Hough Transform

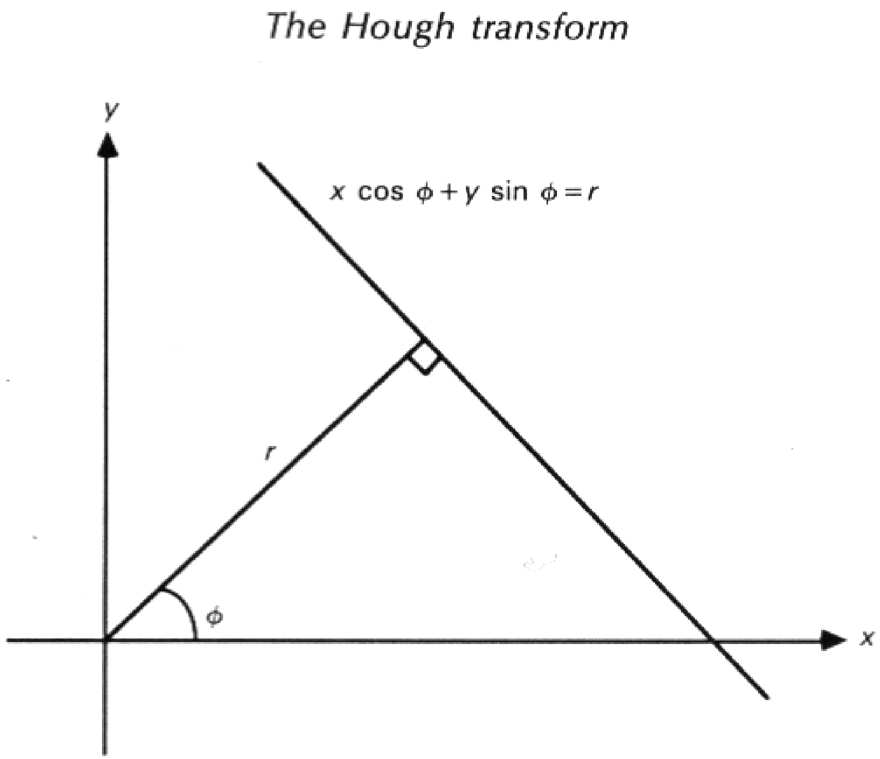
- It is a technique used to isolate curves of a given shape in an image
- In its classical formulation it requires the curve to be specified in some parametric form (usually lines, circles or ellipses)
- also… it can be generalized to arbitrary curved shapes
- Advantage: robust to “gaps” in the object
- Disadvantages:
  - parametric description of the shape
  - depending on the number of parameters might become slow

Hough Transform for lines

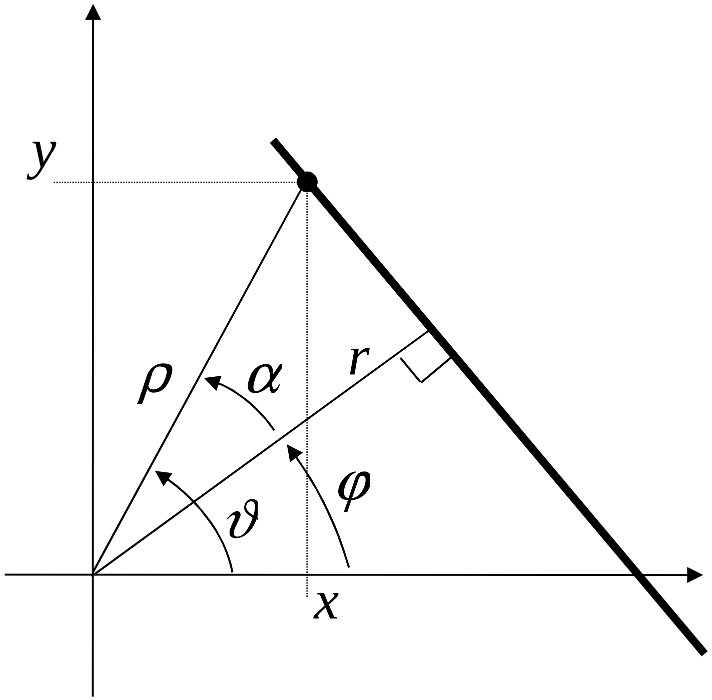
- We want to detect points lying on a straight line
- Start with the equation of a straight line in parametric form:

$$x\cos\varphi + y\sin\varphi = r$$

where *r* is the length of a normal to the line from the origin and φ is its angle with the x-axis



