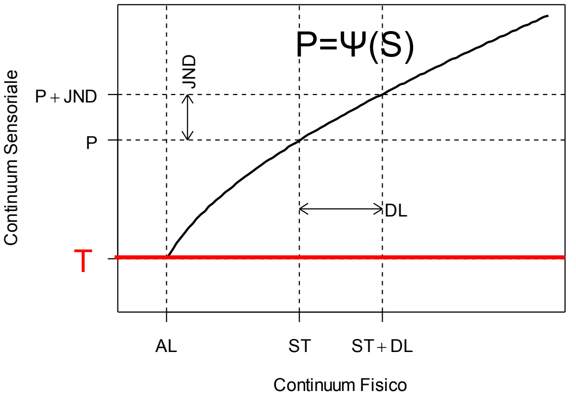
Classical threshold theory

Gabriel Baud-Bovy

Fondamental definitions



- The **psychophysical function Ψ** relates the physical intensity of the stimulus to the corresponding sensation. Note that the sensations are not directly observable.

- The **absolute threshold** (*Absolute Limen, AL*) is the smallest amount of stimulus energy necessary to produce a sensation
- The **difference threshold** (*Difference Limen, DL*) is the amount of changes of a stimulus required to produce a **just noticeable difference** (JND) in the sensation. Note that difference thresholds refer to the stimuli while JND refer to sensations.

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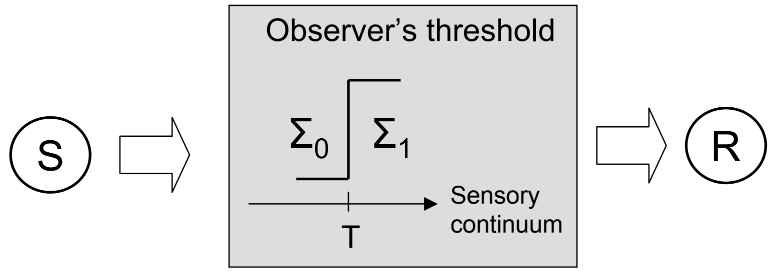
The concept of threshold



- The concept of threshold represents a barrier separating non-observable **internal states** inside the observer

$$\text{if } (P<T) \text{ then internal state } \Sigma = \Sigma_0 \text{ else } \Sigma = \Sigma_1$$

where Σ_0 represents the "Non-detection" state where intensity P along the sensory continuum is below the threshold T and Σ_1 represents the "Detection" sate where intensity P along the sensory continuum is above the threshold T

- **The postulate of internal states is the main characteristics of threshold theory**

- At least originally, threshold theory also assumed that the response of the observer was entirely determined by the observer's internal state: the response was positive if and only if the stimulus was above threshold.

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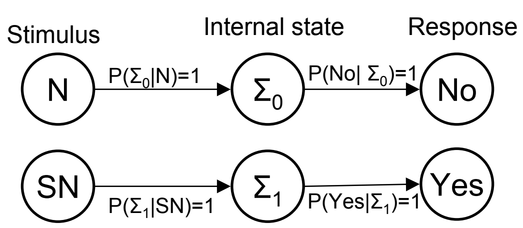
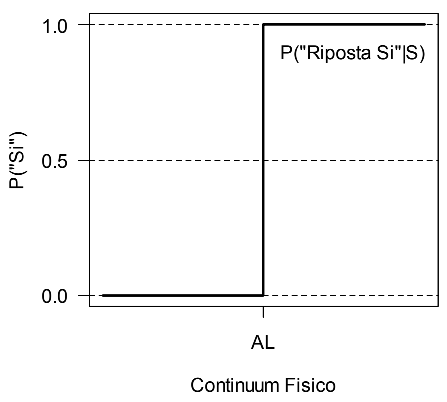
Noiseless (deterministic) observer

Ideal (noiseless) observer



Theoretical response matrix

P(R|S)	R=No	R=Yes
S=N	1	0
S=SN	0	1



- In a detection task, the **psychometric function** represents the proportion of positive detection responses (R=Yes) as a function of the stimulus intensity.

$$F(S) = Prob(R=Yes \mid Stimulus)$$

- If the sensory processes were **deterministic**, the psychometric subject would be a step function

$$P(R = Yes \mid S) = \begin{cases} 1 & if \;\; S = Signal \; (SN) \\ 0 & if \;\; S = Noise \; (N) \end{cases}$$

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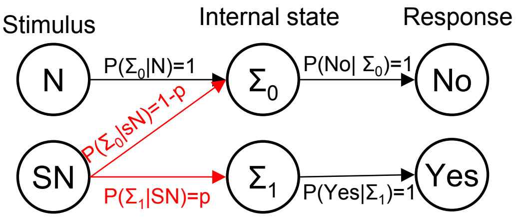
Probabilistic behavior

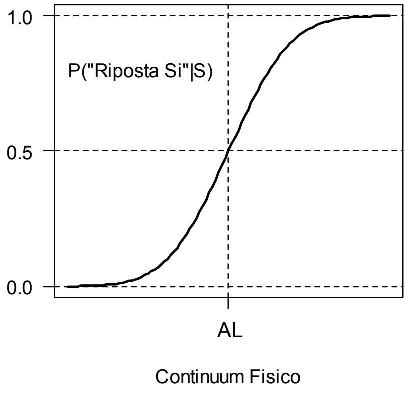
"Classic" threshold theory



Theoretical response matrix

	No	Yes
N	1	0
SN	1-p	p

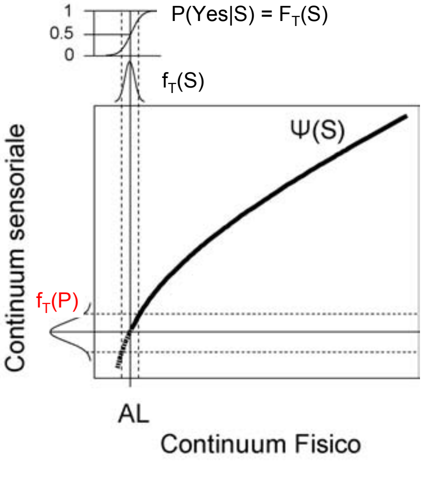
p depends on stimulus intensity



Pr(R=Yes | S) = p

- In reality, subjects exhibit a probabilistic behavior: When the stimulus is near the threshold, the response of the subject vary from trial to trial when presented several times with the same stimulus. In general, the proportion of positive responses increase progressively and monotonicaly with the stimulus intensity, from 0 when the stimulus is clearly below threshold (infraliminal) to 1 when the stimulus is cleary above threshod (supraliminal).
- The **emprical threshold** (absolute limen) is defined as the stimulus value that elicits 50% of positive response

