Consequently

$$\begin{bmatrix} \dot{I}_a \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} {R_a}/{L_a} & {K_E}/{L_a} \\ {K_T}/{J_M + J_L} & {B}/{J_M + J_L} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ \omega \end{bmatrix} + \begin{bmatrix} {-V_{arm}}/{L_a} \\ {\tau_f + \tau_{gr}}/{J_M + J_L} \end{bmatrix}$$

- A linear system of two equations (differential)
- Q: can you write a transfer function from these equations?
- Q: can you transform the equations into a block diagram?

By Laplace-transforming

$$V_{arm}(s) = R_a I_a(s) + L_a I_a(s)s + \omega(s)K_E \Rightarrow I_a(s) = \frac{V_{arm}(s) - \omega(s)K_E}{R_a + L_a s}$$

$$\tau = K_T I_a$$

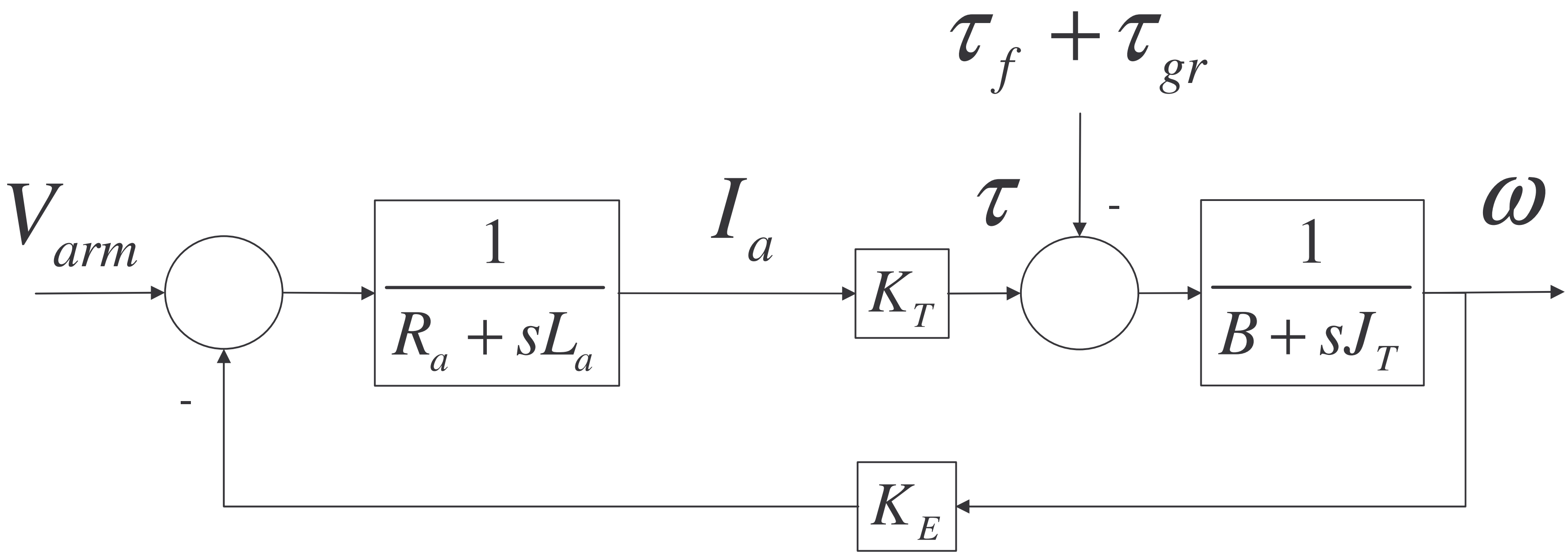
$$K_T \frac{V_{arm}(s) - \omega(s)K_E}{R_a + L_a s} = (J_M + J_L)\omega(s)s + B\omega(s) + \tau_f + \tau_{gr}$$

and finally

$$\frac{\omega(s)}{V_{arm}(s)} = \frac{K_T / L_a J_T}{s^2 + [(R_a J_T + L_a B)/L_a J_T]s + (K_T K_E + R_a B)/L_a J_T}$$

- Considering gravity and friction as additional inputs

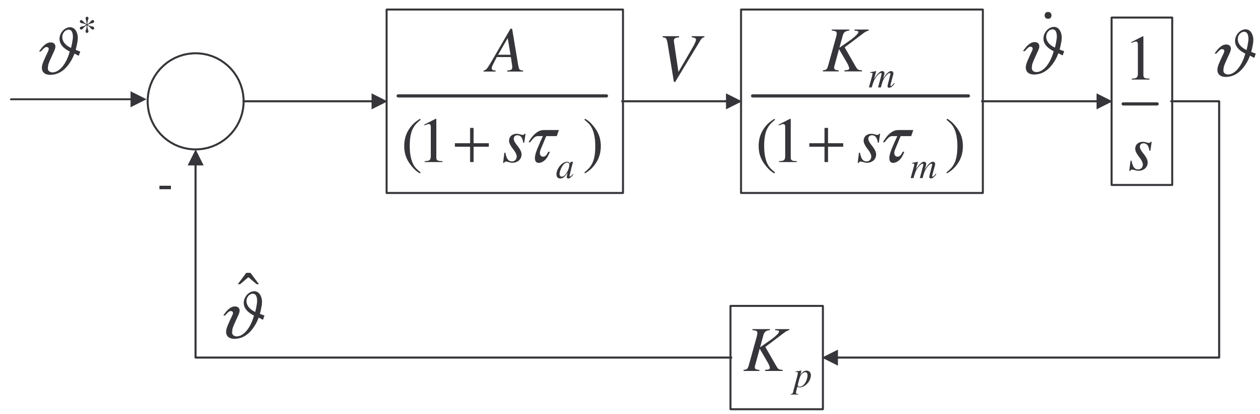
Block diagram



Analysis tools

- Control: determine V_a so to move the motor as desired
- Root locus
- Frequency response

