Robotica Antropomorfa

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Open Questions

- How can we predict the trajectory followed by the system when it is driven by a given force field \( F \)? (Dynamic model of the limb)
- Is there a way of choosing the ‘complete’ set of elementary force fields \( F_k \)? (A trivial solution to the spinal field synthesis problem)
- How should we choose joint torques \( \tau \) so as to obtain a given basis force field \( F_k \)? (The map \( \tau \rightarrow F_k \))
- How do we choose muscle activations so as to obtain a given joint torque?
- Which is the minimum number of elementary force fields that we need to perform a ‘complete’ set of movements?
- Is there a way of choosing the primitives to accommodate different kinematic structures?
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- Is there a way of choosing the primitives to accommodate different kinematic structures?

Control Model of the spinal field experiment

The spinal fields experiment has been modeled in terms of the linear superposition of a finite number of force fields:

\[
F(P, z, \lambda) = \sum_{k=1}^{K} \lambda_k \mathcal{F}_k(P, z)
\]

where \( \mathcal{F}_k \) should be convergent to an equilibrium

Force fields in this model depend also on the velocity of \( P \). This new feature is justified if we want to introduce a certain degree of damping in the system.

Today we use a different notation:

\[
P \leftrightarrow \begin{bmatrix} z \end{bmatrix} \quad \lambda \leftrightarrow \begin{bmatrix} \lambda \end{bmatrix} \quad \mathcal{F}(z, \lambda) = \sum_{k=1}^{K} \lambda_k \mathcal{F}_k(z, \lambda)
\]

Example: 2DOF planar kinematic chain (1/2)

A kinematic chain has the following properties:

- It is composed by \( n \) links \( L_1, \ldots, L_n \)
- \( L_j \) is attached to \( L_{j-1} \) by a 1 DOF rotational joint (non restrictive assumption)
- the joint angle (between \( L_{j-1} \) and \( L_j \)) is denoted \( \theta_j \)
- the end-effector position will be denoted \( z \) and belongs to an \( m \)-dimensional space, with \( m \leq n \).

\[
q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \quad \text{Vector of joint angles} \quad z = \begin{bmatrix} x_P \\ y_P \end{bmatrix} \quad \text{Vector of end-effector position}
\]
Example: 2DOF planar kinematic chain (2/2)

- Direct kinematics:

\[
\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \tau
\]

where

\[
L(q, \dot{q}) = K(q, \dot{q}) - V(q)
\]

**Inertia Matrix**

**Coriolis matrix**

**Gravity effect**

- Jacobian:

\[
\begin{bmatrix}
\dot{x}_P \\
\dot{y}_P
\end{bmatrix} = \begin{bmatrix}
x_P \\
y_P
\end{bmatrix} = A(q)
\]

**Example: dynamics of 2DOF planar chain (1/4)**

- Computing the velocities:

\[
\begin{bmatrix}
x_{m1} \\
y_{m1}
\end{bmatrix} = \begin{bmatrix}
x_{m2} \\
y_{m2}
\end{bmatrix} = \begin{bmatrix}
x_{m3} \\
y_{m3}
\end{bmatrix} = \begin{bmatrix}
x_{m4} \\
y_{m4}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{x}_{m1} \\
\dot{y}_{m1}
\end{bmatrix} = \begin{bmatrix}
\dot{x}_{m2} \\
\dot{y}_{m2}
\end{bmatrix} = \begin{bmatrix}
\dot{x}_{m3} \\
\dot{y}_{m3}
\end{bmatrix} = \begin{bmatrix}
\dot{x}_{m4} \\
\dot{y}_{m4}
\end{bmatrix}
\]

**Example: dynamics of 2DOF planar chain (2/4)**

- Dynamic model of the limb (1/3) “repetita iuvant”

- The dynamic model of a kinematic chain describes the map from applied forces to trajectories of the joint variables. Let the applied forces be the vector of applied torques. Then:

\[
\begin{bmatrix}
\tau_1(t) \\
\tau_2(t)
\end{bmatrix} \in \mathbb{R}^2, \quad t \in [t_1, t_2]
\]

Time evolution of the applied torques

Integration of the dynamic model

Time evolution of the joint angles

**Example: dynamics of 2DOF planar chain (3/3)**

- LEMMA: the matrices of the dynamic model of a kinematic chain satisfy the following two properties:

\[
M(q) = M(q)^T > 0
\]

is skew symmetric

A given matrix \(A\) is skew symmetric if and only if:

\[
A = A^T
\]

Follows from the fact that the kinetic energy is 2 zero and equals zero only at rest

\[
M(q) - 2C(q, \dot{q}) > 0
\]

(positive definite) \(C\) implies that in absence of friction the total energy of the system is conserved

**Dynamic model of the limb (2/3) “repetita iuvant”**

- The dynamic model can be computed following the Lagrangian approach:

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \tau
\]

where

\[
L(q, \dot{q}) = K(q, \dot{q}) - V(q)
\]

Kinetic energy

Potential energy

computations

Integration of the dynamic model

Time evolution of the joint angles

**Example: dynamics of 2DOF planar chain (2/4)**

- Computing the velocities:

\[
\begin{bmatrix}
\dot{x}_{m1} \\
\dot{y}_{m1}
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**Dynamic model of the limb (3/3)**

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Example: dynamics of 2DOF planar chain (3/4)

- Kinetic energy:

\[ K(q, \dot{q}) = \frac{1}{2} \left[ \frac{m_l}{l^2} \dot{q}_1^2 + \frac{m_r}{l^2} \dot{q}_2^2 + 2m_l l \dot{q}_1 \dot{q}_2 \right] \]

This matrix will turn out to be the inertia matrix of the system.