In case you don't trust the equation…



$$r = \rho\cos(\alpha) = \rho\cos(\vartheta - \varphi)$$

$$r = \rho\cos\vartheta\cos\varphi + \rho\sin\vartheta\sin\varphi$$

$$\boxed{r = x\cos\varphi + y\sin\varphi}$$

- Given a point $x_i\, y_i$ on this line we have:

$$x_i \cos\varphi + y_i \sin\varphi = r$$

where r and φ are constant

- Consider now r and φ variable (r_i, φ_i) and $x_i\, y_i$ constant (x,y), the equation describes all possible lines passing through the point, these lines are described by:

$$r_i = x\cos\varphi_i + y\sin\varphi_i \qquad \varphi \in [0, 2\pi]$$

- The *Hough Transform* of the point is the plot of this equation on the (r_i, φ_i) space (*Hough Space*)

we get a sinusoidal curve

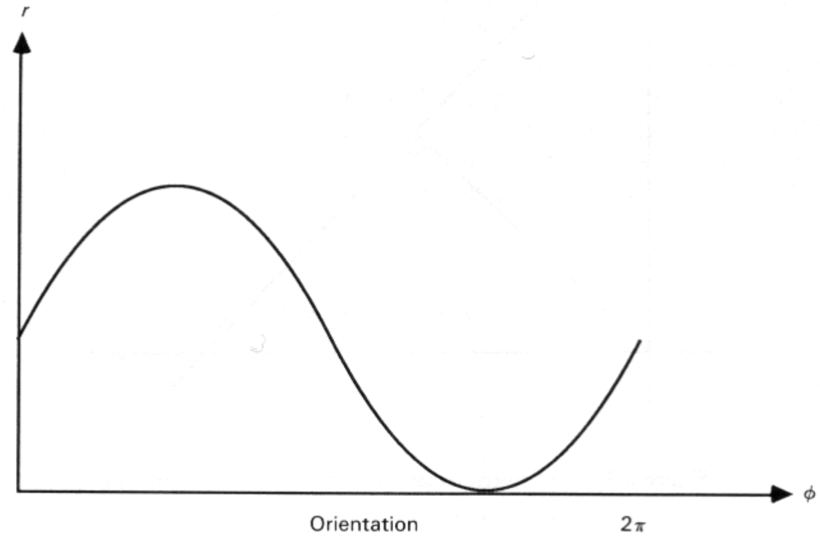
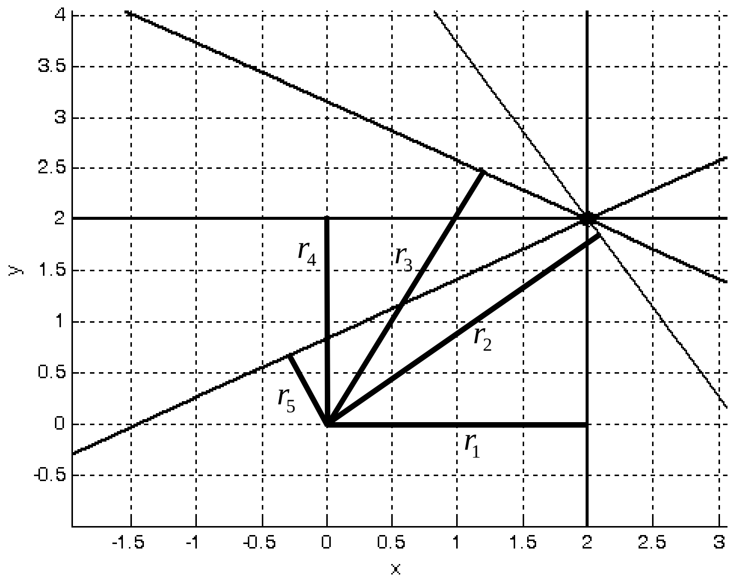
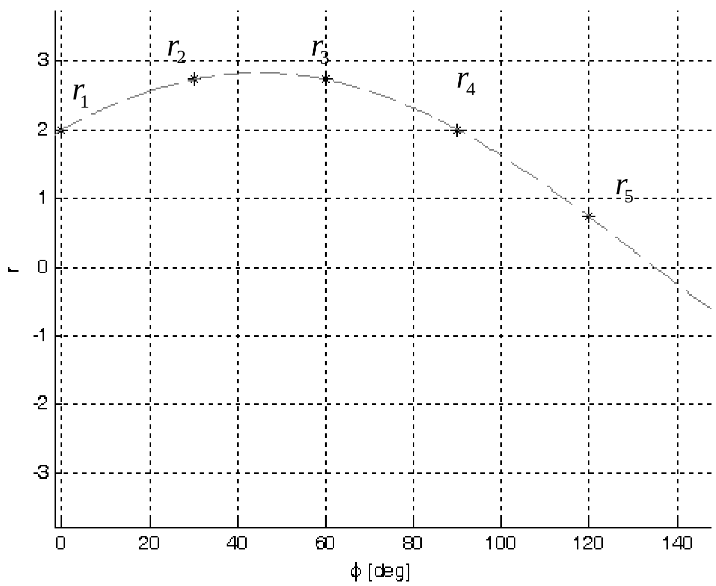