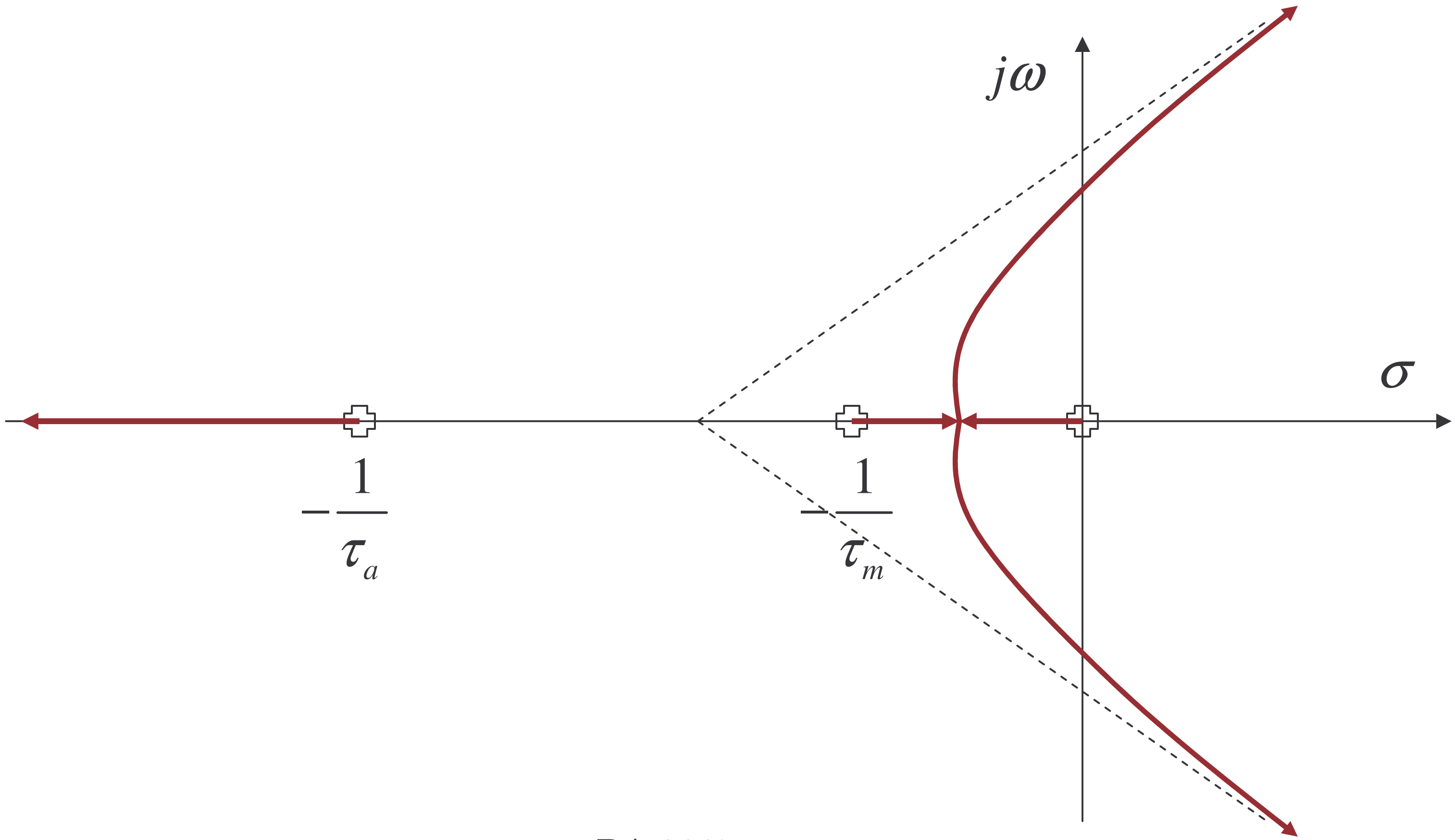
First block diagram



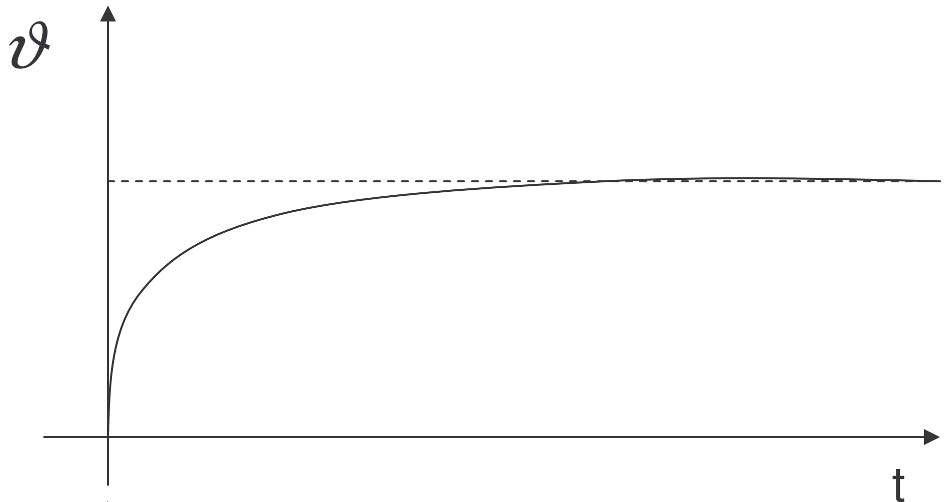
$$H_{open\_loop} = \frac{A}{1+s\tau_a}\frac{K_m}{1+s\tau_m}\frac{K_p}{s}$$

Root locus

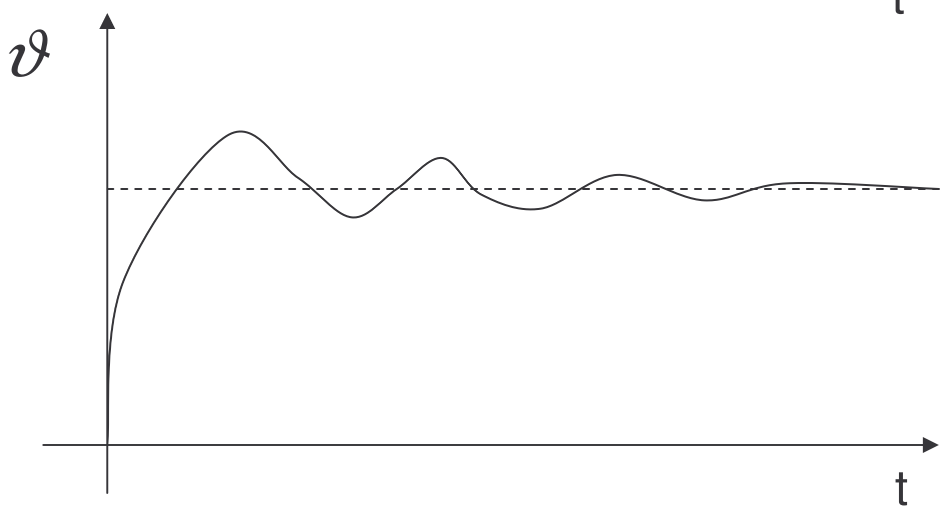
$$H_{open\_loop} = \frac{A}{1+s\tau_a}\frac{K_m}{1+s\tau_m}\frac{K_p}{s} \qquad K = AK_mK_p$$