- Dynamics:

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = \tau \quad \text{where} \quad L(q, \dot{q}) = K(q, \dot{q}) \]

The state space form of the dynamic equation:

\[ M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = \tau \quad \Rightarrow \quad \dot{x} = f(x) + g(x)u \]

Can be rewritten in the standard state space form:

\[ M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = \tau \quad \Rightarrow \quad \dot{x} = f(x) + g(x)u \]

Where:

\[ f(x) = \left[ \begin{array}{c} \dot{q} \\ -M^{-1}(q)[C(q, \dot{q}) + G(q)] \end{array} \right] \]

PD control of a kinematic chain

Without loss of generality let us assume \( G(q) = 0 \). If this is not the case let us assume that it has been compensated choosing:

\[ \tau = \dot{q} + G(q) \quad \Rightarrow \quad M(q) \ddot{q} + C(q, \dot{q}) \dot{q} = \dot{\tau} \]

- FACT: the following PD (proportional + derivative) control law:

\[ \tau = -K_p(q - q_e) \]

Is such that the corresponding system has a unique equilibrium point \((q_e)\) which is globally asymptotically stable.

- PROOF: (sketch) try to use the following Lyapunov function:

\[ V(q, \dot{q}) = \frac{1}{2} \dot{q}^T M(q) \dot{q} + \frac{1}{2} (q - q_e)^T K_p(q - q_e) \]

and take advantage of the passivity property.

Back to Bizzi’s experiment

In case of non-redundant manipulators, we have the following equivalences:

\[ \tilde{F} \leftrightarrow \tau \quad \tilde{z} \leftrightarrow q \quad \tilde{\xi} \leftrightarrow \dot{q} \]

And therefore the spinal field model can be rewritten as follows:

\[ \tilde{F}(z, \xi) = \sum_{k=1}^{K} \lambda_k \tilde{F}_k(z, \xi) \quad \Rightarrow \quad \tau(q, \dot{q}) = \sum_{k=1}^{K} \lambda_k \tilde{F}_k(q, \dot{q}) \]

PRB: How should we choose the elementary control actions so that:

1. Each elementary controller should drive the system towards a unique (globally asymptotically stable) equilibrium point
2. The combinations of the elementary controllers should be capable of driving the system to any desired configuration

A trivial solution to the synthesis problem (1/2)

HINT:

\[ \tau_i = -K_p(q - q_{d,i}) \quad \text{Convergent to the equilibrium} \]

And impose the following for all admissible \( q_d \):

\[ \sum_{i=1}^{K} \lambda_i \tau_i(q, \dot{q}) = -K_p(q - q_{d}) \]

Which can be rewritten:

\[ \sum_{i=1}^{K} \lambda_i \left( K_p(q - q_{d}) - K_p(q - q_{d,i}) \right) = K_p(q - q_{d}) \]

Which is verified if:

\[ \sum_{i=1}^{K} \lambda_i = 1 \quad \text{and} \quad \sum_{i=1}^{K} \lambda_i q_{d,i} = q_d \]
A trivial solution to the synthesis problem (2/2)

Rewriting the previous expression:

\[
\begin{bmatrix}
\mathbf{q}_{d,1} & \cdots & \mathbf{q}_{d,K}
\end{bmatrix}
\begin{bmatrix}
\lambda_1 & \cdots & \lambda_K
\end{bmatrix} = \begin{bmatrix}
\mathbf{q}_d
\end{bmatrix}
\]

Which has a solution for any \(q_d\) if and only if the matrix on the left has full row rank. This observation gives a criterion to choose the equilibrium point realized by the elementary controls.

Moreover, we can have full row rank only if:

\[
K \geq n + 1
\]

This can be proven to be the minimum number of primitives necessary to control an \(n\)-DOF kinematic chain.

Back to the end-effector space

In the redundant case we can go back:

\[
\tau(q, \dot{q}) = \sum_{k=1}^{K} \lambda_k \tau_k(q, \dot{q}) \quad \Rightarrow \quad \mathbf{F}(z, \dot{z}) = \sum_{k=1}^{K} \lambda_k \mathbf{F}_k(z, \dot{z})
\]

Using the following equations:

- \(z = \mathbf{N}(q) \quad \Rightarrow \quad q = \mathbf{N}^{-1}(z)\)
- \(\dot{z} = \mathbf{J}(q)\dot{q} \quad \Rightarrow \quad \dot{q} = \mathbf{J}^{-1}(q)\dot{z} \quad \Rightarrow \quad \dot{q} = (\mathbf{J}(\mathbf{N}(z)))^{-1}\dot{z}\)
- \(\tau(q, \dot{q}) = \mathbf{J}^T(q)\mathbf{F} \quad \Rightarrow \quad \mathbf{F}(q, \dot{q}) = \mathbf{J}^T(q)\mathbf{r}(q, \dot{q}) \quad \Rightarrow \quad \ldots \quad \Rightarrow \quad \mathbf{F}(z, \dot{z}) = \mathbf{J}^T(\mathbf{N}(z))\mathbf{r}(\mathbf{N}(z), \mathbf{J}(q)\dot{z})
\]

Graphical representation of the fields (2DOF chain)

\(\tau_k(q, \dot{q})\)

Can be graphically represented (null velocities): \(\tau_k(q, \dot{q}) = 0\)

\(\mathbf{F}_k(z, \dot{z})\)

Looks quite different in the Cartesian space: \(\mathbf{F}_k(z, \dot{z})\)

Interested?

Check out my web page
(http://www.dei.unipd.it/~iron)

and have a look at Bizzi Lab web site
(http://web.mit.edu/bcs/bizzilab/)