Random threshold



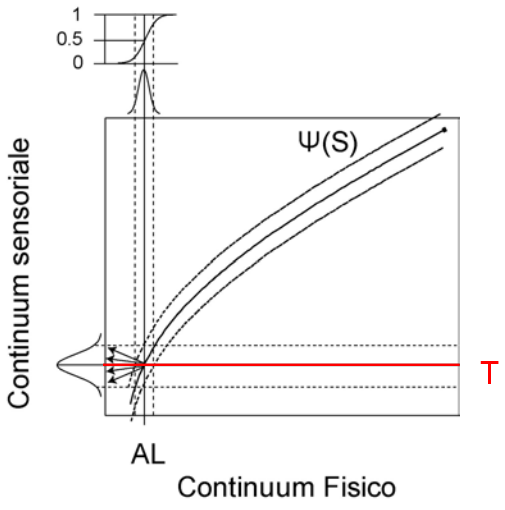
- Classically, this probabilistic behaviour is explain in terms of random variation of the threshold T along the sensory continuim. Often, it is assumed that the threshold has a normal distribution $f_T(P)$.
- The relationship between the distribution of the threshold along the sensory continuum and the psychometric function depends on the slope of the psychophysical function near the threshold

$$\Pr(\text{"Yes"} \mid S) = F_T(S) = \int_{-\infty}^{S} f_T(s)\,ds$$

where

$$f_T(S) = f_T(P)\left(\frac{d\psi}{dS}\right)^{-1}$$

Discriminal process



- An alternative explanation for the probalistic behavior of the subject is that the threshold T is fixed but the effect P of the stimulus S on the central nervous system varies randomly around a central value. In other words, the stimulus is internaly represented by a distribution (“discriminal process” according to Thurstone’s terminology).
- The two explanations cannot be distinguished experimentally.

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Response bias and false alarms

- Early on, experimenters realized that there were problems with threshold theory:
  1. **Response biases**: There are differences between observers that appear *unrelated with the strenght of the stimulus*. For example, cautious observers need more evidence to respond positively, they have a negative response bias.
  2. **False alarms**: Observers respond can respond positively in absence of the stimulus (**blank trials**). This observation is difficult to interpet if one assumes that activity is entirely caused by the stimulus.
- The challenge is double:
  - How to handle individual response biases at the practical level?
  - How to account for positive respones in blank trials, individual biases, and the influence of non-sensory factors at the theoretical level?

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Eliminating the response bias

- One approach to handle these problems was to try to eliminate biases
  1) Using expert subjects. However this approach was not very successful because it remains difficult to know whether subjects respond really without bias in detection experiments.
  2) Giving feedback to the subjects at the end of each trial. However, there is the risk that the subject's threshold changes during the experiment (perceptual learning) as a result of this procedure.
  3) Using tasks where the subject is less likely to exhibit bias such as the Standard Two Alternative Forced Choice Task.

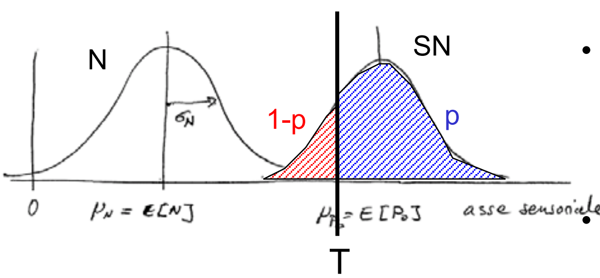
- An alternative approach is to take into account the bias at theoertical level and possibly derivate measures of the sensitivity of the sensory system that are not influenced by it

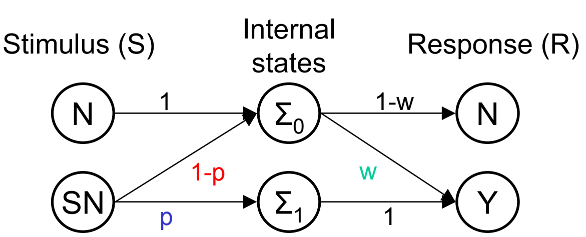
  → **Blackwell's high-treshold theory** (correction for chance)

  → Luce's low treshold theory

  → **Signal detection theory**

Blackwell's high-threshold theory





Theoretical response matrix

P(R|S)	No	Yes
N	1-w	w
SN	(1-p)(1-w)	(1-p)w+p

- Let's represent the sensation during the blank and real trials by two gaussian distributions along the sensory continuum.
- The high-threshold theory states that the threshold must be at least three standard deviations above the mean of the noise distribution so that it is reasonable to assume that the threshold is never exceeded during the presentation of blank trials ($P(\Sigma_1|N)=0$).
- The high-threshold theory assumes that the observer in state Σ_0 responds positively in a small percentage of trials ($P(Y|\Sigma_0)=w$)
- $p = P(\Sigma_1|SN)$ is the true probability of detecting a stimulus.

The correction for chance

P(Σ|S)	Σ_0	Σ_1
N	1	0
SN	1-*p*	*p*

P(R|Σ)	N	Y
Σ_0	1-w	w
Σ_1	0	*1*

- Probability of a positive answer in a blank trial:

$$P(Y|N) = P(Y|\Sigma_0)\,P(\Sigma_0|N) + P(Y|\Sigma_1)\,P(\Sigma_1|N)$$
$$= w\times 1 + 1\times 0 = w$$

The weight w is the proportion of False Alarms P("Yes"|N).

- Probability of a positive response In the case of a real presentation,

$$P(Y|SN) = P(Y|\Sigma_0)\,P(\Sigma_0|SN) + P(Y|\Sigma_1)\,P(\Sigma_1|SN)$$
$$= w \times (1-p) + 1 \times p$$

- The correction for chance: Substitutin w_A with P("Yes" | Σ_0) yields

$$P(\text{"Yes"}|SN) = P(\text{"Yes"}|N)\,(1-p) + p$$

that is

$$p = \frac{P(\text{"Yes"}|SN) - P(\text{"Yes"}|N)}{1 - P(\text{"Yes"}|N)}$$

where p = P(Σ_1|SN) is the true probability of detecting a stimulus. It can be computed from the response matrix. This formula, known as the correction for change, eliminates some positive answers due to the non-zero probability P("Yes"|SN) in the computation of the "true" probability *p* of detectinng the stimulus which should be used to compute the threshold.

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Testing the high-threshold theory

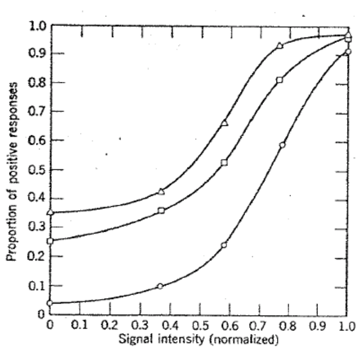

FIG. 5-2*a* Psychometric functions showing false-alarm proportions of 0.35, 0.25, and 0.04.

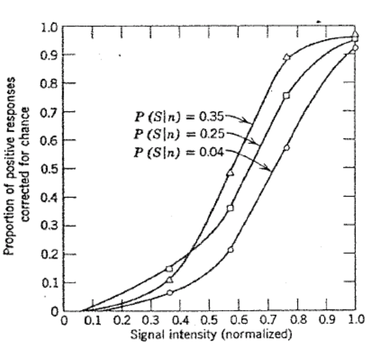

FIG. 5-2*b* The psychometric functions of Fig. 5-2*a* corrected for chance success, by Eq. 5.2.

- The plot on the left shows the psychometric functions measured in three conditions where the false alarm rate was manipulated by the experimenter. The plot on the right show the same psychometric function corrected for chance. If the high-threshold theory was correct, the three curves should overlap exactly, which is not the case.