Changing K

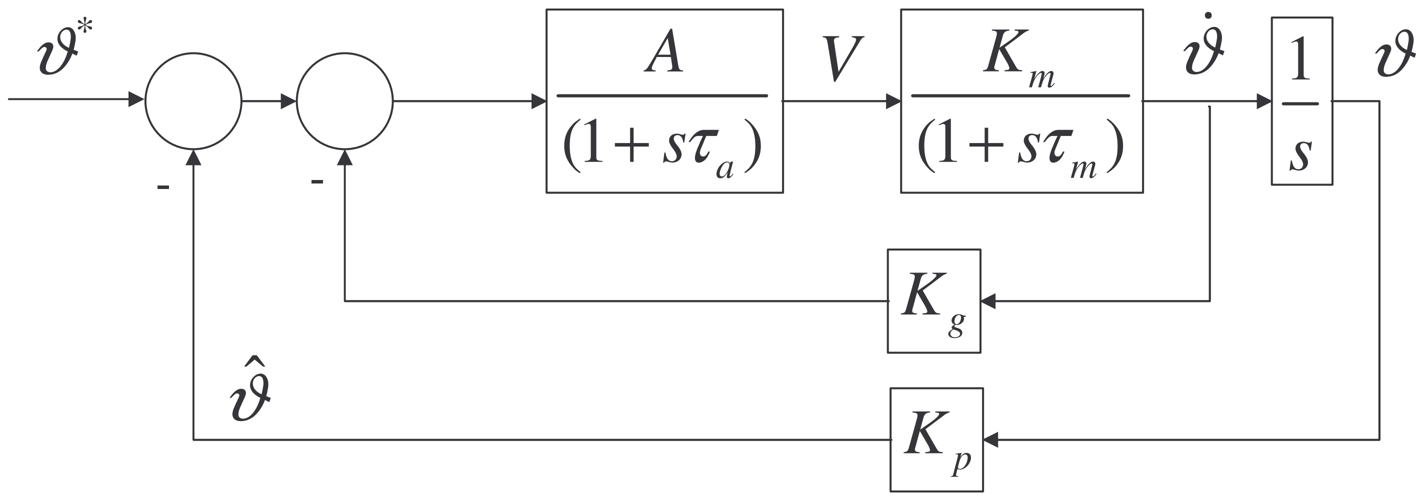


small K



higher K

Let's add something second diagram



$$H_{open\_loop} = \frac{AK_m(K_p + sK_g)}{(1+s\tau_a)(1+s\tau_m)s}$$

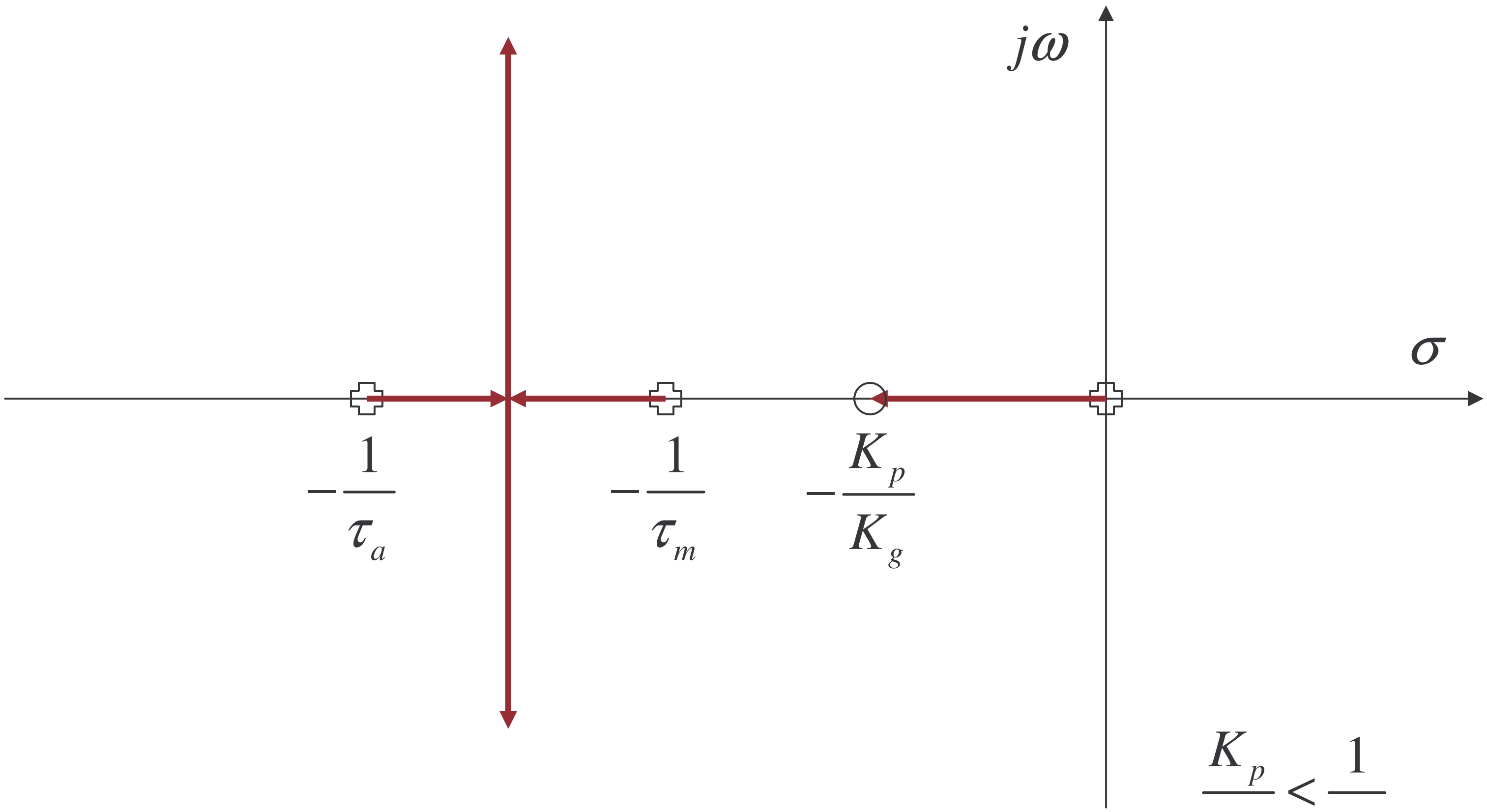
Analysis

$$H_{open\_loop} = \frac{AK_m K_p (1 + s \, {K_g}/{K_p})}{(1 + s\tau_a)(1 + s\tau_m)s}$$

$$K = AK_m K_p$$

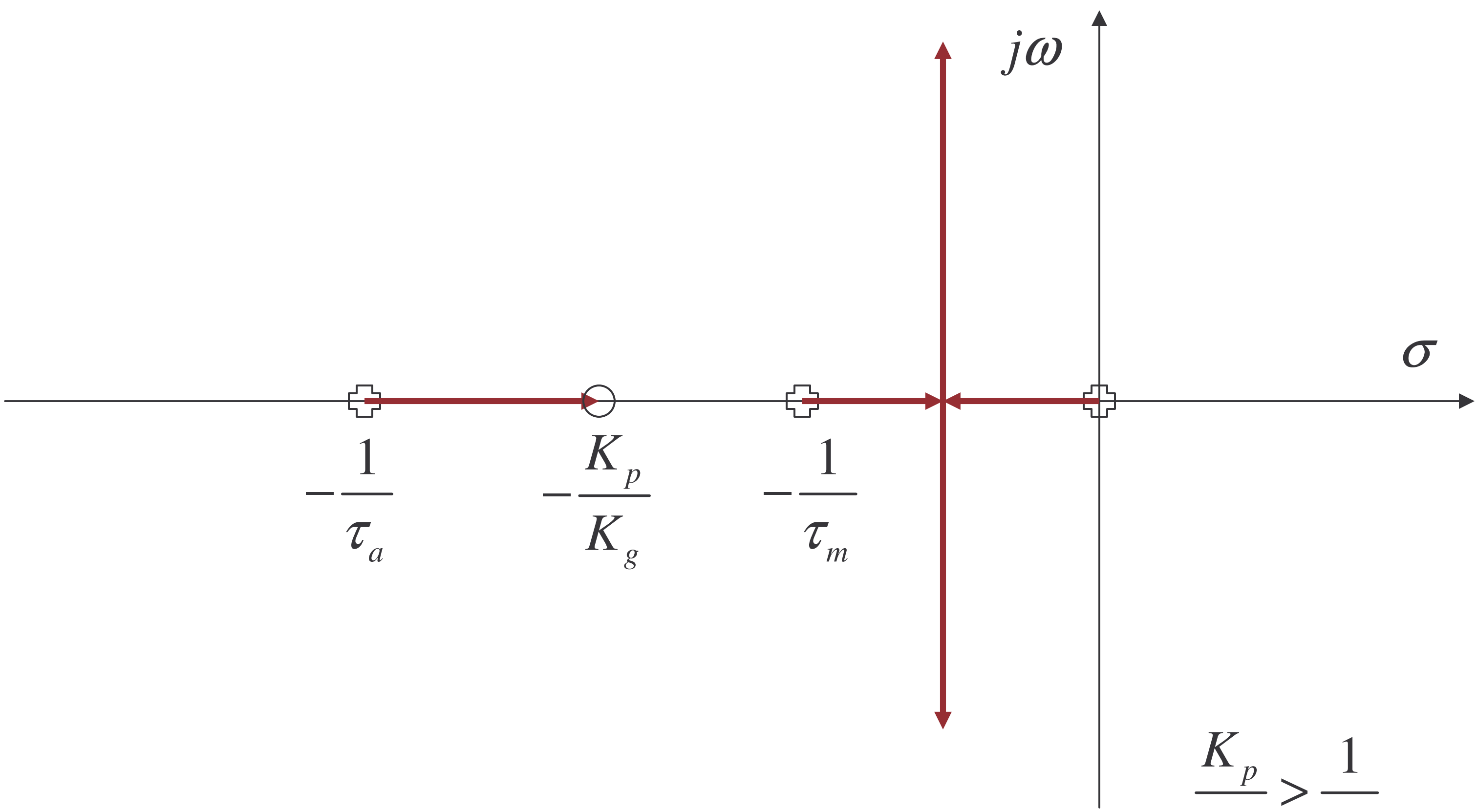
$$Z_{feedback} = \frac{K_g}{K_p}$$

Root locus (case 1)



