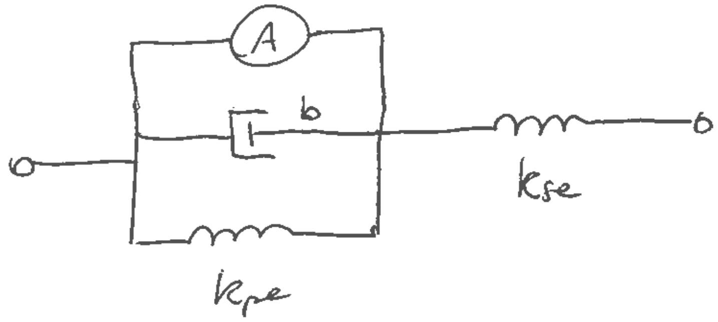


$$x = x_1 + x_2$$

$$x^* = x_1^* + x_2^*$$

$$x - x^* = (x_1 - x_1^*) + (x_2 - x_2^*)$$

(1) $T = k_{se}\,(x_1 - x_1^*)$

(2) $T = k_{pe}\,(x_2 - x_2^*) + b\,\dot{x}_2 + A$

$$T = k_{pe}\left[(x - x^*) - (x_1 - x_1^*)\right] + b\,\dot{x}_2 + A$$

$$= k_{pe}(x - x^*) - k_{pe} T / k_{se} + b\,\dot{x}_2 + A$$

$$= k_{pe}(x - x^*) - k_{pe}/k_{se} \cdot T + b(\dot{x} - \dot{x}_1) + A$$

$\longrightarrow$ deriv. (1)

$$= k_{pe}(x - x^*) - k_{pe}/k_{se} \cdot T + b\dot{x} - b\,\dot{T}/k_{se} + A$$

$$T = k_{pe}(x - x^*) - \frac{k_{pe}}{k_{se}} T + b\dot{x} - b\frac{\dot{T}}{k_{se}} + A$$

$$\downarrow$$

$$\dot{T} = \frac{k_{se}}{b}\left(k_{pe}\Delta x + b\dot{x} - \left(1 + \frac{k_{pe}}{k_{se}}\right)T + A\right)$$

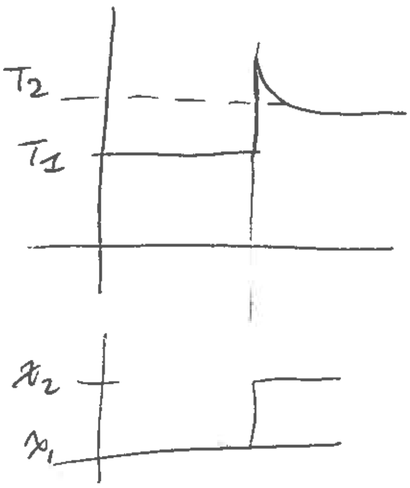
(3)

$$k_{se} \cong \frac{150}{1.1} = 136\ g/cm$$

For k_{pe} ?

$$T = k_{pe}\,(x - x^*) - k_{pe}\,\frac{T}{k_{se}} + b\left(\dot{x} - \frac{\dot{T}}{k_{se}}\right) + A$$

Before stretch :



$$T_1\left(1 + \frac{k_{pe}}{k_{se}}\right) = k_{pe}\left(x_1 - x^*\right) + A$$

$$T_2\left(1 + \frac{k_{pe}}{k_{se}}\right) = k_{pe}\left(x_2 - x^*\right) + A$$

$$\Downarrow$$

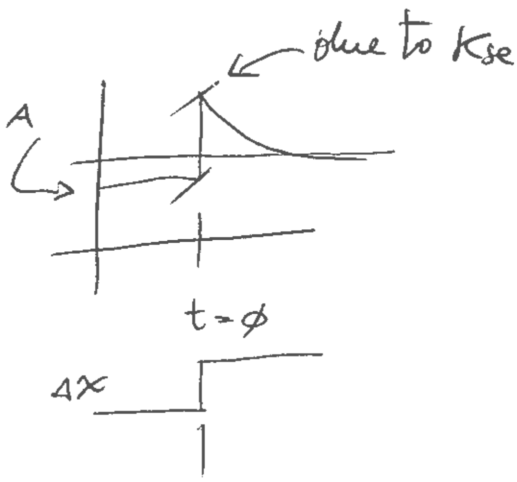
$$\Delta T\left(1 + \frac{k_{pe}}{k_{se}}\right) = k_{pe}\,\Delta x$$

$$\Downarrow$$

$$\frac{k_{se} + k_{pe}}{k_{pe}\,k_{se}} = \frac{\Delta x}{\Delta T} \implies k_{pe} = 75\ g/cm$$

