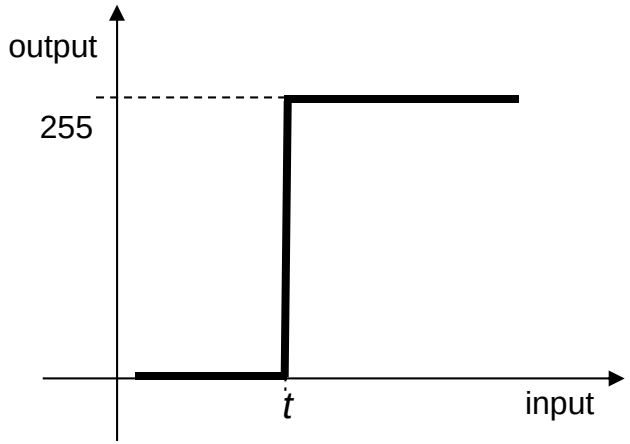
Negative

```
function S=negative(A)

R=size(A,1);
C=size(A,2);

%prepare image
S=zeros(R,C);
...
```

```
...
for r=1:R
    for c=1:C
        S(r,c)=255-double(A(r,c));
    end
end
```

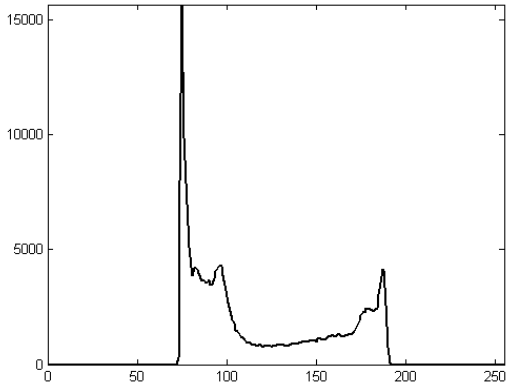
Threshold

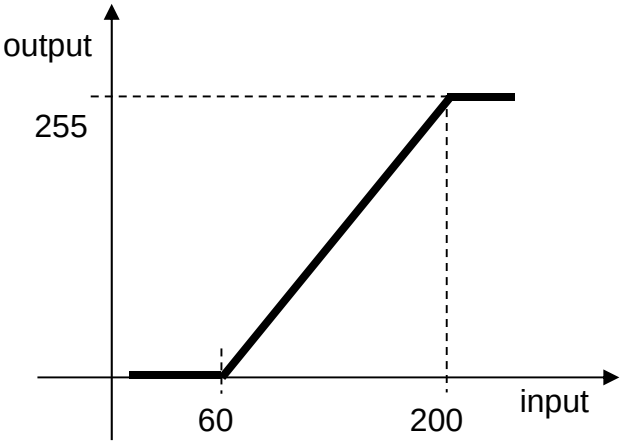
- Produces a two-level image
- We pick a threshold t, we set to 255 all pixels whose value > t, 0 all the others

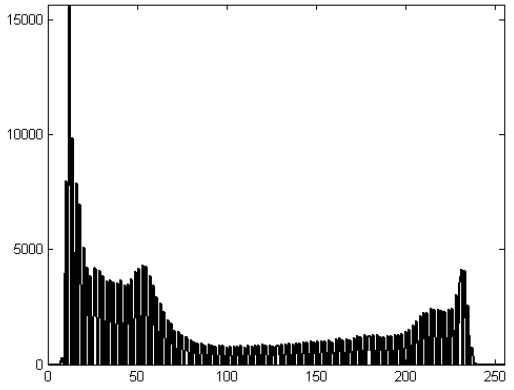


Histogram Stretch

- From the histogram it is possible to see if there are levels in the image that are not used
- We can map the levels of the image to expand the histogram