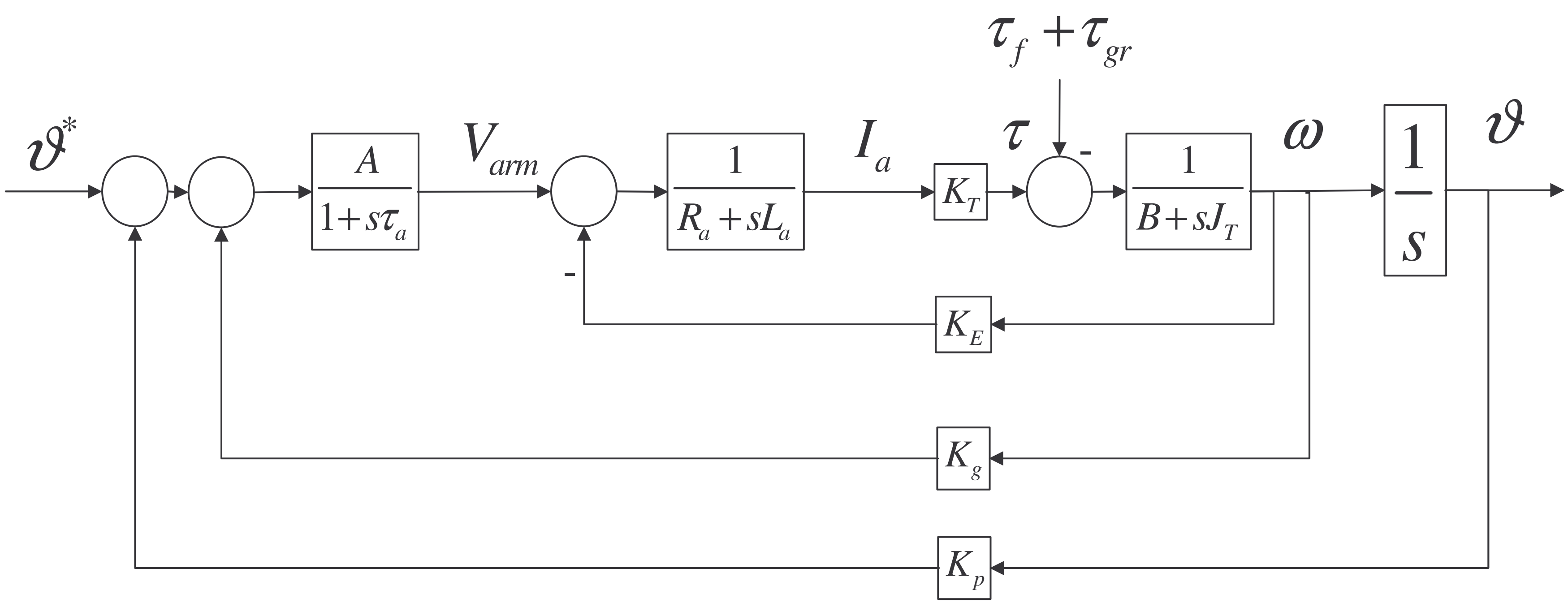
$$\frac{K_p}{K_g} < \frac{1}{\tau_m}$$

Root locus (case 2)



$$\frac{K_p}{K_g} > \frac{1}{\tau_m}$$

Overall…



ϑ^*
$\frac{A}{1+s\tau_a}$
V_{arm}
$\frac{1}{R_a+sL_a}$
I_a
K_T
τ
$\tau_f+\tau_{gr}$
-
$\frac{1}{B+sJ_T}$
ω
$\frac{1}{s}$
ϑ
-
K_E
K_g
K_p

Error and performance

$$\vartheta = \frac{\vartheta_d}{s} \qquad M(s) = \frac{K_T}{(R_a + sL_a)(B + sJ_T) + K_E K_T}$$

closed loop (velocity)

$$\omega(s) = \frac{\frac{A}{1+s\tau_a} M(s)}{1 + \frac{A}{1+s\tau_a} M(s) K_g}$$

$$\vartheta(s) = \frac{1}{s}\omega(s)$$

closed loop (position)

$$\vartheta(s) = \frac{\frac{1}{s}\omega(s)}{1 + \frac{1}{s}\omega(s) K_p}$$

finally

$$\lim_{s\to 0} sH(s) = \lim_{t\to\infty} h(t)$$

$$\Rightarrow \lim_{s\to 0} s\frac{\vartheta_d}{s}\vartheta(s) = \lim_{s\to 0}\frac{s\frac{1}{s}\frac{\vartheta_d}{s}\omega(s)}{1+\frac{1}{s}\omega(s)K_p} = \frac{\vartheta_d}{K_p}$$

- For zero error K must be 1 or the control structure must be different

Same line of reasoning

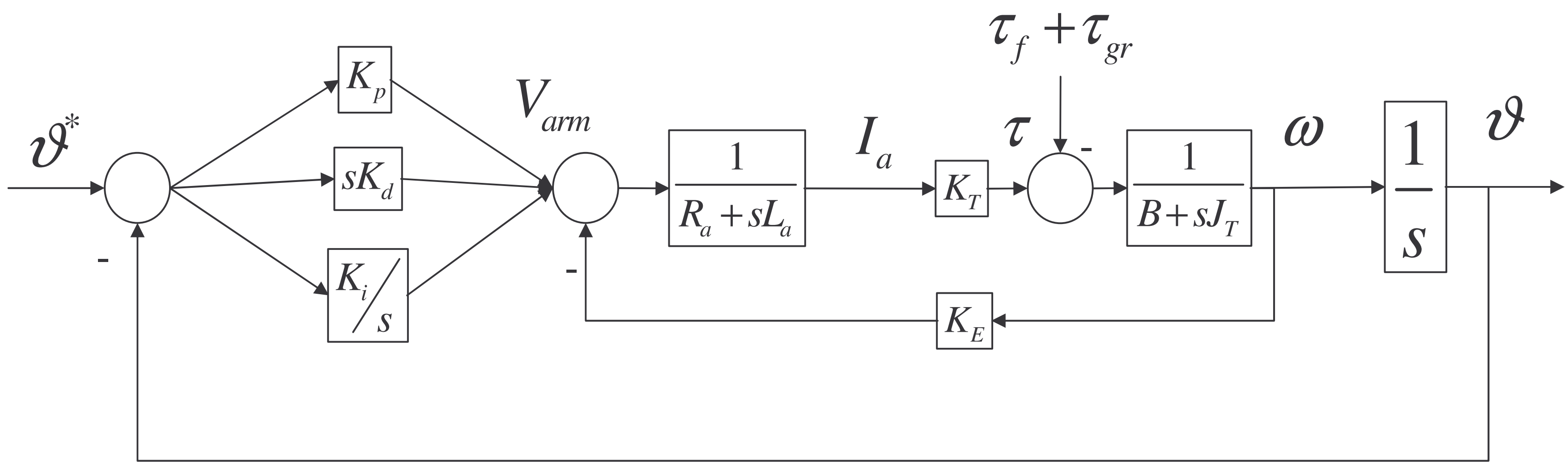
$$\vartheta_{final} = -\frac{\tau_{gr} R_a}{A K_T K_p}$$

- Final value due to friction and gravity

$$\left| \frac{\tau_{gr} R_a}{A K_T K_p} \right| \leq \vartheta_{\max} \Rightarrow K_p \geq \frac{\tau_{gr} R_a}{A K_T \vartheta_{\max}}$$

$$K_{p\min} = \frac{\tau_{gr} R_a}{A K_T \vartheta_{\max}}$$

PID controller



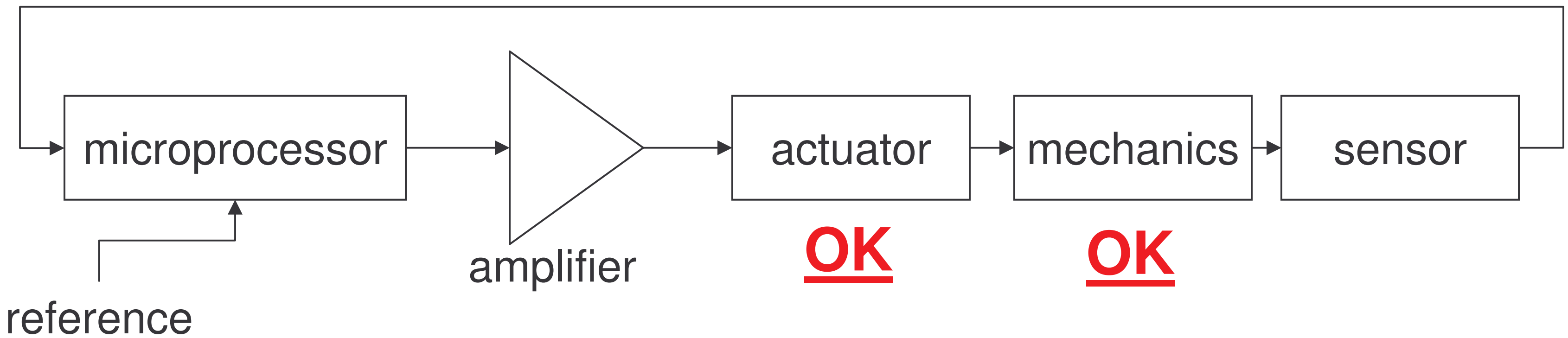
PID controller

- We now know why we need the proportional
- We also know why we need the derivative
- Finally, we add the integral
  - Integrates the error, in practice needs to be limited