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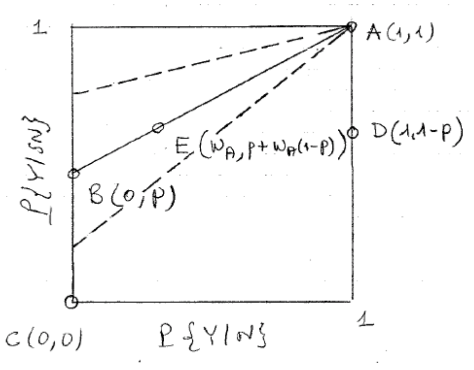
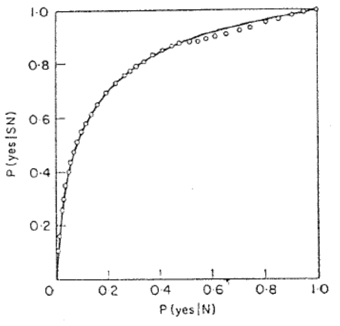
Testing the high-threshold theory





- The high-threshold theory predicts a linear relationship between the hit rate (P("Yes"|SN) and the false alarm rate P("Yes"|N) :

$$P(\text{"Yes"}|SN) = P(\text{"Yes"}|N)\ (1-p) + p$$

- Experiments have shown that this relation is not linear. The high-treshold theory does not explain well the data
- There are other theories such as Luce's low threshold theory (1963) or von Bekesy's quantal theory (1930) that we will not discuss here.

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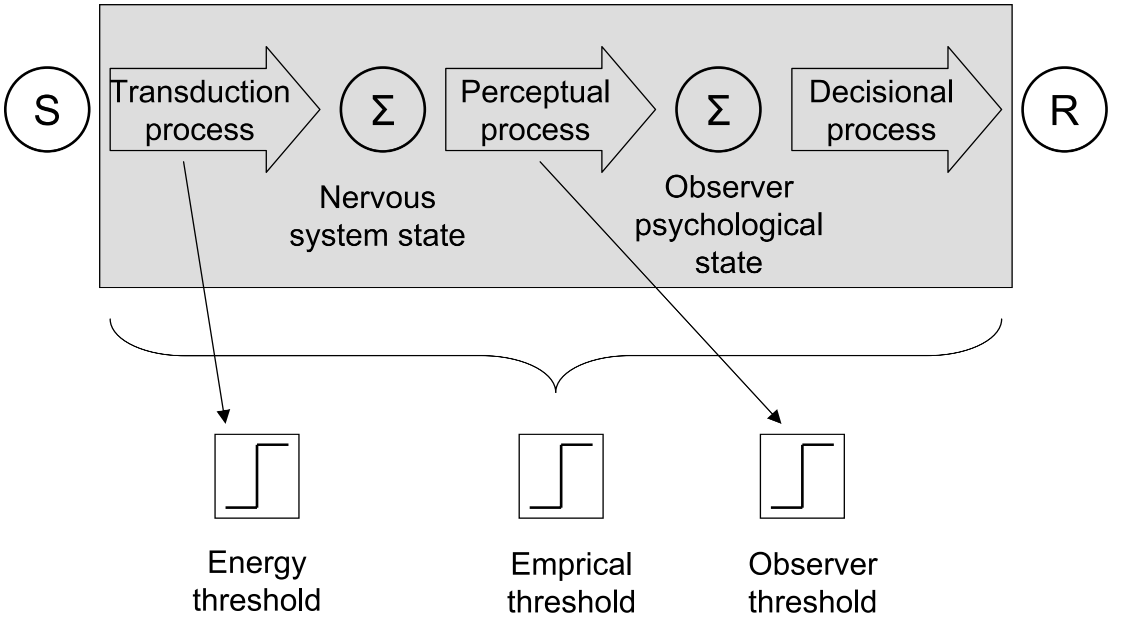
A modern view of thresholds



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Energy threshold

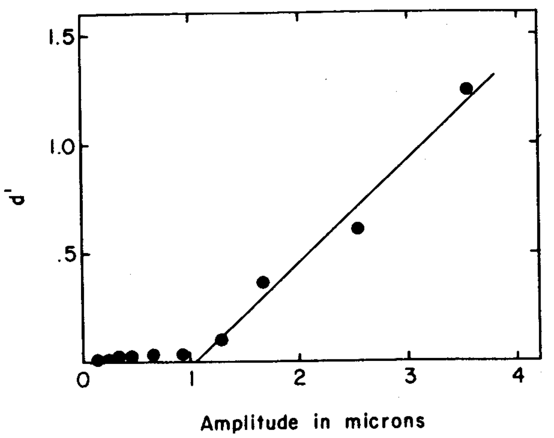


FIG. 6.9. The relation between *d'* and signal amplitude for detecting 60-Hz vibration on the fingertip. The data indicate an energy threshold corresponding to a signal amplitude of 1.0-μm peak-to-peak displacement of the stimulator. (From Gescheider, Wright, Weber, & Barton, 1971.)

Signal detection theory

Signal detection theory

- Signal Detection Theory was developed in 1950s by engineers and mathematicians. Its use in Psychophysics is due to Tanner, Swets and colleagues.
- SDT assumes the existences of a sensory and decision processes but no threshold. Each process is characterized by one parameter:
  - The ***sensitivity*** for the sensory process
  - The ***bias*** or ***response criterium*** for the decision process
- Definition of the sensitiviy and response criterium depends on the task.

Psychophysical Procedures

Yes-No Procedure

- The observer must say whether the single stimulus on each trial was a signal or noise.

- Rating Procedure
  - The observer is presented with a single stimulus and is instructed to rate on a n-point scale the likelihood (confidence) that the observation was caused by the signal
- Force-Choice Procedure (n-Alternative Force Choice Task)
  - The signal is presented in one of two or more intervals (usually temporal, but sometimes spatial) and the observer is instructed to select the interval that he believes most likely to have contained the signal.
- The rating and forced-choice procedures require some adjustments to the formulae used to compute the sensitivity and response criterion. See Chapter 2 and Appendix III of Green & Swets (1966) for details of experimental procedures .

The yes-no detection task

- The starting point of detection theory is the analysis yes-no detection task, were the experimenter presents either the stimulus (signal) or nothing (noise). The observer must decide whether the stimulus was present or not.

Stimulus	Response	
	"Yes"	"No"
Present (SN) (Signal+Noise)	P("Yes" |SN) (Hit)	P("No"|SN) (Miss)
Absence (N) (Noise)	P("Yes"| N) (False alarm)	P("No"|N) (Correct rejection)

- An observer can make to type of errors:
  - Respond positively when the signal is absent (**False alarm**)
  - Respond negatively when the signal is present (**Miss**)

Sensory processes

- In SDT, the sensory process are assumed to be noisy. The activiation along the sensory continuum in absence of stimulus reflects the noise (N) properties. When the signal (stimulus) is present, the signal is combined with noise (hence the notation SN).

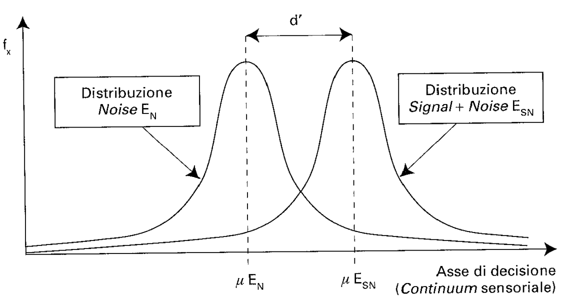


Figura 3.1

- This idea is similar to Thurstone's concept of "discriminal dispersion" that the effects of stimulation can be represented by a random variable. The stimulus has a mean value on the relevant sensorial or psychological scale but its exact value fluctuates from moment to moment.