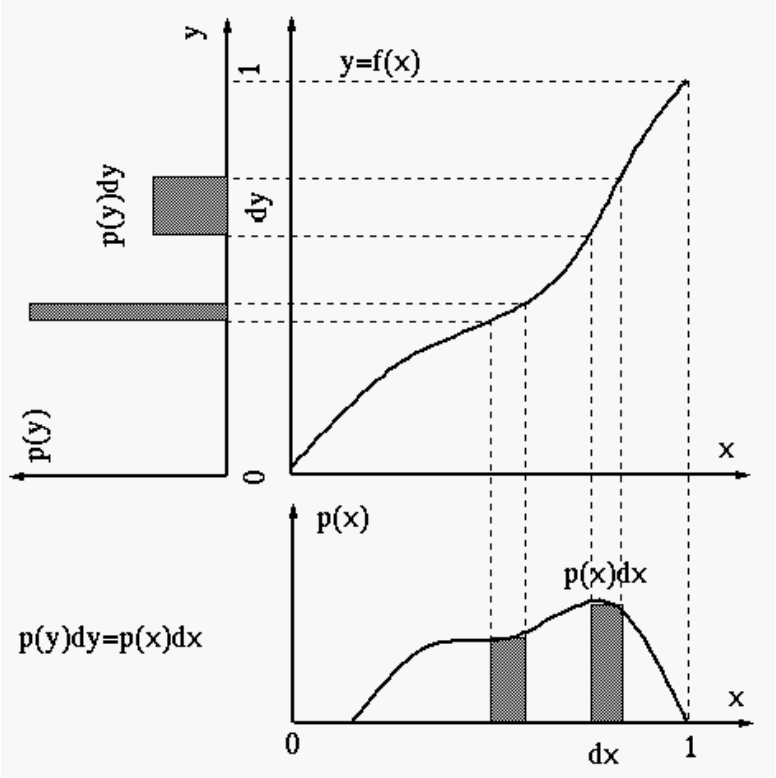
Histogram stretch: sample code

```
function S=stretchHisto(A, min, max)

%%%%% build look up table
lut=zeros(1,256);
for i=0:255
    if (i<min)
        lut(i+1)=0;
    elseif (i>max)
        lut(i+1)=255;
    else
        lut(i+1)=(i-min)*255/(max-min);
    end
end
%%%%%
….
```

```
…
R=size(A,1);
C=size(A,2);

%prepare image
S= zeros(R,C);

for r=1:R
    for c=1:C
        index= A(r,c)+1;
        S(r,c)=lut(index);
    end
end
```

Histogram equalization

- Equally use all gray levels
- Find a transformation y=f(x) to “flatten” the histogram



#pixels is unchanged:

$$p(y) \cdot dy = p(x) \cdot dx$$

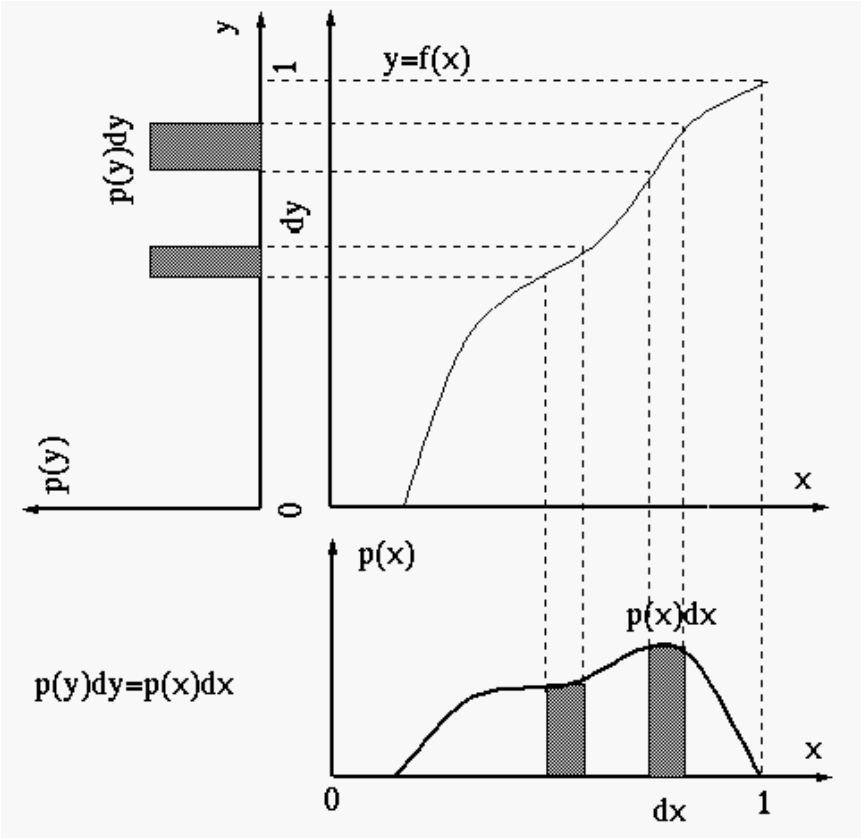
Assume x and y between 0 and 1
We would like p(y) = constant = 1