Interpreting the PID

- Proportional: to go where required, linked to the steady-state error
- Derivative: damping
- Integral: to reduce the steady-state error

Global view

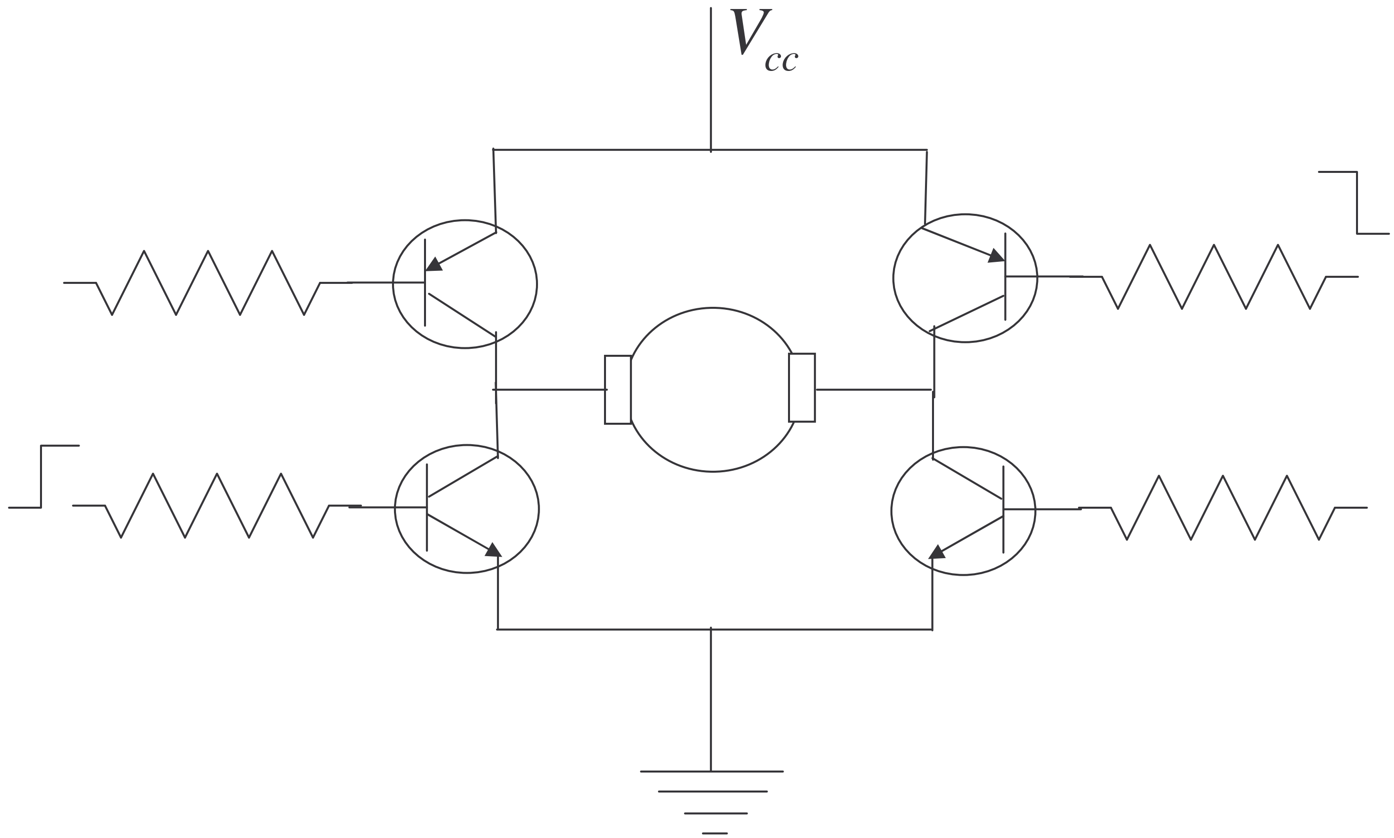


microprocessor
actuator
mechanics
sensor
reference
amplifier
OK
OK

About the amplifiers

- Linear amplifiers
  - H type
  - T type
- PWM (switching) amplifiers

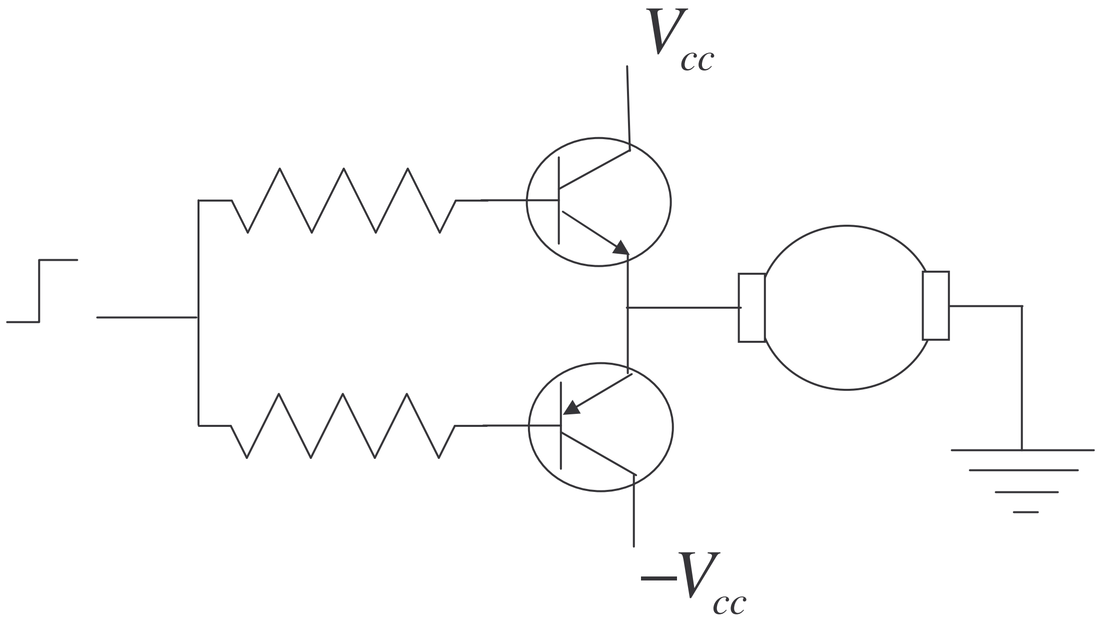
Let's consider the linear as a starting point



H-type

- The motor doesn't have a reference to ground (floating)
- It's difficult to get feedback signals (e.g. to measure the current flowing through the motor)