Figure 6.8 Hough transform of a point (x_i, y_i).

Hough Transform of one point





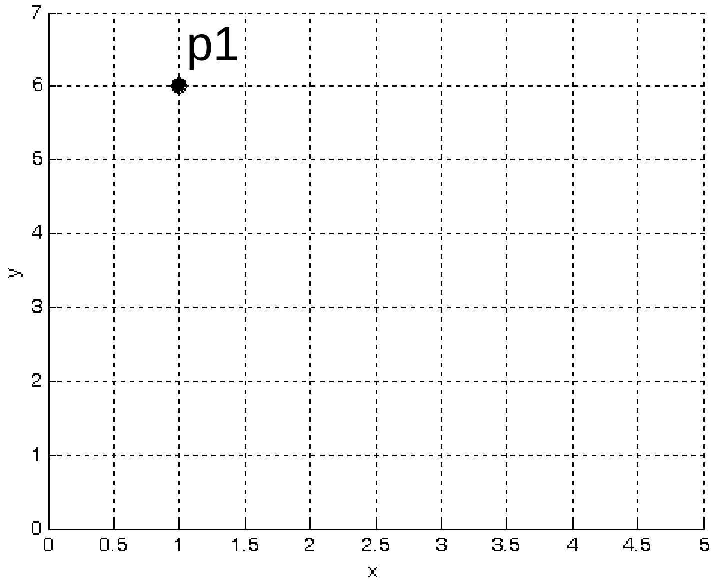
Hough Transform

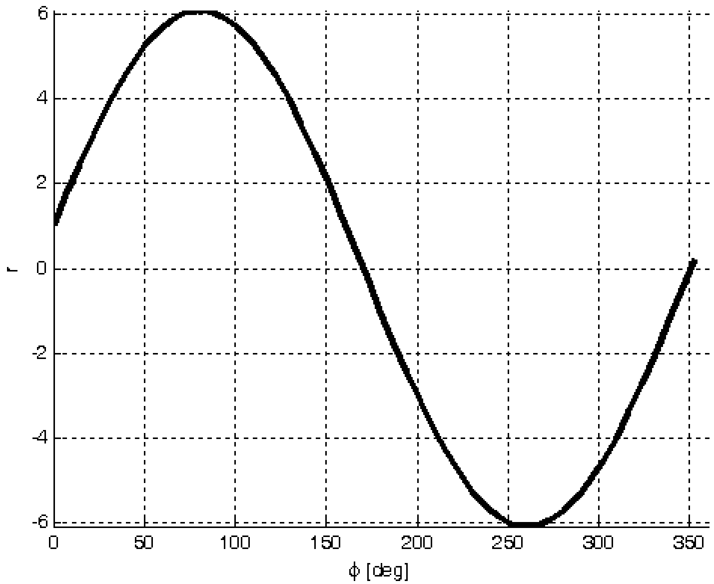
- Define the Hough Space: range for r (example: 0-255) and the angular resolution of the sampling of φ (example: 6 degrees)
- This gives a Hough Space (HS) of *256x60* points
- Each point in the image "vote" for a set of lines passing through it; all these votes are accumulated in the HS
- For each (x,y) increment all accumulator cells (r,φ) which satisfy the equation:

$$r = x\cos\varphi + y\sin\varphi$$

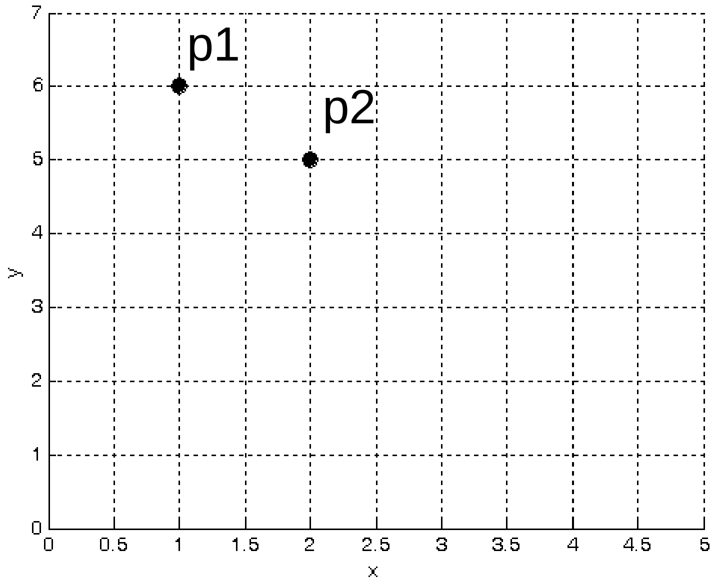
- At the end we scan the accumulator searching for cells which have high count: they correspond to lines for which there are many points in the image plane

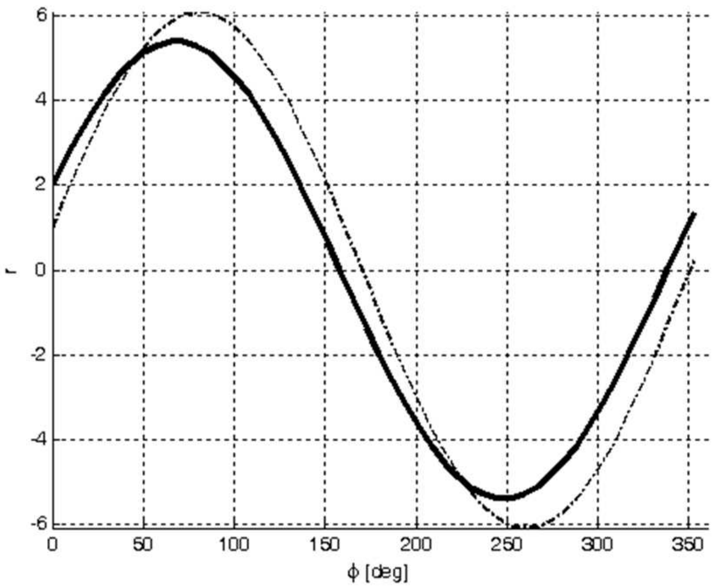
Example: start with one point (pixel)