$$p(y)\cdot dy = p(x)\cdot dx$$

$$dy = p(x)\cdot dx \Rightarrow \frac{dy}{dx} = p(x)$$

$$y(x) = \int_0^x p(u)du$$

cumulative probability distribution



- low p(x) → smooth f(x) → narrow d(y)
- large p(x) → steep f(x) → large d(y)

Discrete case

- x and y assume discrete values between [0,L-1] (often L=256)

$$y(x) = \int_0^x p(u)du \longrightarrow y'(x) = \sum_0^x P_i$$

$$P_i = \frac{n_i}{N}$$

n_i: number of pixels that have value i

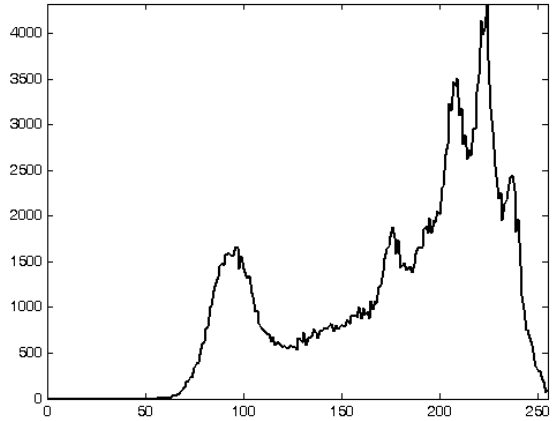
N: total number of pixels (HxW)

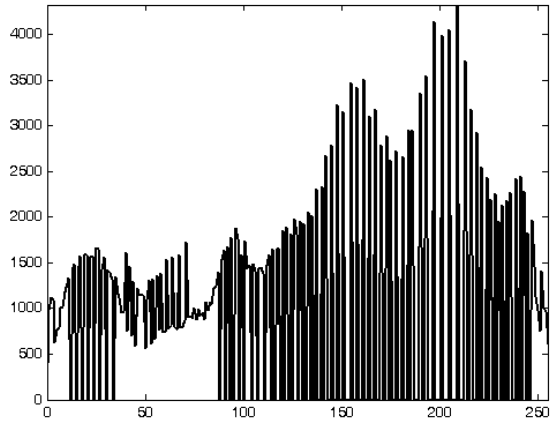
- y' assumes values between [0,1] → needs to be scaled to [0, L-1]:

$$y = \left\lfloor \frac{y' - y'_{\min}}{1 - y'_{\min}} (L-1) + 0.5 \right\rfloor$$