- The distance between the two curves corresponds to the "sensitivity" of the sensory system.

$$d' = \frac{\mu_{SN} - \mu_N}{\sigma}$$

Sensitivity (d')

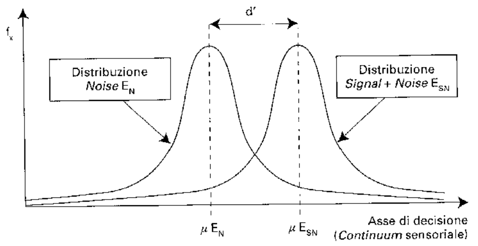
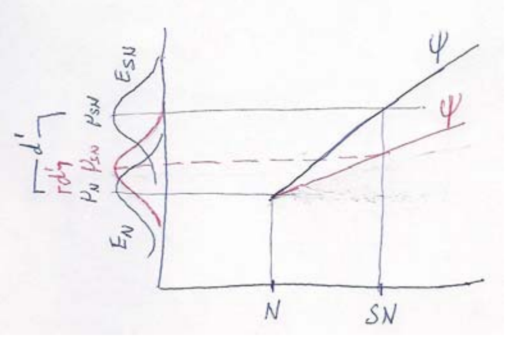


Figura 3.1



- Following Thurstone, it is often assumed that the noise and signal-plus-noise distributions are gaussian and that both the noise and the signal have the same variance (discriminal dispersion $\sigma=\sigma_N=\sigma_{SN}$).
- The distance d' between the two curves corresponds to the "sensitivity" of the sensory system. Conventionally, the distance d' is expressed in standard deviation unit

$$d' = \frac{\mu_{SN} - \mu_N}{\sigma}$$

- Like the difference threshold, d' depends on the slope of the psychophysical function. A larger d' (in blue) correspond to a more sensitive sensory process (steeper psychophysical function)

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The decision criterion (c)

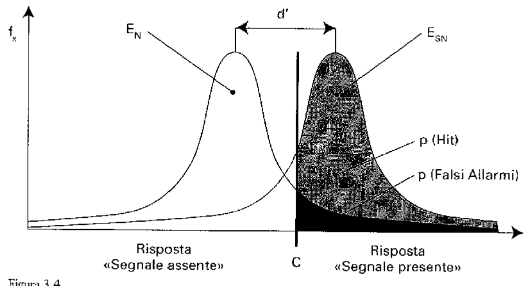

Figura 3.4

$$\text{Response} = \begin{cases} Yes & if \quad e \geq c \\ No & if \quad e < c \end{cases}$$

- The SDT assumes that the observer responds positively if the sensation (or state of excitation *e* of the sensory system) is above some value c and negatively otherwise. The value c is the decision criterion is under control of the subject who can adjust it to maximize, for example, the number of correct responses, or to maximize the number of hits, or to minimize the number of false alarm.
- The optimal value of c depends on the context.
- In classical psychophysical theory, the value *c* corresponds to the threshold and is thought as being a fixed property of the sensory system

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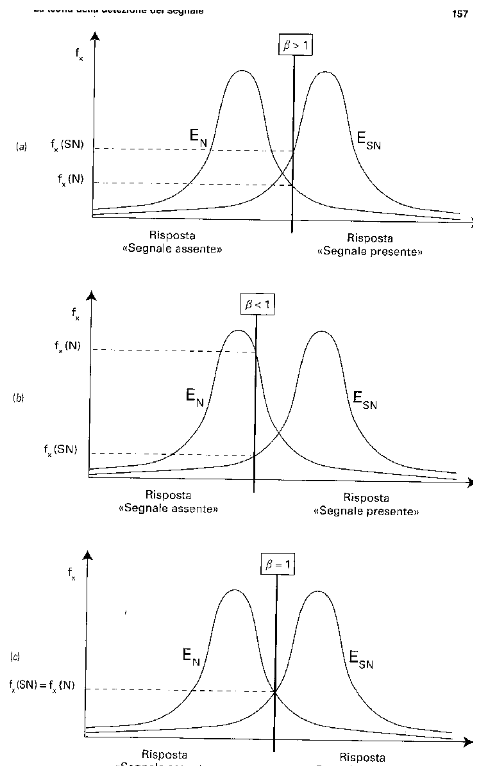
The bias



- The bias is a measure that is closely related to the decision criterion.

$$\beta = LR = \frac{f_{SN}(c)}{f_N(c)}$$

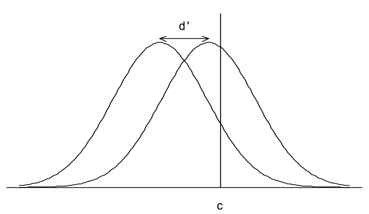
- It corresponds to the ratio of the values of the gaussian probability density functions that represent the signal and the noise at the point c
- If the noise and the signal are gaussian and have the same variance, then

$$\log \beta = d'c$$

Computation of d' and c

Stimulus-Response Matrix		Response	
		Yes	No
Signal	SN	0.4	0.6
	N	0.1	0.9

z-transform		Response	
		Yes	No
Signal	SN	-0.253	0.253
	N	-1.282	1.282



$$d' = -0.253 - (-1.282) = 1.029$$

$$c = -\frac{-0.253 - 1.282}{2} = 0.7675$$

$$\beta = \frac{0.386}{0.175} = 2.206$$

$$\log \beta = d'c = 0.79$$

- When the two noise and signal-plus-noise distributions are gaussian and have the same variance ($\sigma_N = \sigma_{SN}$):

$$d' = z_H - z_{FA}$$

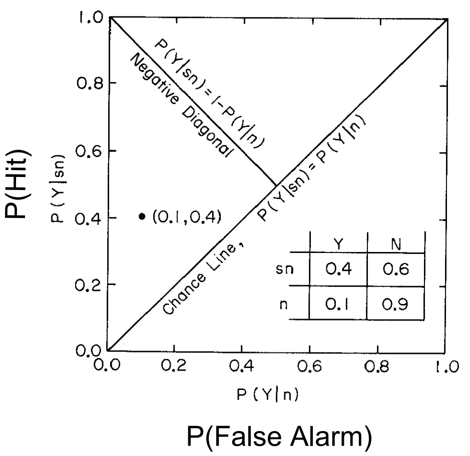
$$c = -\frac{z_H + z_{FA}}{2}$$

- This definition is valid only for the yes-no detection task under the equal variance assumption.

The ROC graph

- The results of a yes-no detection task conducted with one stimulus (SN) and blank trials (N) is ofte represented by a point the ROC graph, with the false the false-alarm rate along the horizontal axis and the hit rate along the vertical axis.