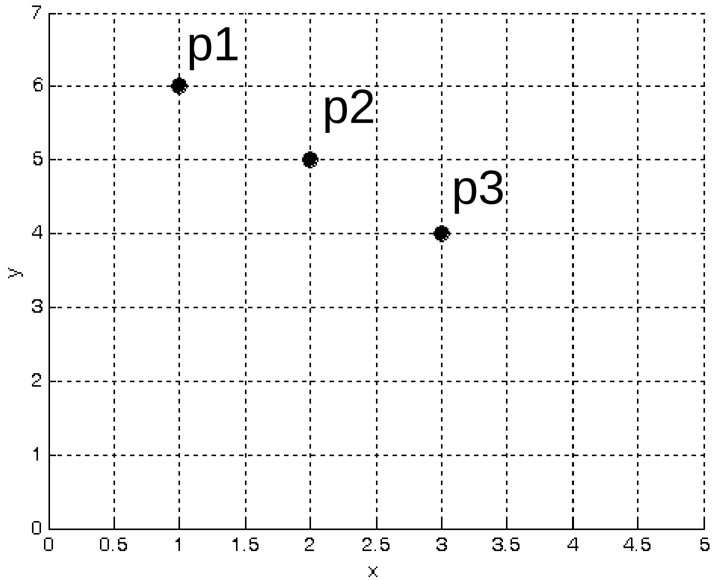


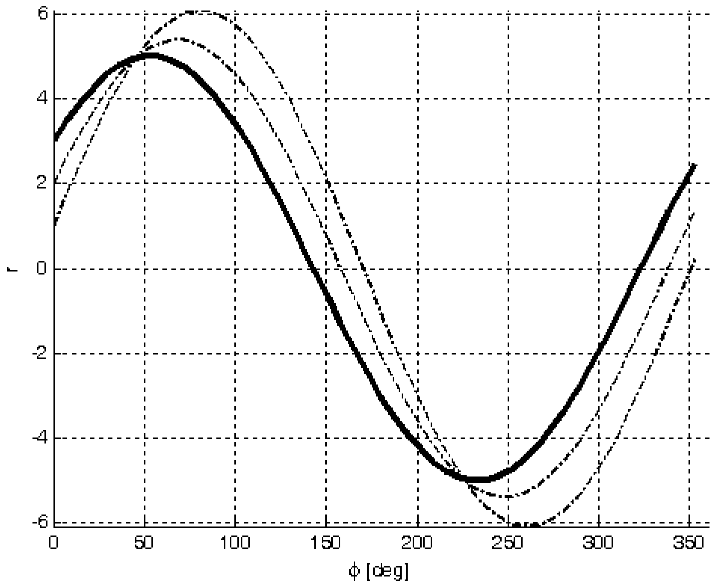
Example: add another pixel



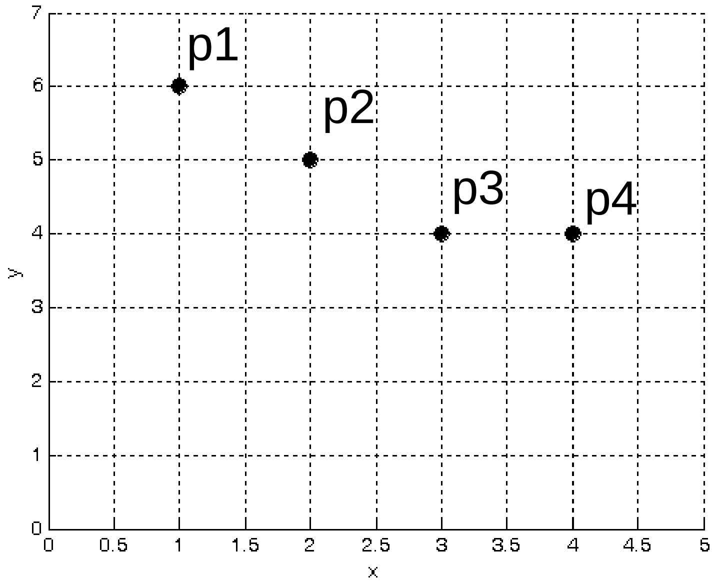


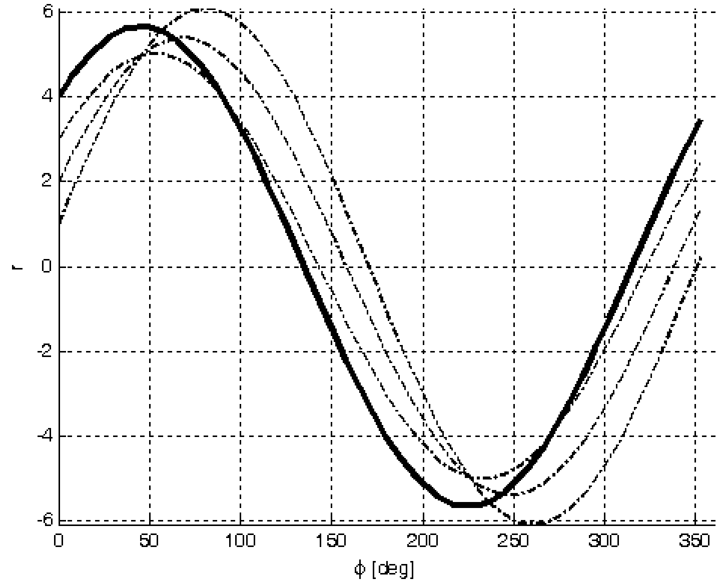
Example: now we have three pixels



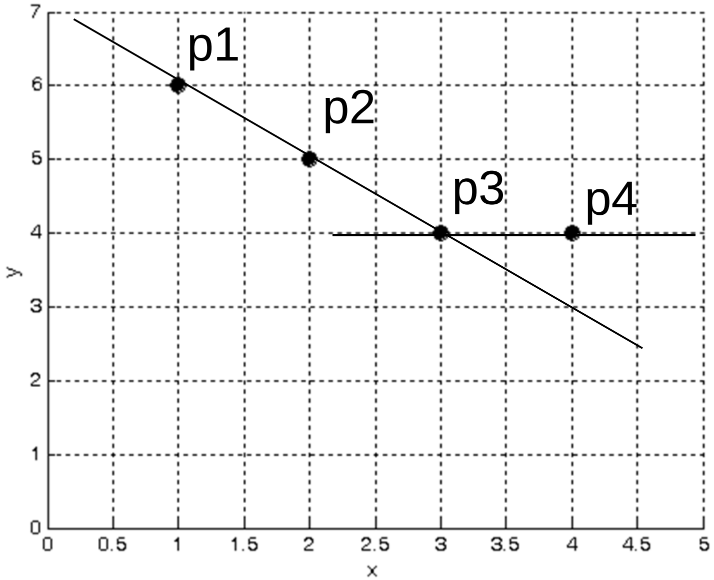


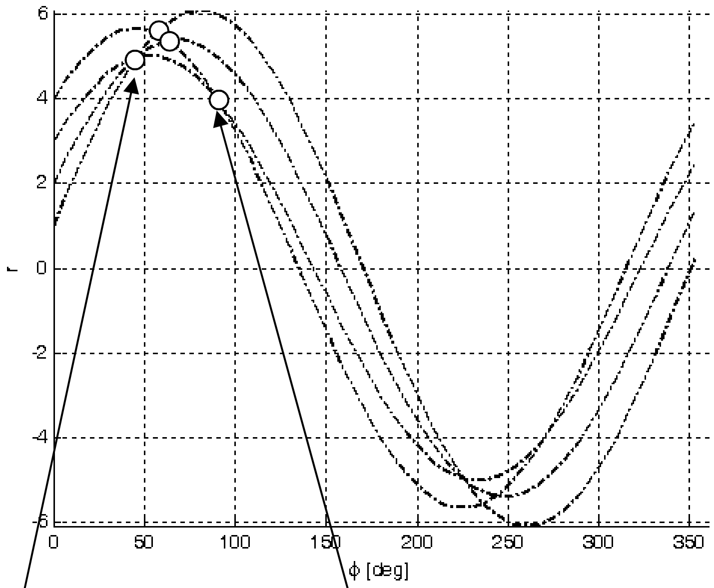