T-type



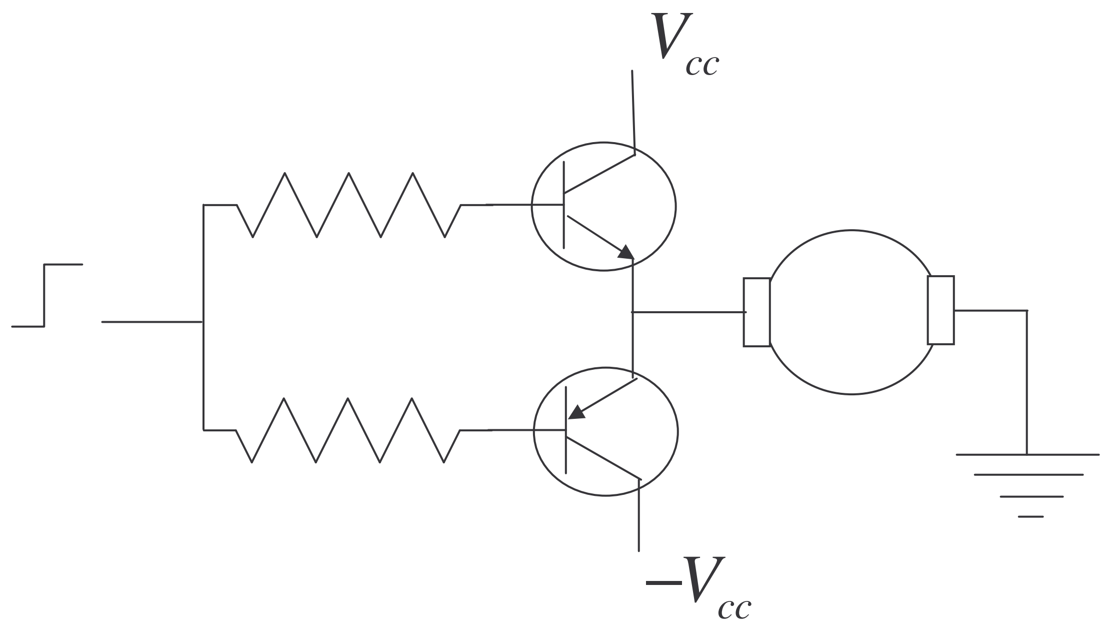
On the T-type

- Bipolar DC supply
- Dead band (around zero)
- Need to avoid simultaneous conduction (short circuit)

Things not shown

- Transistor protection (currents flowing back from the motor)
- Power dissipation and heat sink
  - Cooling
- Sudden stop due to obstacles
  - High currents $\rightarrow$ current limits and timeouts

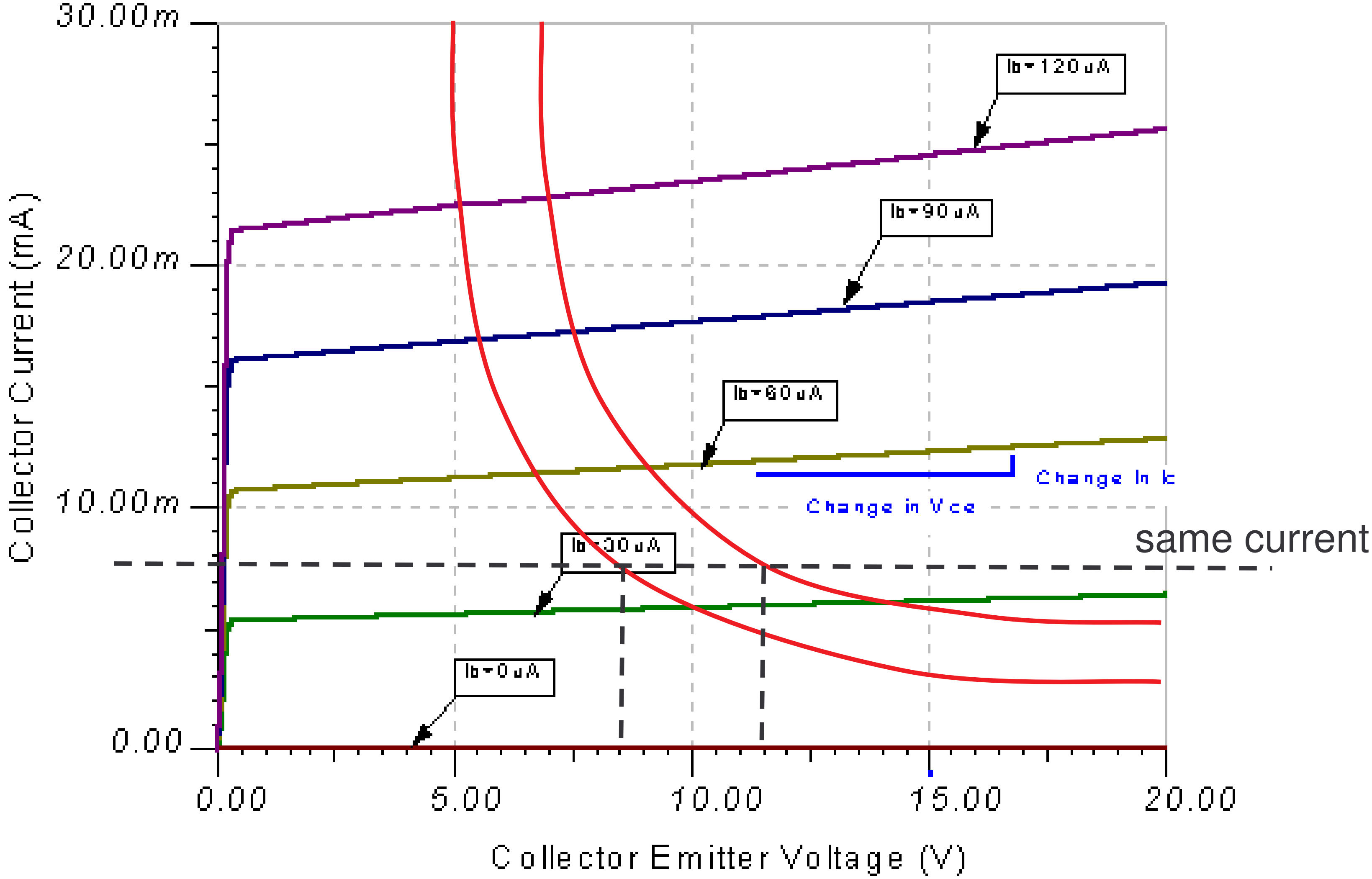
T-type



$$I_c \approx \frac{V_{cc}}{R_{transisor} + R_{motor}}$$

PWM amplifiers

$$P = V_{ce} I_c$$



PWM signal

$$P = V_{ce} I_c$$

- Transistors either “on” or “off”
  - When off, current is very low, little power too
  - When on, V is low, working point close to (or in) saturation, power dissipation is low

Comparison

- 12W for a 6A current using a switching amplifier
- 72W for a corresponding linear amplifier

Why does it work?