$\lfloor \cdot \rfloor$: integral part of a real number

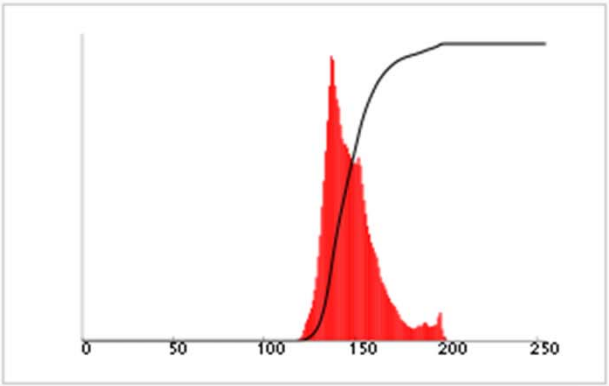
Example





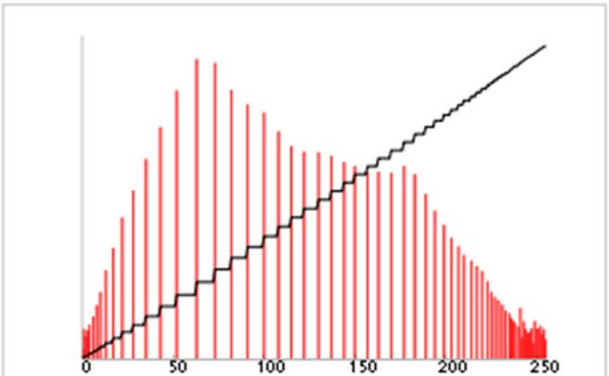
Note: the resulting histogram is not perfectly flat because we cannot separate pixels having the same gray level, the resulting cumulative distribution, however, would approximate a linear ramp

An unequalized image



Corresponding histogram (red) and cumulative histogram (black)

The same image after histogram equalization



Corresponding histogram (red) and cumulative histogram (black)

Detect Changes

- Take the difference between each pixel in two images *A* and *B* (grayscale):

  B="background"

  A=new image

  D=abs(A-B)

- Extend the concept to a sequence of images
- At each instant in time we take the difference between the current frame and the previous one:

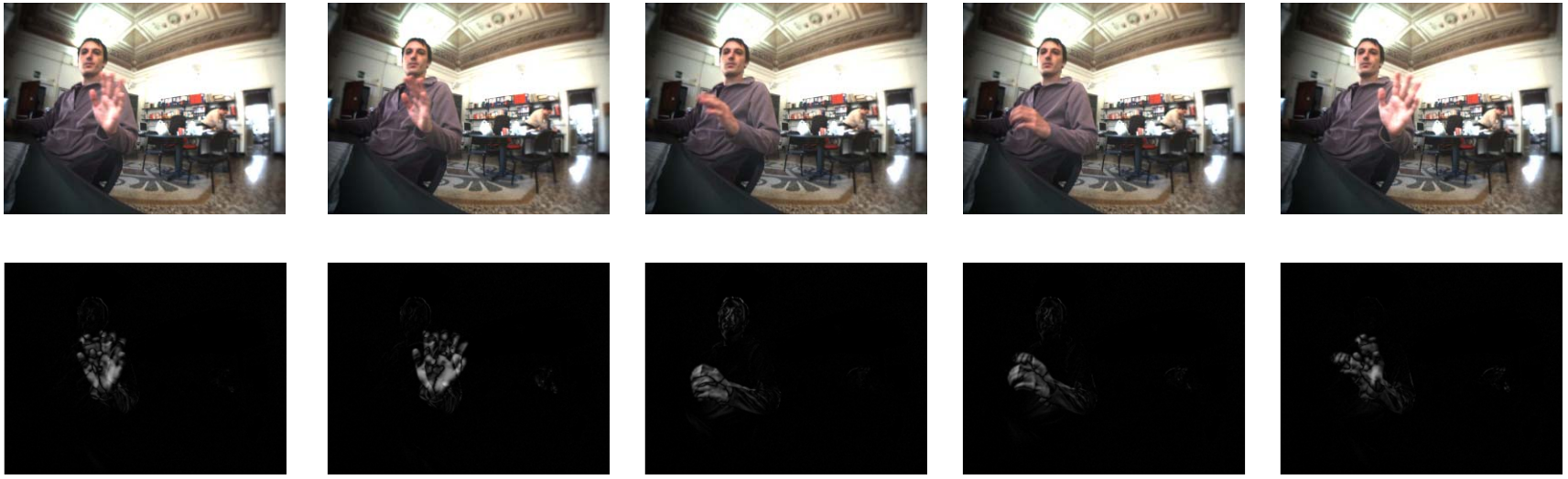
  D=abs(A(t)-A(t-1))

  Detection can be done by thresholding:

  Out=threshold(D,th);