Viscosity

$\dot{x} = 0$ after the stretch $\Rightarrow$ T decreases in spite of $\dot{x} = \phi$



$$T(0) = K_{se}\,\Delta x + A$$

(3)

$\dot{x} = \phi$

$$\dot{T}(t) = \frac{K_{se}}{b}\left(K_{pe}\,\Delta x + A\right) + - \frac{K_{se}}{b}\left(1 + \frac{K_{pe}}{K_{se}}\right) T(t)$$

Eq diff in T

$$\begin{cases} \dot{T} = a_1 - a_2 T \\ T = a_1/a_2 + c\,e^{-a_2/t} \end{cases}$$

$$T = \frac{K_{se}\left(A + K_{pe}\,\Delta x\right)}{K_{pe} + K_{se}} + \left[\frac{K_{pe} A + K_{se}^2 \Delta x}{K_{pe} + K_{se}} \cdot \exp\left(-\frac{K_{pe} + K_{se}}{b} t\right)\right]$$

$\exp(-t/\tau) \Rightarrow$ τ represents when the exp has declined 63%

$\Rightarrow \tau = 0{,}23\,s$

$$\tau = \frac{b}{K_{pe} + K_{se}} \Rightarrow b \simeq 50\ g\,s/cm$$

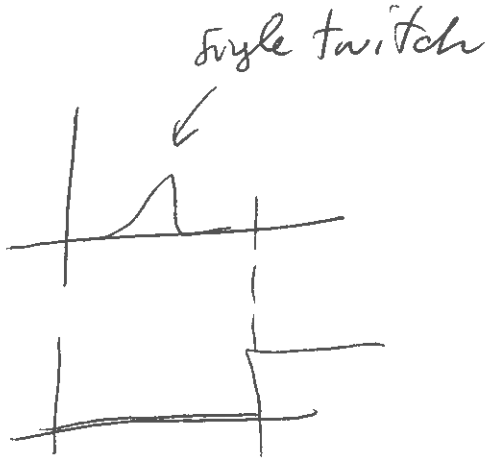
— Single twitch

for $\dot{x} = \emptyset$

from eq (3):

$$\dot{T} = \frac{k_{se}}{b}\Big(\underbrace{k_{pe}\Delta x}_{const\ =\ \emptyset} - \Big(1 - \frac{k_{pe}}{k_{se}}\Big)T + A\Big)$$

$$\frac{b}{k_{se}}\dot{T} + \Big(1 - \frac{k_{pe}}{k_{se}}\Big)T = A$$



↓

→ Fit T from data

→ then calculate A

since the input was a single "short" twitch

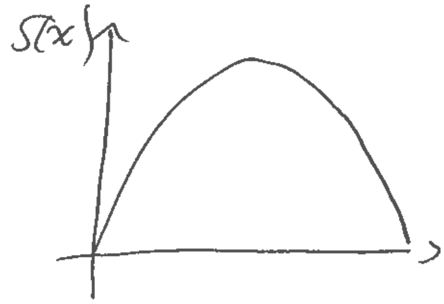
→ A(t) is the impulse response of the contractile element

$$\rightarrow \quad A(t) = \int_{-\infty}^{t} u(\tau)\,h(t-\tau)\,d\tau$$

$$u(t) = \delta(t-t_1) + (t-t_2) + \dots$$

Further, dependency on length

$$A(x,t) = s(x) \int_{-\infty}^{t} u(\tau)\, h(t-\tau)\, d\tau$$



Torque vs. force (for muscles)

$$\tau \Delta\theta = -f \Delta\lambda$$

$$\Rightarrow \quad \tau = -f \left(\frac{\Delta\lambda}{\Delta\theta}\right)$$

Jacobian

$$\Downarrow$$

$$\tau = -\frac{d\lambda}{d\theta} \cdot f$$

law of cosines

$$\lambda = \sqrt{a^2 + b^2 - 2ab\cos\theta}$$

$$\underbrace{\frac{d\lambda}{d\theta} = \frac{+\cancel{2}ab\sin\theta}{\cancel{2}\sqrt{a^2+b^2-2ab\cos\theta}} = \frac{ab\sin\theta}{\sqrt{a^2+b^2-2ab\cos\theta}}}_{J}$$