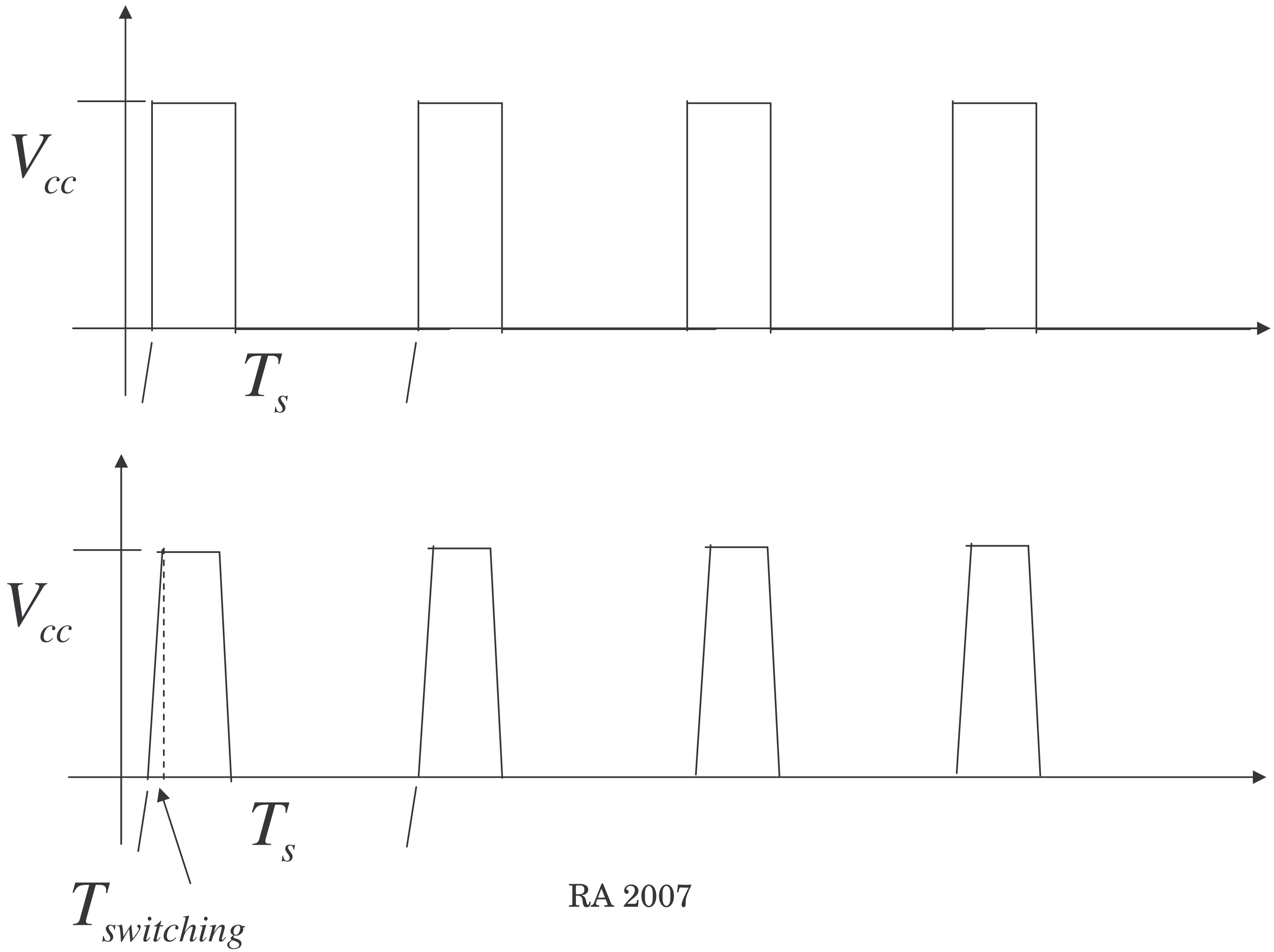
$$\frac{\omega(s)}{V_{arm}(s)} = \frac{K_T / L_a J_T}{s^2 + [(R_a J_T + L_a B)/L_a J_T]s + (K_T K_E + R_a B)/L_a J_T}$$

- In practice the motor transfer function is a low-pass filter

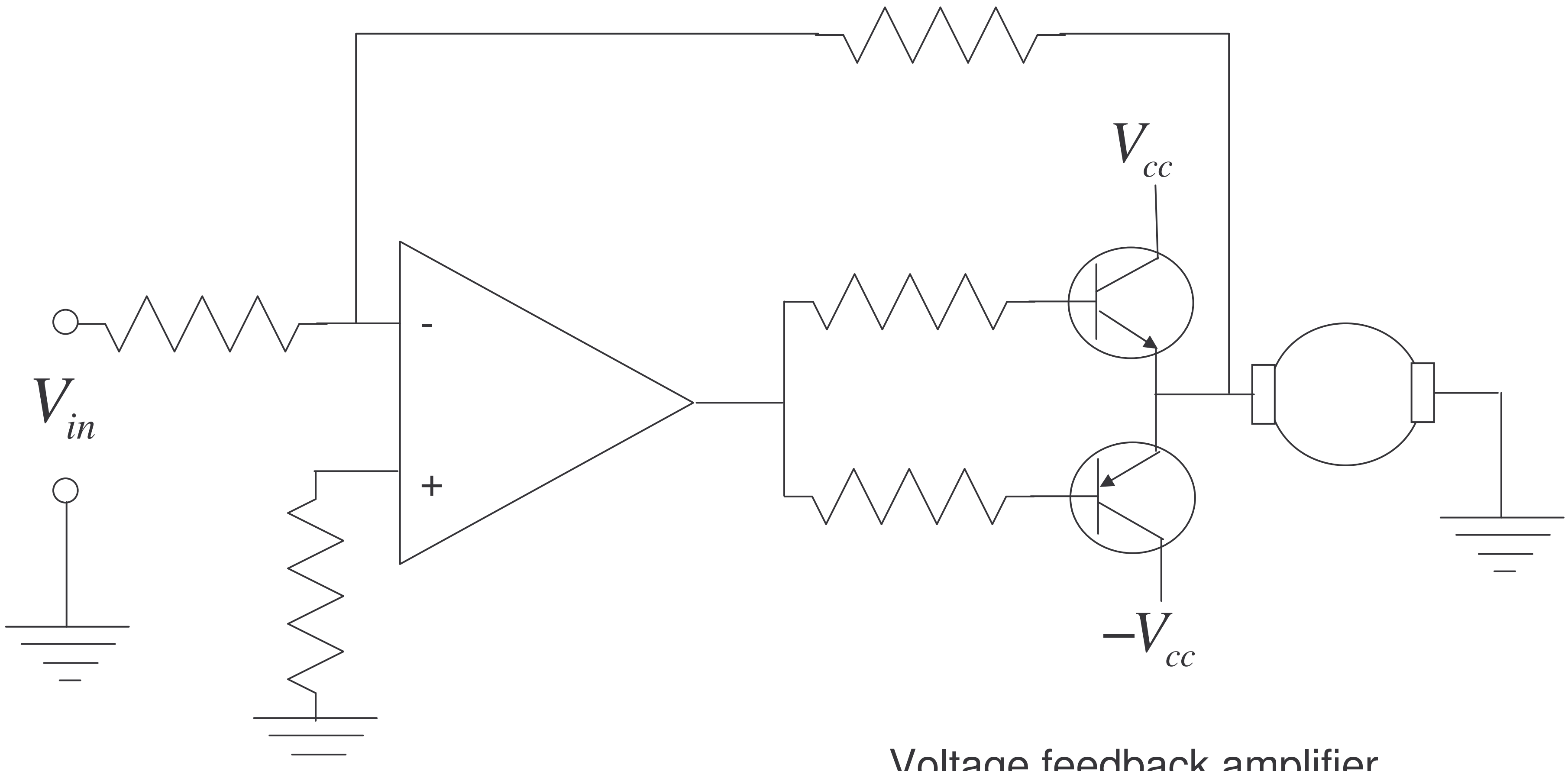
$$T_s \quad \text{with} \quad f_s \gg f_E (f_s > 100 f_E)$$