Image Difference

```
function imageDiff(basename, start, last)

cFrame=sprintf('%s%d.ppm', basename, start);
A=imRead(cFrame);
PREV=rgb2Gray(A);

for i=start:last
   cFrame=sprintf('%s%d.ppm', basename, i);
   % read new image
   A=imRead(cFrame);

   % convert to grayscale
   G=rgb2Gray(A);

   % take the difference between the current frame and the previous one
   D=double(G)-double(PREV);
   % compute the abs value
   D=abs(D);
   % threshold
   diff_th=im2bw(uint8(D),50/255);

   % store frame
   PREV=G;

   %%%% PLOT
   figure(1), subplot(1,2,1), imShow(uint8(A)), drawnow;
   figure(1), subplot(1,2,2), imShow(uint8(255*diff_th)), drawnow;

   pause(0.05); %%wait some time
end
```



Another option

- Model the background by taking into account more than a single frame:

$$B = a * A(t-1) + (a-1) * B$$
$$D = abs(A(t) - B)$$

a determines how fast we update the background:

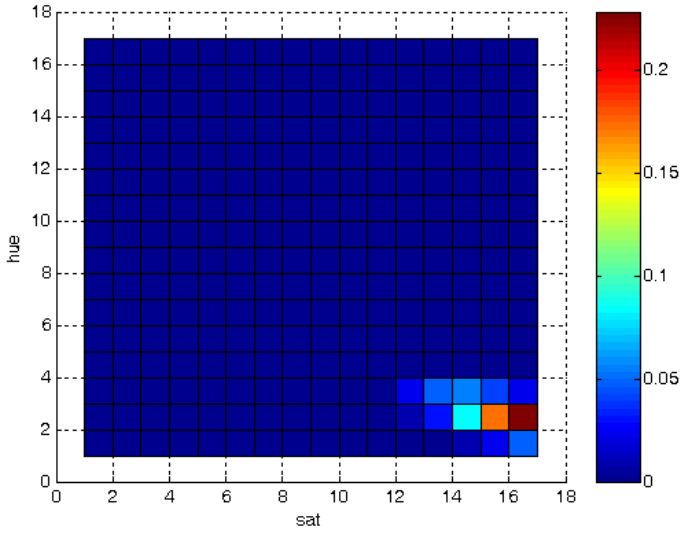
a=1 → image difference

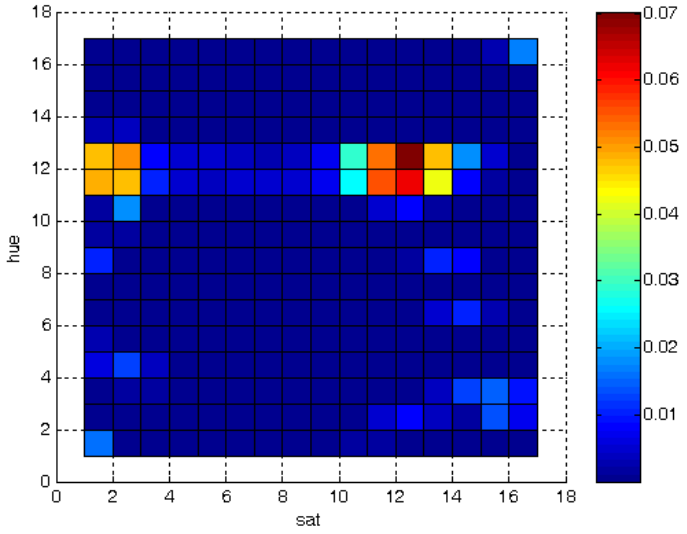
a=0 → persistent background (never updated)

Color Histograms

- Count the color of the pixels of the images
- It is a statistical description of the color of the image, useful to characterize a particular object
- Appealing because invariant to translation and rotation, slowly changing with scale and view point
  - r,g,b → 3D function, intensity dependent, easily too large (es: 256x256x256x32 ~ 64MB)
  - discard luminance, use H,S or r,g → 2D