Example: the last one…





Finally: search intersections in the HT





p1,p2 and p3 intersect at the orientation of 45°

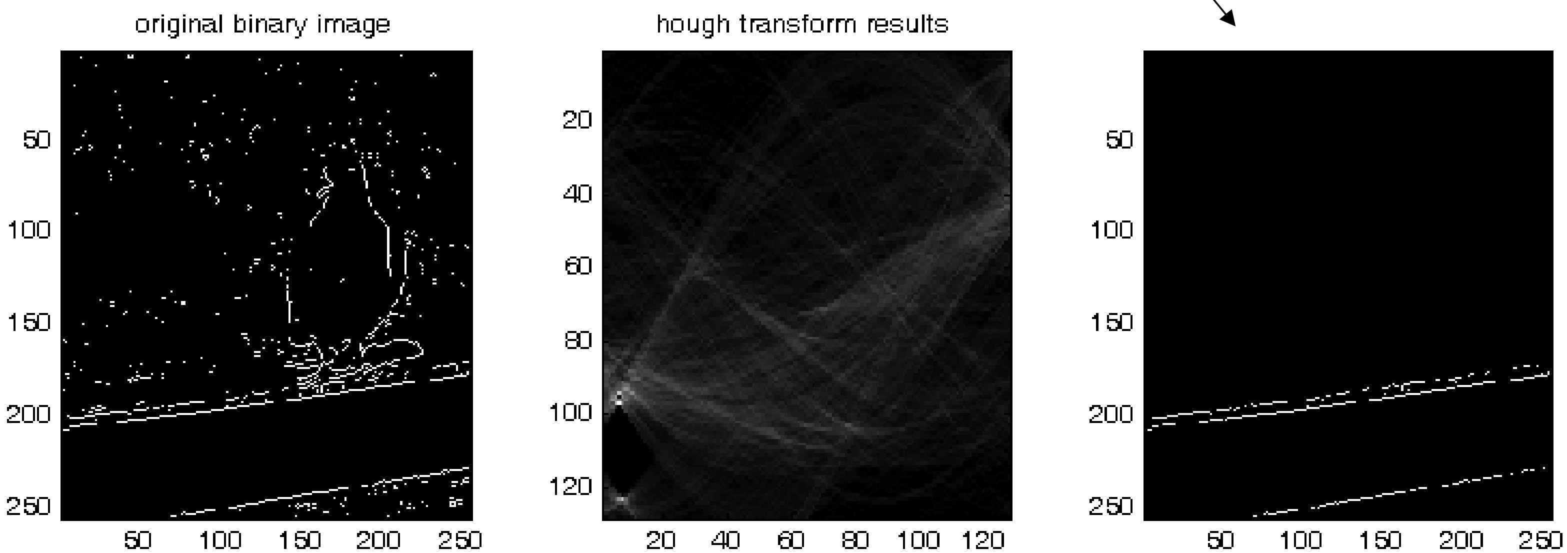
p3 and p4 intersects at 90°, p4 intersects the othe r points around 60°

Improvements:

- Usually start with the output of an edge detector, to reduce the number of points in the image