- **The position of the point on this graph reflects the behaviours of the observer:**



1. The top left angle of the graph corresponds to an observer who detects the stimulus perfectly, i.e., P(Hit)=P(Y|SN)=1 and P(False Alarm)=P(Y|N)=0.
2. When the observer answers randomly, P(Y|SN)=P(Y|N), the point must be on the main diagonal where P(Hit) = P(False Alarm). For this reason, the main diagonal is called the "chance line".
3. The point on the bottom left (P(Hit) = P(FA) = 0) corresponds to an observer who responds always negatively and the point on the top right (P(Hit) = P(FA) = 1) corresponds to an observers who responds always positively

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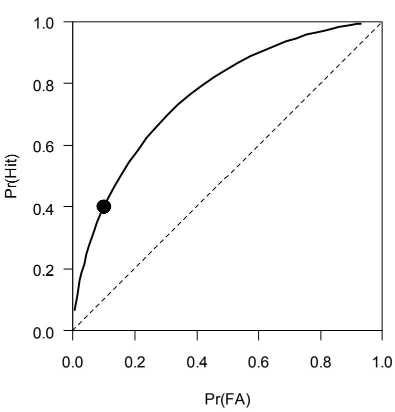
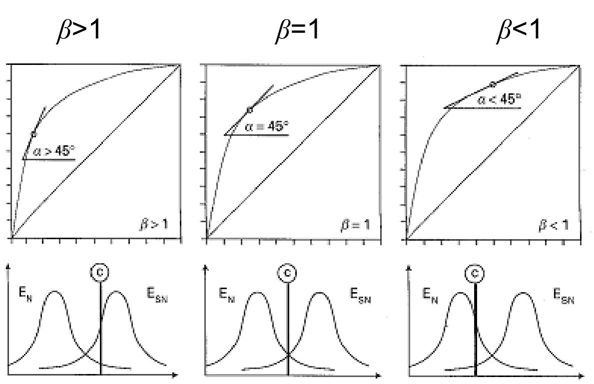
The ROC curve

- The ROC curve shows how the proportions of hits and false alarms vary as a function of the criteria of responses for a fixed d'.



Example. ROC curve for d'=1.029 when c varies from –2.0 to +2.0 ($\sigma_N=\sigma_{SN}=1$).



- It is possible to show that β is equal to the slope of the ROC curve for a given response criterion

- At the experimental level, a ROC curve is obtained by manipulating the response bias of the observer.

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Manipulating the response bias

- Example of non-sensory factors that influence observer's responses:

1. **Verbal instructions**
   - use "lax", "medium" or "strict" criterion (Egan, Schylmna, & Greenberg, 1959): "strict" means "press the yes key only when you are certain that the signal was presented", "lax" means press the yes key each time there was the slightest indication that a signal was presented".
2. **Cost and rewards**
   - Varying the costs or rewards associated with the correct detection of the signal
3. **A-priori probability of signal occurrence**

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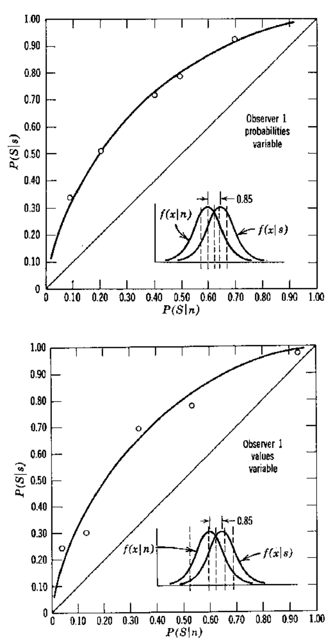
Manipulation of the reponse criterion

- SDT is validated by experimental results showing that manipulation of non-sensory factors change only the value of the response criterion



Tanner, Swets, Green (1956)

- Auditory experiment: Detection of a tone burst in a background white noise.
- 600 trials/point
- Top plot: **Variation of a priori probability** of signal (p=0.1,0.3,0.5,0.7,0.9 for points from the left to the right)
- Bottom plot: **Manipulation of rewards** (points from left to right correspond to an increase of the reward for a correct response when signal was presented relative the a correct response when the signal was absent)
- For both plots, the solid line shows the theoretical curve assuming normal probability functions of equal variance.
- The d'=0.85 (under the assumption that the standard deviation of the noise distribution is unity)

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Rating procedure

- The rating procedure is usefull to obtain ROC curves faster than by repeating the experiment under various conditions where the response criterium is manipulated.
- In the rating procedure, the observer is presented with a single stimulus (signal or blank trial) and is instructed to rate on a n-point scale the likelihood (confidence) that the observation was caused by the signal.

Stimulus response matrix (6-point scale)

P(1|SN)	P(2|SN)	P(3|SN)	P(4|SN)	P(5|SN)	P(6|SN)
P(1|N)	P(2|N)	P(3|N)	P(4|N)	P(5|N)	P(6|N)

- The various rating are assumed to correspond to different criteria and can be used to define ROC curve (see Gescheider, p. 150).
- The d' can be computed by pooling together the three first and three last responses.

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ROC curves and d'

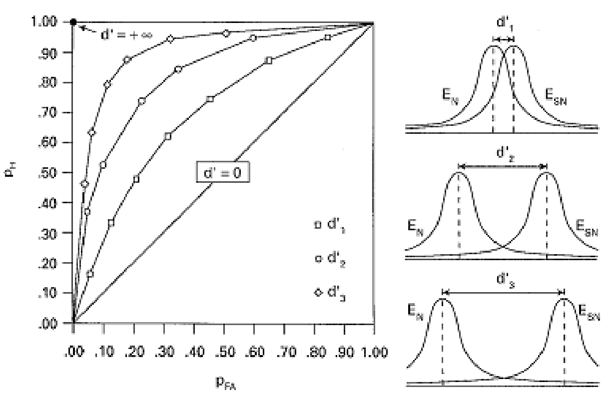

Figura 3.12

- The distance between the two distributions is expected to increase when a more intense signal is used
- When the two distributions are most distant (larger d'), the probability of a Hit can increase and the probability of a False Alarm can decrease at the same time.
- Accordingly, the ROC curve is more distant to the main diagonal (chance line) which corresponds to two overlapping distribution (d'=0)

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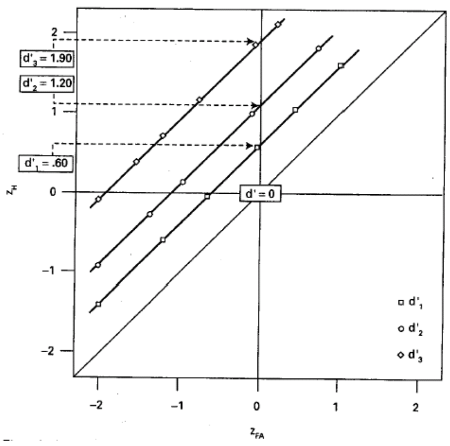
Rectified ROC curves

The rectified ROC graph plot the z values corresponding to the false alarm and hit rates. If the underlying distribution are gaussians, the rectified ROC curves should be straight lines.



- When the variability of the noise (σ_N) and signal-plus-noise (σ_{SN}) distributions are equal, then the rectified ROC :

1. the slope of the rectified ROC is parallel to the diagonal (slope = 1)

$$b = \frac{\sigma_N}{\sigma_{SN}} = 1$$

2. the intercept corresponds to d'

$$a = d'$$

In experiments where the response criterium has been manipulated, it is common to fit a straight line to compute the d' and to test the equality of variance hypothesis (see next slides).