Color Histogram: examples





bin size: 16x16

Comparing Histograms

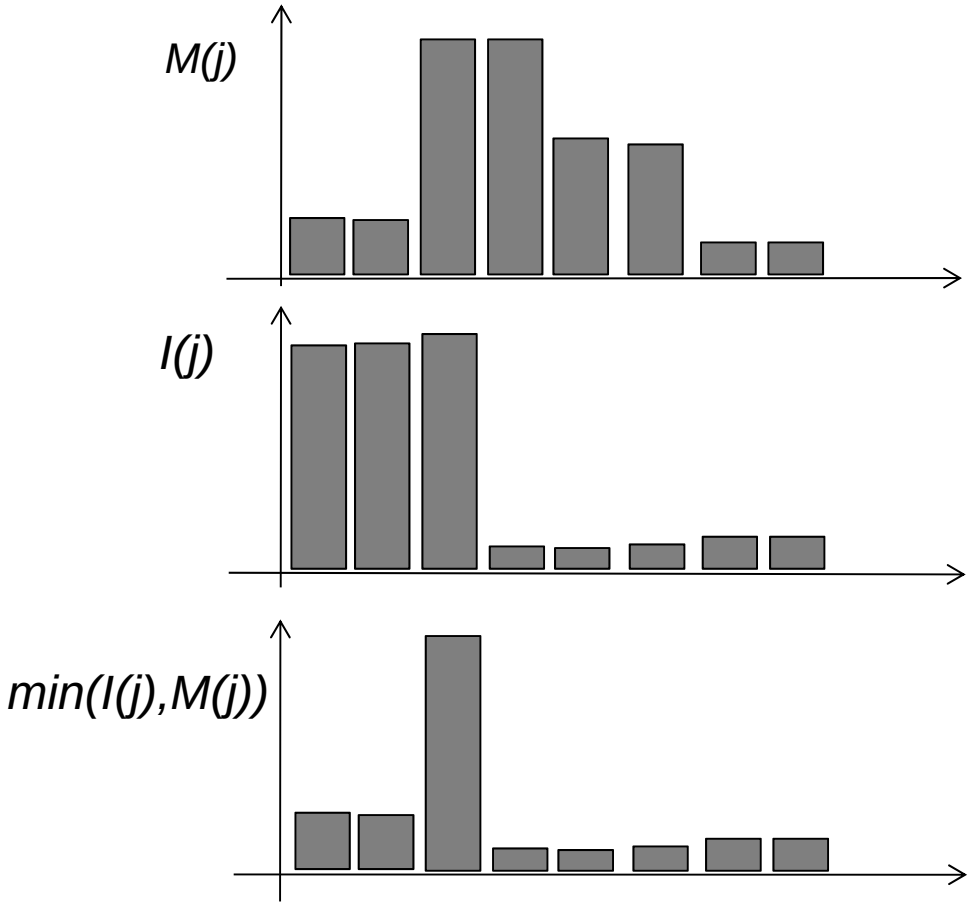
- Suppose we want to compare two histograms I and M, each with *n* bins
- Useful to solve the *identification problem*: compare two images M and I and decide if they are similar
- Intersection, the number of pixels from the model that correspond to pixels of the same color in the image, formally:

$$\sum_{j=1}^{n} \min(I_j, M_j)$$

- Normalize by the number of pixels in the histogram M:

$$H(I, M) = \frac{\sum_{j=1}^{n} \min(I_j, M_j)}{\sum_{j=1}^{n} M_j}$$

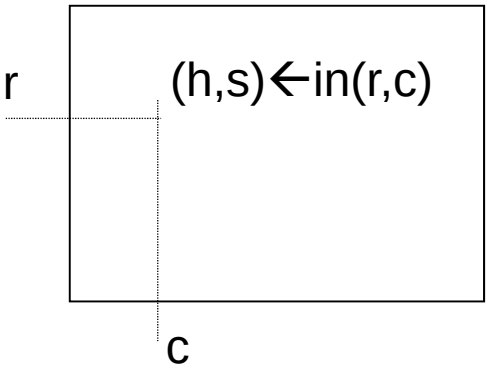
Swain and Ballard 1991

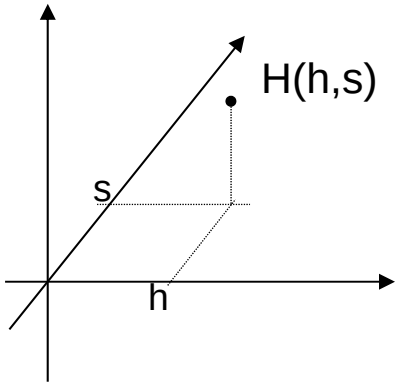

M(j)
I(j)
min(I(j),M(j))