- Search along directions perpendicular to the image gradient

Hough Example

extract 5 strongest lines



houghdemo.m from http://homepages.cae.wisc.edu/~ece533/matlab/index.html

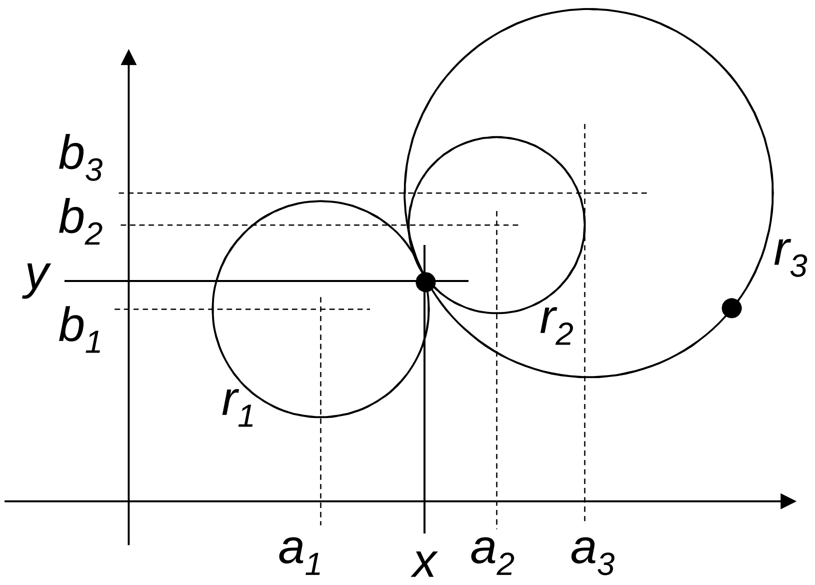
Circular Hough Transform

- Detect circles in the image
- Parametric equation of a circle:

$$(x-a)^2 + (y-b)^2 = r^2$$

where *(a,b)* are the coordinates of the center of the circle and *r* its radius

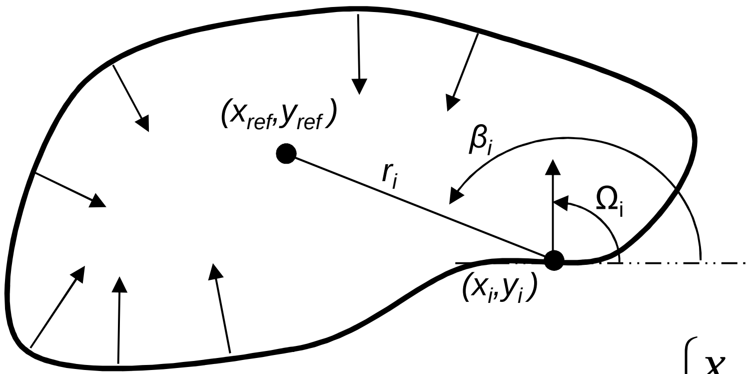
- *a,b* and *r* define the parameter space, the accumulator is three-dimensional



a_1,b_1,r_1
a_2,b_2,r_2
a_3,b_3,r_3
…

Generalized Hough Transform

- Suppose we don't have a simple analytical equation for the object
- Instead we use a LUT defining the relationship between the coordinates of the point, its orientation (the orientation of the local gradient) and the Hough parameters

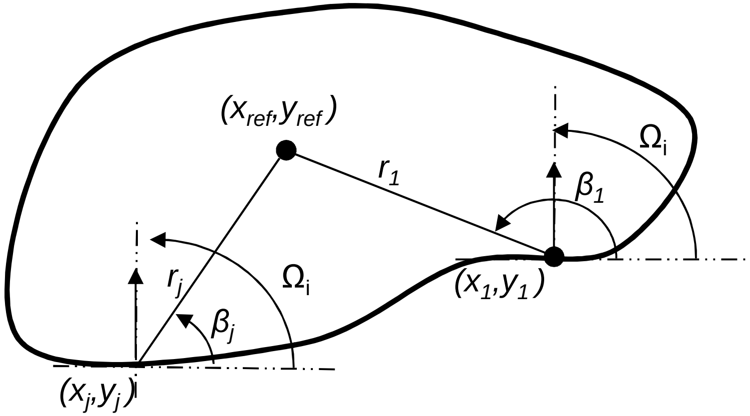


$$\begin{cases} x_{ref} = x_i + r_i \cos \beta_i \\ y_{ref} = y_i + r_i \sin \beta_i \end{cases}$$

Build the LUT:

1. Select an arbitrary reference point (x_{ref},y_{ref})
2. For all points of the boundary (x_i,y_i)
    - draw a line from (x_i,y_i) to (x_{ref}, y_{ref})
    - Measure (β_i,r_i)
    - Compute the orientation of the boundary Ω_i
    - Add (β_i,r_i) to a table indexed by Ω_i

- Probably there will be more than one occurrence of a particular orientation…



$$\begin{cases} x_{ref} = x_i + r_i \cos \beta_i \\ y_{ref} = y_i + r_i \sin \beta_i \end{cases}$$

Ω_0	…
Ω_1	…
…	…
Ω_i	(r_1,β_1), (r_2,β_2) …
…	…

R-table

- Once we have the R-table for the object, we can perform the Hough Transform of the image
- For each point in the image (x_i,y_i), we compute the point (x_{ref},y_{ref}) from:

$$\begin{cases} x_{ref} = x_i + r_i \cos \beta_i \\ y_{ref} = y_i + r_i \sin \beta_i \end{cases}$$

- where (r_i,β_i) are derived from the R-table, starting from the orientation of the point Ω_i
- We accumulate the Hough Space in (x_{ref},y_{ref}):

$$A(x_{ref}, y_{ref})++$$

- Finally search for local maxima in A to identify the center(s) of the object(s)

- It is easy to extend the search for different object orientations φ and scales *S*:

$$\begin{cases} x_{ref} = x_i + S \cdot r_i \cos(\beta_i + \varphi) \\ y_{ref} = y_i + S \cdot r_i \sin(\beta_i + \varphi) \end{cases}$$

- In this case we explore and accumulate a four-dimensional space:

$$A(x_{ref}, y_{ref}, \varphi, S){+}{+}$$