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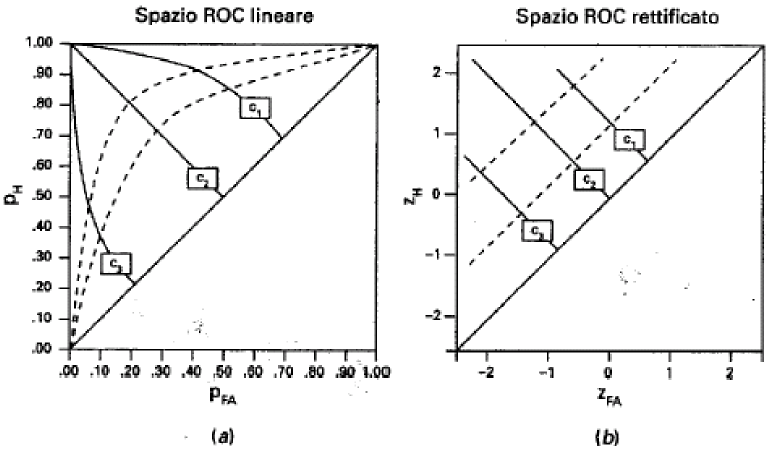
Isocriterion ROC curves



- The solid line in the figure on the left represents an ***isocriterion curve***, i.e., a curve that has been obtained by varying the intensity of the signal for a fixed response criterion.
- The z-transformed isocriterion ROC curves are also straight lines which are perpendicular to the diagonal (chance linec when variances of E_N and E_{SN} are equals ($\sigma_N = \sigma_{SN}$).

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d’ and the psychometric function

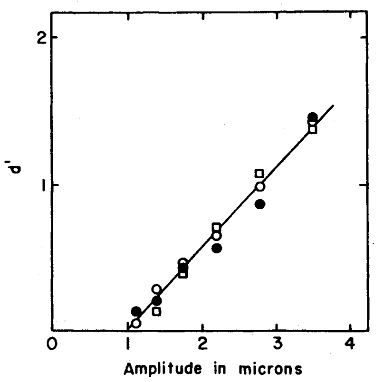

FIG. 7.8. The relation between *d'* and vibration amplitude. The open and filled points represent *d'* values obtained from the yes–no procedure when the observer's criterion was low and high, respectively. The squares are *d'* values from the rating procedure. (From Gescheider, Wright, & Polak, 1971.)

- The psychometric function is a probability of detecting the signal P(*"Yes"*|*S*) as a function of the stimulus intensity *S*.
- In the SDT framework, it is common to plot the sensisitivy parameter d’ against the stimulus intensity.
- The absolute threshold might be (arbitrarily) defined as the stimulus S that yields d’(S) = 1.

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d' and the correction for chance

- In the framework of the SDT, d' can viewed as mode of correcting the probability of detecting the response by taking into account the bias of the observer (false alarms).

$$d'(S) = z(\text{Hit}) - z(\text{FA}) = \Phi^{-1}(P(\text{Yes}|S)) - \Phi^{-1}(P(\text{Yes}|N))$$

- Note that this formula where false alarm rates are substracte is akin to the previously seen "correction for guessing" but done between *z* scores instead of probabilities:

$$p = \frac{P(\text{"}Yes\text{"}|\,SN) - P(\text{"}Yes\text{"}|\,N)}{1 - P(\text{"}Yes\text{"}|\,N)}$$

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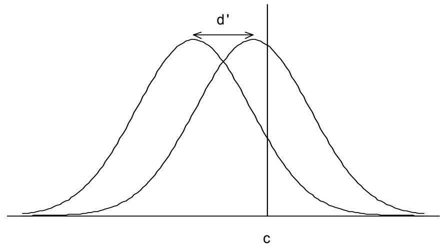
Optimal response criterion

- What is the optimal value for the response criterion? The response depends on how optimal is defined:
  1. Maximize number of correct responses.
  2. Maximize the expected gain.
  3. Maximize the number of hits while maintaining the number of false alarms under a predetermined value (Neuman-Pearson objective).



- In the following example, we will assume that the two underlying gaussian distributions with the same variance ($\sigma_N=\sigma_{SN}=1$) and that d'=μ_{SN}-μ_N=1.029 (i.e., μ_N=-d'/2 and μ_{SN}=d'/2).

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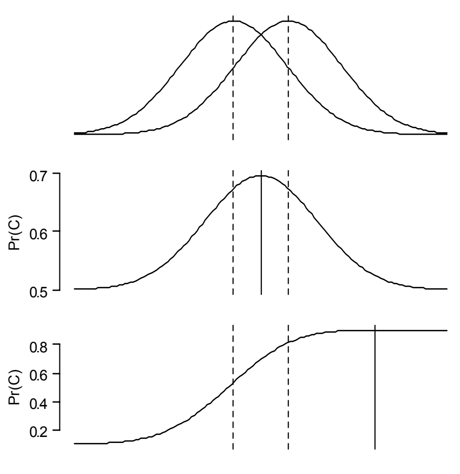
Maximizing correct responses



The two lower plots represent the probability of a correct response P(C) as a function of c. The middle plot assume P(SN)=0.5 and the lower plot assume P(SN)=0.1.

- In a detection task, the observer responds correctly when he responds positively when the signal is present and negatively when the signal is absent.
- The probability of responding correctly depends on the probability of a signal P(SN), i.e. the proportions of trials in which the signal was present.

$$P(C) = P\big((\text{Yes and } SN) \text{ } or \text{ } (\text{No and } N)\big)$$
$$= P(\text{Yes} \mid SN)P(SN) + P(\text{No} \mid N)P(N)$$

Note that $\Pr(N) = 1 - \Pr(SN)$

- The optimal bias and response criterion depend on the probabilities P(SN) and P(N):

$$\beta_0 = \frac{\Pr(N)}{\Pr(SN)}$$

$$c = \frac{\log(\beta_0)}{d'}$$

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Payoff matrix and expected gain

- To make a rational decision, it is also often necessary to know what is the gain or cost associated with all possible outcomes.
- The payoff matrix specify the gains and costs associated with a correct response or an error:

Pay-Off Matrix		Response	
		Yes	No
Signal	SN	V_H	V_M
	N	V_{FA}	F_{CR}

- The values V_X in the payoff matrix can be either positive (gains) or negative (costs).

- The expected gain E[G] is:

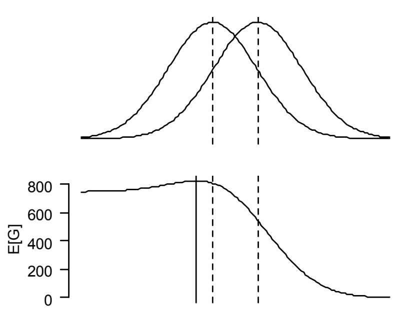
$$\begin{aligned} \mathrm{E[G]} &= V_H \Pr(\mathit{Yes} \text{ and } SN) + V_{FA} \Pr(\mathit{Yes} \text{ and } N) + V_M \Pr(\mathit{No} \text{ and } SN) + V_{CR} \Pr(\mathit{No} \text{ and } N) \\ &= V_H \Pr(\mathit{Yes} \mid SN)\Pr(SN) + V_{FA} \Pr(\mathit{Yes} \mid N)\Pr(N) + V_M \Pr(\mathit{No} \mid SN)\Pr(SN) + V_{CR} \Pr(\mathit{No} \mid N)\Pr(N) \end{aligned}$$

- The value of the bias that maximize the expected gain is

$$\beta_0 = \frac{(V_{CR} - V_{FA})\Pr(N)}{(V_H - V_M)\Pr(SN)} \qquad c = \frac{\log(\beta_0)}{d'}$$

Maximizing the expected gain

Payoff matrix		Response	
		Yes	No
Signal	SN	+2000	-500
	N	-500	+500



The bottom figure show the expected gain E[G] as a function of the response criterion c.