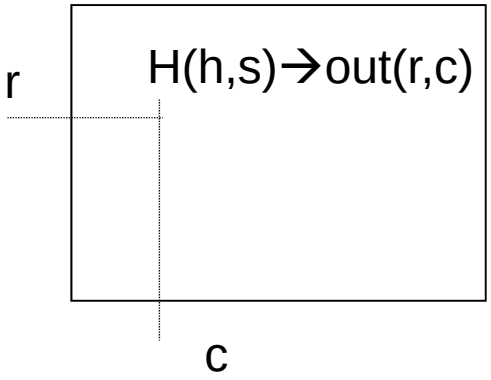
Histogram Backprojection

- Assume we have a model of an object (its color histogram)
- *Localization problem*: where in the image are the colors of the object being looked for?
- The histogram gives the probability of occurrence of the colors of the object, or *p(color/object)*
- We can approximate:

$$p(\text{object}\,|\,\text{color}) = \frac{p(\text{color}\,|\,\text{object}) \cdot p(\text{object})}{p(\text{color})} \propto p(\text{color}\,|\,\text{object})$$





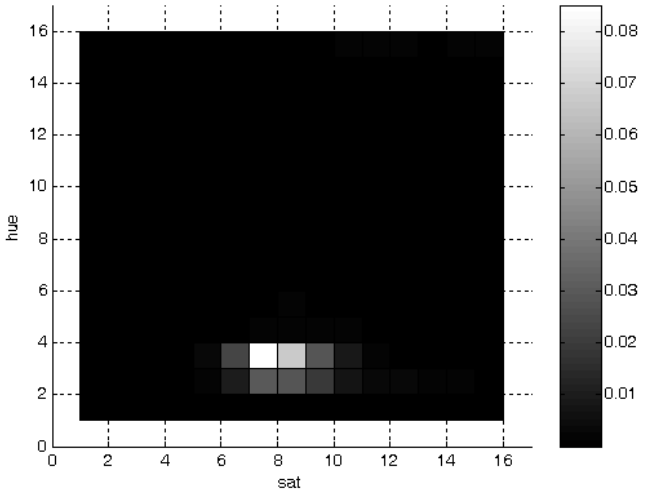


- Similar approach, compute the “ratio histogram” (Swain and Ballard, 1991):

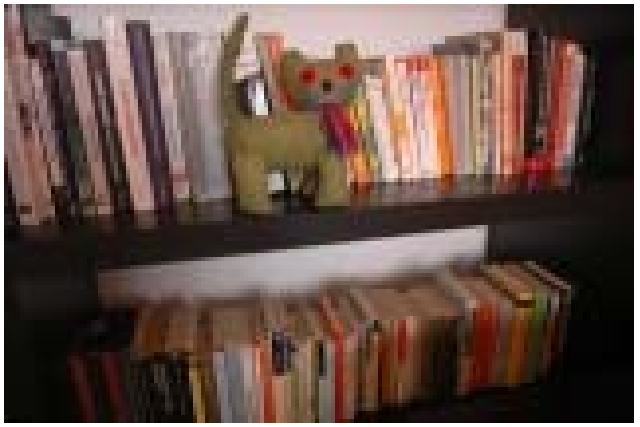
$$R_i = \min\left(\frac{M_i}{I_i}, 1\right)$$

- Perform backprojection of R into the image
- Heuristic to deemphasize colors that are not in the object looked for (for which I>M)
- Search for a uniform region whose size matches the one of the object

Compute histogram



Backprojection
(ratio histogram)



Examples:

- Swain and Ballard 1991, use color histograms to recognize objects
- Skin detection, preprocessing for face detection…
  - Example (Peer 2003)

Assume (r,g,b) space (and daylight illumination)
classify (r,g,b) as skin if:

$r > 95$ and $g > 40$ and $b > 20$,
$Max\{r,g,b\} - min\{r,g,b\} > 15$, and
$|r-b| > 15$ and $r > g$ and $r > b$