More general J

(−) because of the conventions on angles and lenghts.

$$\tau = -J^T f$$

$$c = \sqrt{a^2 + b^2 + 2ab\cos\theta_2}$$

$$\beta = \sin^{-1}\left(\frac{b\sin\theta_2}{c}\right)$$

$$\lambda = \sqrt{d^2 + c^2 + 2dc\cos(\beta + \theta_1)}$$

$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$

$$\lambda = \sqrt{d^2 + c^2 + 2dc\left[\cos\beta\cos\theta_1 - \sin\beta\sin\theta_1\right]}$$

→ insert + $\cos(\sin^{-1}(a)) = \sqrt{1-a^2}$

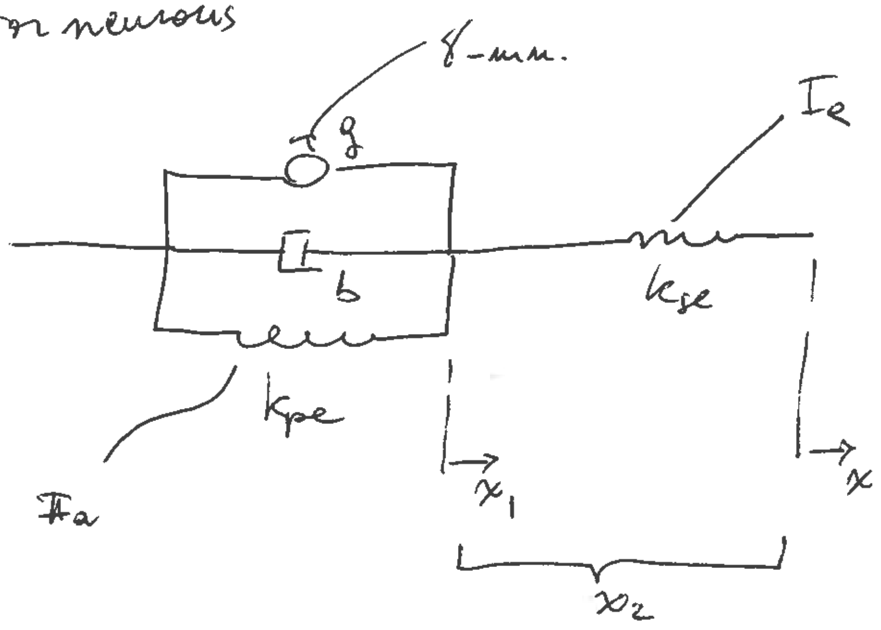
$$\lambda = \sqrt{d^2 + c^2 + 2dc\left(\sqrt{1 - \frac{b^2\sin^2\theta_2}{c^2}}\cos\theta_1 - \frac{b\sin\theta_2}{c}\sin\theta_1\right)}$$

$$J = \frac{d\lambda}{d\theta} = \underbrace{\begin{bmatrix} \frac{d\lambda}{d\theta_1} & \frac{d\lambda}{d\theta_2} \end{bmatrix}}_{(2\times1)}$$

$$\Rightarrow \underset{(2\times1)}{\tau} = \begin{bmatrix} \frac{d\lambda}{d\theta_1} \\ \frac{d\lambda}{d\theta_2} \end{bmatrix} \cdot f \leftarrow \text{scalar} \qquad \tau = [\tau_1, \tau_2]$$

Extend the model from the extrafusal to intrafusal fibers.