- The value of the bias that maximize the expected gain is

$$\beta_0 = \frac{(V_{CR} - V_{FA})\Pr(N)}{(V_H - V_M)\Pr(SN)}$$

- The optimal value depnds on the a priori probabilitues P(SN) and P(N) and on the values in the payoff matrix

- The corresponding value for the response criterion can be derived from the bias as previously.

$$c = \frac{\log(\beta_0)}{d'}$$

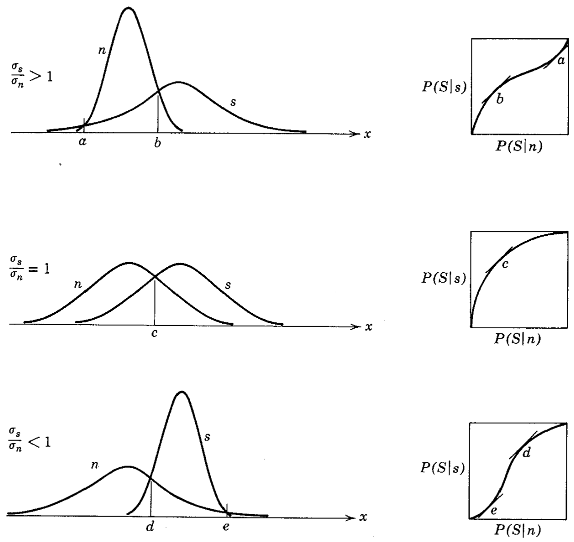
- **Exercise.** Compute β_0 and c using the payoff matrix on the left and assuming P(SN)=0.5.

$$\beta_0 = \frac{1000 \times 0.5}{2500 \times 0.5} = 0.4$$

$$c = \frac{\log(0.4)}{1.029} = \text{-}0.891$$

Unequal variance case ($\sigma_{SN} \neq \sigma_N$)

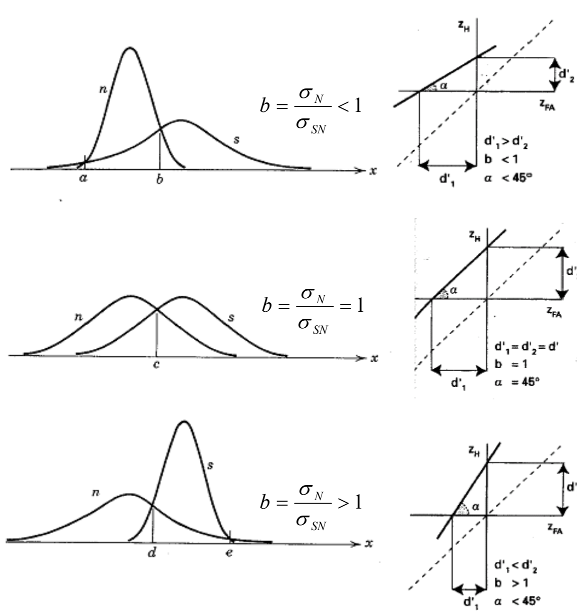
- The shape of the ROC curve depend on the relative size of the variances of the signal and of the noise. The ROC curve is symmetric about the inverse diagonal if and only if the signal and the noise have the same variance.



- Case $\sigma_{SN}/\sigma_N > 1$: variance of signal larger than variance of noise
- Case $\sigma_{SN}/\sigma_N = 1$:

  => ROC curves are symmetric about the inverse diagonal
- Case $\sigma_{SN}/\sigma_N < 1$

Testing equality of variance



- If the distributions are gaussians, then the rectified ROC curve becomes straight line:

$$z_H = a + b\, z_{FA}$$

where

$$a = \frac{\mu_{SN} - \mu_N}{\sigma_{SN}}$$

$$b = \frac{\sigma_N}{\sigma_{SN}}$$

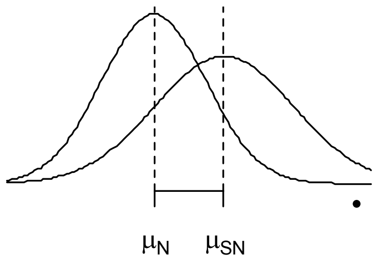
- The slope of the rectified ROC curve is 1 if and only if the two variances are equal.
- Proof: By definition $d' = z_H - z_{FA}$ when $\sigma_N = \sigma_{SN}$, therefore

$$z_H = d' + (1)\, z_{FA}$$

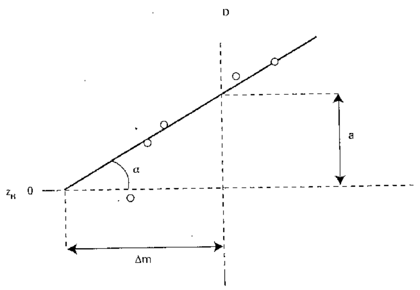
which implies a = d' and b = 1.

- It is possible to test whether the slope of the rectified ROC curve is different from 1 to test the assumption that both variances are equal.

Sensitivy parameter ($\sigma_{SN} \neq \sigma_N$)



- When the variances of the noise (σ_N) and of the signal-plus-noise (σ_{SN}) distributions are different, it is possible to define the sensitivity index d' in various ways according to the unit in which to express it.
- One possibility would be to use the intercept of the rectified ROC curve as index of sensitivity



$$a = \frac{\mu_{SN} - \mu_N}{\sigma_{SN}}$$

The intercept corresponds to the distance between the two means in the unit of the standard deviation (σ_{SN}) of the signal-plus-noise distribution.

However, it is more common to use the indices Δm, d_a, or d_e'.

Δm, d_a, d'_e

- The first indice of sensitifity corresponds to the distance between the two means when it is expressed in the unit of the noise distribution σ_N

$$\Delta m = \frac{\mu_{SN} - \mu_N}{\sigma_N} = \frac{a}{b}$$

- The index of sensitifity d_a corresponds to the distance between the two means expressed in the unit of the pooled variance

$$d_a = \frac{\mu_{SN} - \mu_N}{\sqrt{\frac{\sigma_{SN}^2 + \sigma_N^2}{2}}} = \frac{|a|}{\sqrt{\frac{1+b^2}{2}}}$$

- The index of sensitifity d_e' also gives equal weight to the units of the noise and signal-plus-nois distributions:

$$d'_e = \frac{\mu_{SN} - \mu_N}{\left(\frac{\sigma_N + \sigma_{SN}}{2}\right)} = 2\left|\frac{a}{b+1}\right|$$

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Response Criteria ($\sigma_{SN} \neq \sigma_N$)

- Equal variance $c = -0.5\,(z_H + z_{FA})$

- Unequal variances

$$c_e = -\left(\frac{2b}{(1+b)^2}\right)(z_H + z_{FA})$$

$$c_a = -\left(\frac{b\sqrt{2}}{(1+b)\sqrt{1+b^2}}\right)(z_H + z_{FA})$$

Note that both unequal variance response criteria are equivalent to the equal variance response criterion when the variance of EN and ESN are equal (b=1).