γ motor neurons



$$S_{Ia} = a x_2 = a(x - x_1)$$

firing rate

$$S_{II} = a x_1$$

(1) $T = k_{se}(x - x_1)$

$$\Downarrow$$

$$x_1 = x - \frac{T}{k_{se}} \quad \Rightarrow \quad \dot{x}_1 = \dot{x} - \frac{\dot{T}}{k_{se}}$$

(2) $T = k_{pe} x_1 + b\dot{x}_1 + g$

$$T = k_{pe}\left(x - \frac{T}{k_{se}}\right) + b\left(\dot{x} - \frac{\dot{T}}{k_{se}}\right) + g$$

(3) combining (1) and (2)

$$\dot{T} = \frac{k_{se}}{b}\left(b\dot{x} + k_{pe}x + g\right) - \left(\frac{k_{se} + k_{pe}}{b}\right)T$$

~~[illegible]~~

After a sudden stretch $\begin{cases} \dot{x} = \phi \\ T(0) = k_{se}\, x + g \end{cases}$

$$T = e_1(x) + e_2(x)\, e^{-t/\tau}$$

$$T = \frac{k_{se}\,(g + k_{pe}\, x)}{k_{se} + k_{pe}} + \frac{g\, k_{pe} + k_{se}^2\, x}{k_{se} + k_{pe}}\, \exp\left(-\frac{k_{se} + k_{pe}}{b}\, t\right)$$

$$S_{Ie} = a\,(x - x_1) = a\,\frac{T}{k_{se}}$$ from $T = \cancel{a}\, k_{se}\underbrace{(x - x^*)}_{\Delta x}$

$$S_{Ie}(t \to \infty) = a\,\frac{(g + k_{pe}\, x)}{k_{se} + k_{pe}}$$ → using the solution of the diff. eq.

$$S_{II} = a\, x_1 = a\left(x - \frac{T}{k_{se}}\right)$$

$$S_{II}(t \to \infty) = a\left(x - \frac{g + k_{pe}\, x}{k_{se} + k_{pe}}\right)$$

role of γ motor neurons.

$x_d(t)$ is the desired trajectory

α-γ coactivation

eq(3)

$$\dot{T} = \frac{k_{se}}{b}\left(b\dot{x} + k_{pe}x + g\right) - \left(\frac{k_{se}+k_{pe}}{b}\right)T$$

we want:

$\dot{T} = \phi \qquad x \equiv x_d$

no change in T for spindles

$$g(t) = g(0) - b\dot{x}_d - k_{pe}(x_d - x_0)$$

$\uparrow$ gets the term in T

$\uparrow$ some starting point

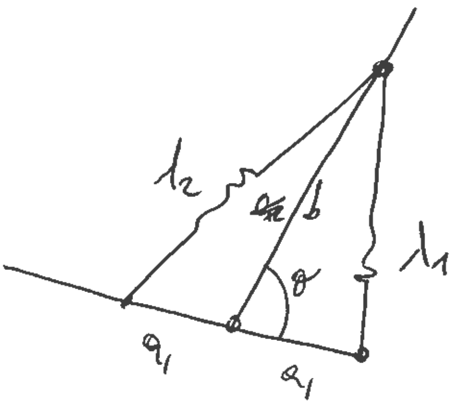
$\downarrow$

γ-motor neurons activation to keep $\dot{\Phi} = \phi$

Limb stability

- spring-like properties
- damping (intrinsic)

Moving a joint requires a pair of muscles.



example:

from J^T we can write:

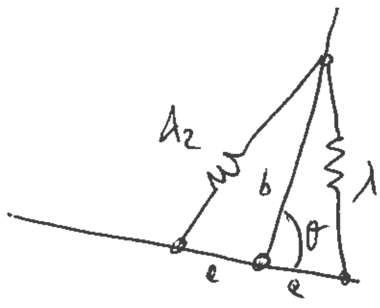
$$\frac{d\lambda_1}{d\theta} = \frac{ab\sin\theta}{\sqrt{a^2+b^2-2ab\cos\theta}}$$

$$\frac{d\lambda_2}{d\theta} = \frac{-ab\sin\theta}{\sqrt{a^2+b^2+2ab\cos\theta}}$$