2AFC detection task

- **Two alternative forced choice task**: The experimenter presents the stimulus during one of two intervals (time-separated presentations) or at one of two possible locations (space-separated presentations) and the subject must indicate in which interval or location the stimulus is present.

  The position of the stimulus in space or in time must be randomized between trials.

- Unlike the yes-no task, the 2AFC task is little influenced by the response bias because the response is based on the *comparison* of two stimuli. Any bias present in the evaluation of the magnitude of a stimulus will affect both stimuli equally and is thus cancelled when the two stimuli are compared.

2AFC Response Matrix and d'

2AFC Response Matrix

Stimulus	Response	
	Signal First	Signal Second
<Signal, Noise>	P(First |<S,N>) (correct)	P(Second|<S,N>)) (incorrect)
<Noise, Signal>	P(First|<N,S>) (incorrect)	P(Second|<N,S>) (correct)

- Sensitivity parameter in the 2AFC

$$d' = \frac{1}{\sqrt{2}}[z_1 - z_2]$$ where $$z_1 = \Phi^{-1}(\mathrm{P(First} \,|< \mathrm{S,N} >)) \quad z_2 = \Phi^{-1}(\mathrm{P(Second} \,|< \mathrm{S,N} >))$$

- Bias is computed in the same manner as the Yes-No task

$$c = 0.5[z_1 + z_2]$$

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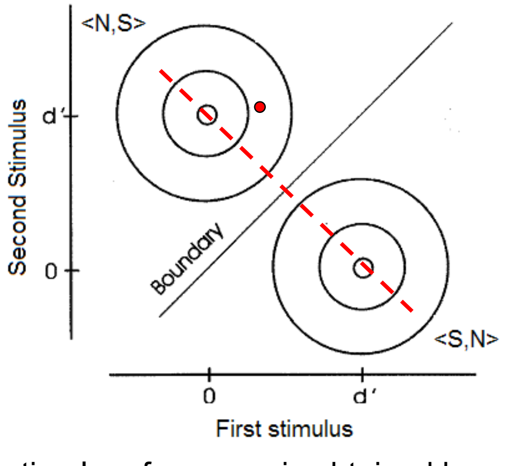
Analysis of 2AFC task

- Let d' be the distance between the noise and signal. We also assume that the two stimuli are evaluated separately. The internal effect of a single experimental trial is a point in a two-dimensional plane.



- For a trial <N,S>, the expected level excitation is represented by a bivariate normal distribution centered on <0,d'>
- For a trial <S,N>, the expected level excitation is represented by a bivariate normal distribution centered on <d', 0>

- Optimal performance is obtained by projecting the internal effect on the decision axis that connects the two bivariate distribution.
- On this axis, the two distributions are separated by √2 d' which also the value estimated by computing $z_1 - z_2$. Therefore, on must divide $z_1 - z_2$ by √2 to obtain d'.

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Some remarks on 2AFC

- Green & Swets (1966, pp. 45 ff) have demonstrated that the proportion of correct in 2AFC by an *unbiased* observer equals the area under the yes-no isosentivity curve. Thus, it is possible to transform the proportion of correct in a d' prime measure if one assumes that the observed is unbiased:

$$d' = \sqrt{2}\Phi^{-1}\left(P(C)\right)$$

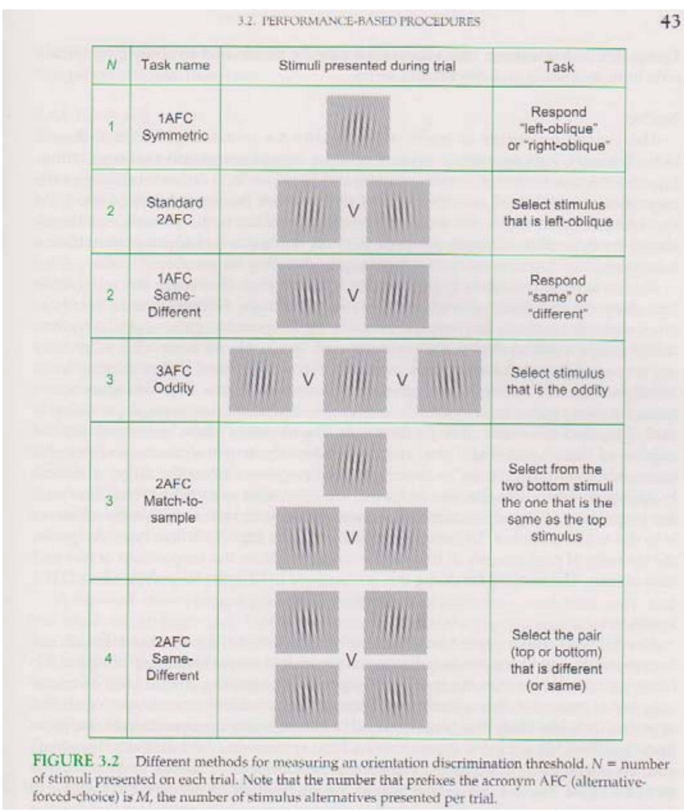
  This result is sometimes cited as a justification for using the proportion of correct as a measure of discrimination performance (see MacMillan & Creelman, 2005).

- Several studies have found similar estimate for d' in auditory and visual Yes-No and 2AFC *detection* tasks. However, this seems not to be the case in *discrimination* tasks, where the 2AFC was a factor of 2, rather than √2, better than Yes-No.

- When time separate the two intervals, there is tendency to overestimate the stimulus in the second interval, which has been interpreted to show decay of a central representation of the stimulus over time.

- The interstimulus interval has also been found to influence the sensitivity.

Other SDT tasks



3.2. PERFORMANCE-BASED PROCEDURES 43

N	Task name	Stimuli presented during trial	Task
1	1AFC Symmetric		Respond "left-oblique" or "right-oblique"
2	Standard 2AFC	V	Select stimulus that is left-oblique
2	1AFC Same-Different	V	Respond "same" or "different"
3	3AFC Oddity	V V	Select stimulus that is the oddity
3	2AFC Match-to-sample	V	Select from the two bottom stimuli the one that is the same as the top stimulus
4	2AFC Same-Different	V	Select the pair (top or bottom) that is different (or same)

FIGURE 3.2 Different methods for measuring an orientation discrimination threshold. N = number of stimuli presented on each trial. Note that the number that prefixes the acronym AFC (alternative-forced-choice) is M, the number of stimulus alternatives presented per trial.

Why use SDT and d'?

- In theory, d' prime is a ***bias-free*** measure of performance.
- In theory, d' is a ***procedure-free*** measure of performance.
  - remember that d' is computed differently for each procedure to take into account difference between them
  - remember that empirical tests did not always confirm the theory (e.g. in discrimination tasks)
- The relation between d' and the stimulus intensity (psychometric function) is often linear

Some Guidelines

- The ***2AFC procedure*** is very popular for two good reasons:
  - the procedure discourage the bias
  - the performance levels as measured by P(C) are high
- A popular alternative to the 2AFC procedure is the ***1AFC symmetric*** procedure
  - computation are essentially like in the Yes-No tasks
  - like 2AFC, the procedure also discourages bias
  - the task is simple for the subject and has a small cognitive load.
- If you use the Yes-No task, it is probably a good idea to use the rating procedure to check the equal-variance assumption.