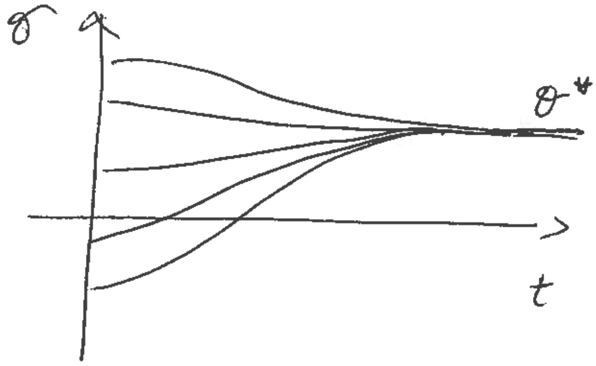
$$\begin{matrix}\tau_1 \\ \tau_2\end{matrix} \quad \rightsquigarrow \quad \tau = \tau_1 + \tau_2$$

more on the model (single joint)

$$I\ddot{\theta} - \nu\dot{\theta} = \tau$$



$\downarrow$ integrate (any method)



stimulation $\rightarrow$ T (force)

$T \rightarrow \tau$

$\tau \rightarrow$ into θ given

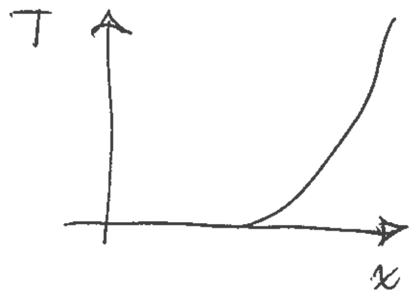
θ_0 $\dot{\theta}_0$

Note that T depends also on θ (configuration

and T is determined by the solution of a diff. equation

We said that

passive

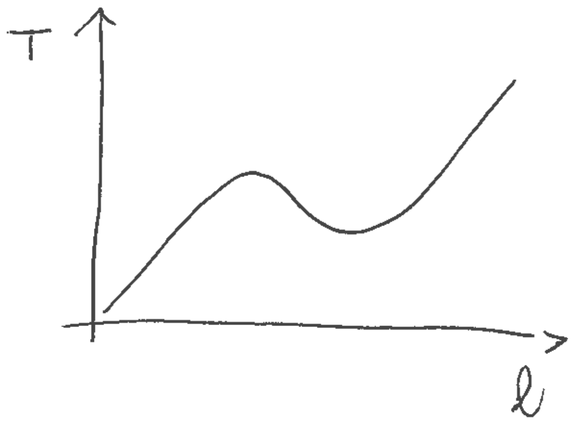


due to materials (structural)

active

due to the # of active sites that bind the myosin heads.

→ determine the equilibrium



Polit & Bizzi exp.

- Equilibrium point.

- results
  - muscle properties (in the absence of feedback) can bring the hand to a remembered position
  - different initial configurations lead to errors → no adaptation in absence of feedback.

→ Mussa-Ivaldi & Bizzi

- Frog experiment (from paper)
- field approximation (from paper)

→ Ideas of motor primitives: controversial.

- Questions
  - Stimulation of the motor cortex for 500 ms → duration of reaching
  - obtain coordinated movement (again primitives, controversial)

Mussa-Ivaldi

basis fields

$$D(q\dot{q}\ddot{q}) = \sum_{i=1}^{k} c_i \, \phi_i(q,\dot{q},t)$$

arm dynamics

supraspinal command

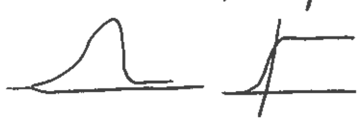
---

gradient of Gaussian

$$\chi(q\dot{q}) = k(q-q_0)\, e^{-(q-q_0)k(q-q_0)} + B\dot{q}$$

$$\phi(q\dot{q}t) = f(t) \times \chi(q\dot{q})$$

impulse / step



$\} \longrightarrow$ from stimulation experiments

Qualitatively consistent w/ empirical observations

$$\sum_i c_i \phi_i = D(q \dot{q} \ddot{q})$$

$$\sum_i \Phi_{ji} \, c_i = \Lambda_j$$

$$\Phi_{ji} = \int \phi_j(q_d, \dot{q}_d, t) \cdot \phi_i(q_d, \dot{q}_d, t) \, dt$$

$$\Lambda_j = \int \phi_j(q_d, \dot{q}_d, t) \cdot D(q_d, \dot{q}_d, t) \, dt$$

$\longrightarrow$ non-negative least-square method

that is:

solve $\min_{c_i} \left\| \sum_i \Phi_{ji} c_i - \Lambda_j \right\|$ s.t $c_i > 0$