- Switching frequency must be high enough

PWM signal

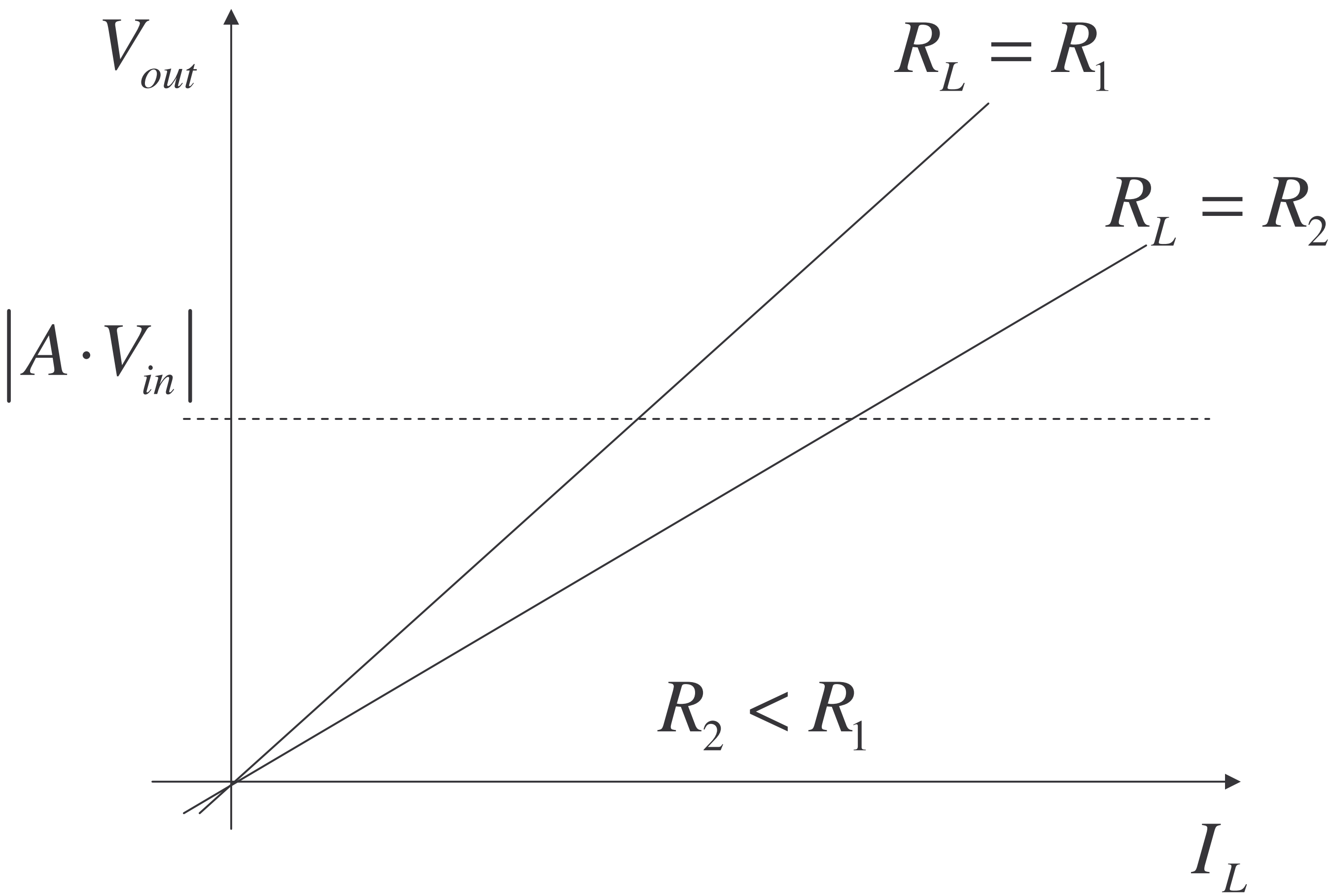


Feedback in servo amplifiers

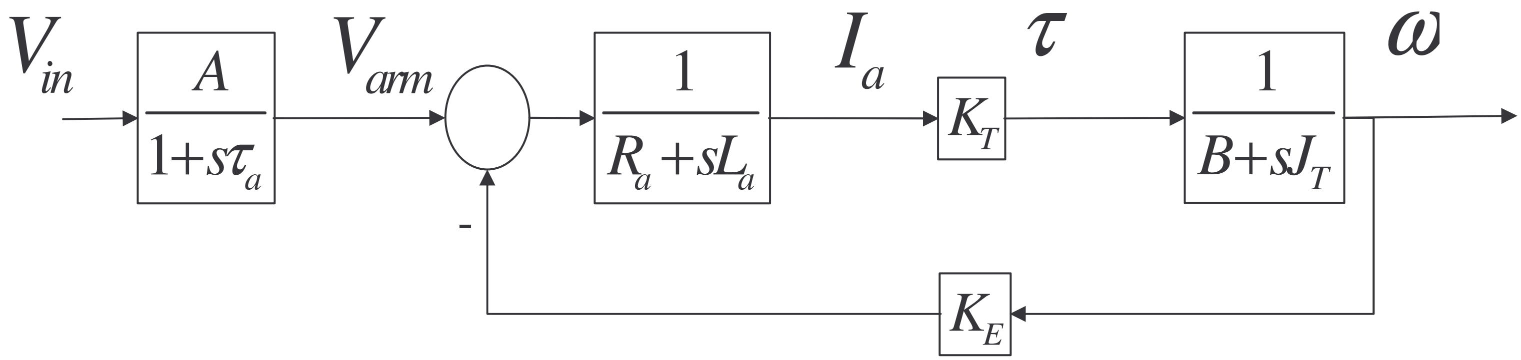


Voltage feedback amplifier

Operating characteristic

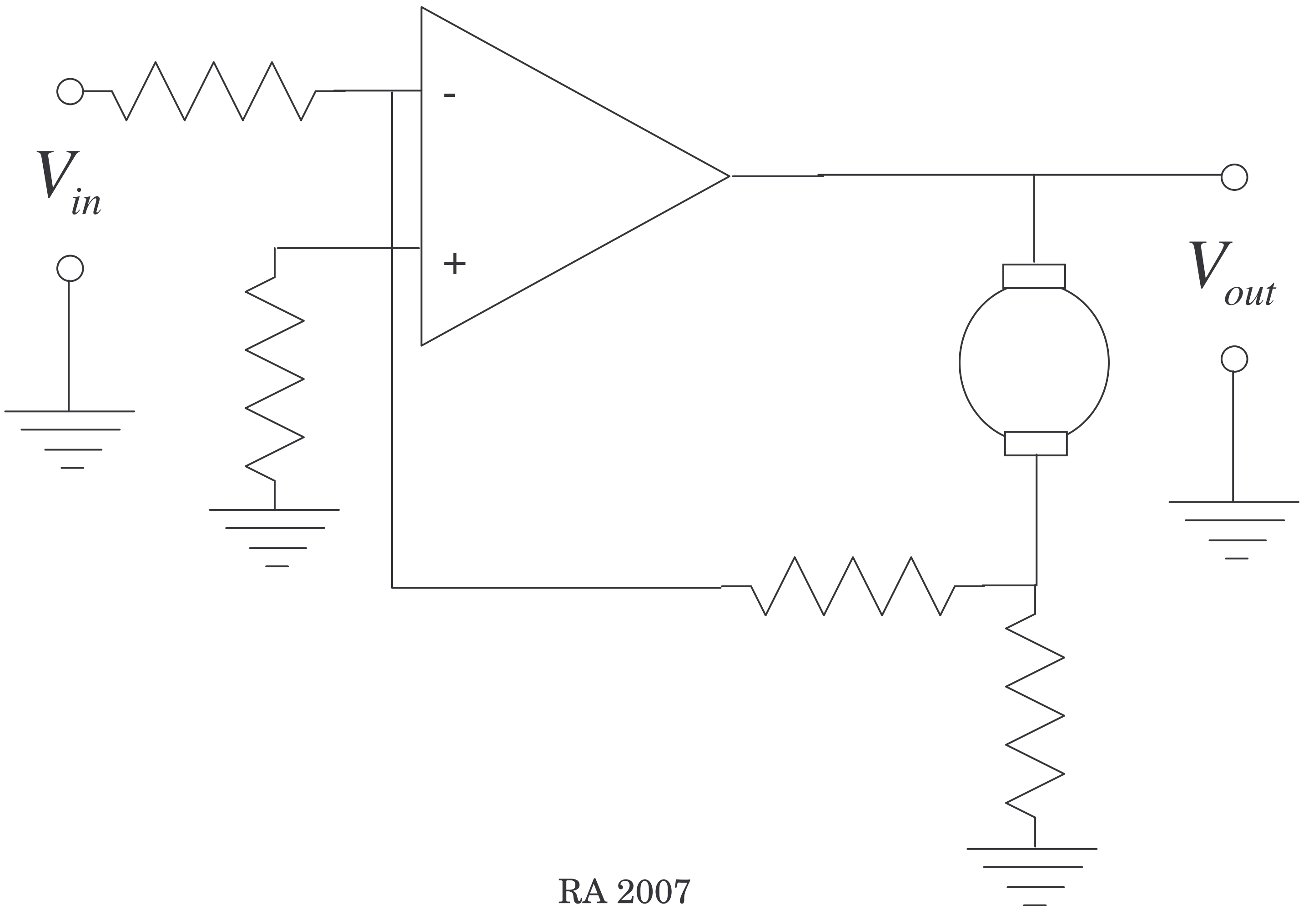


We've already seen this

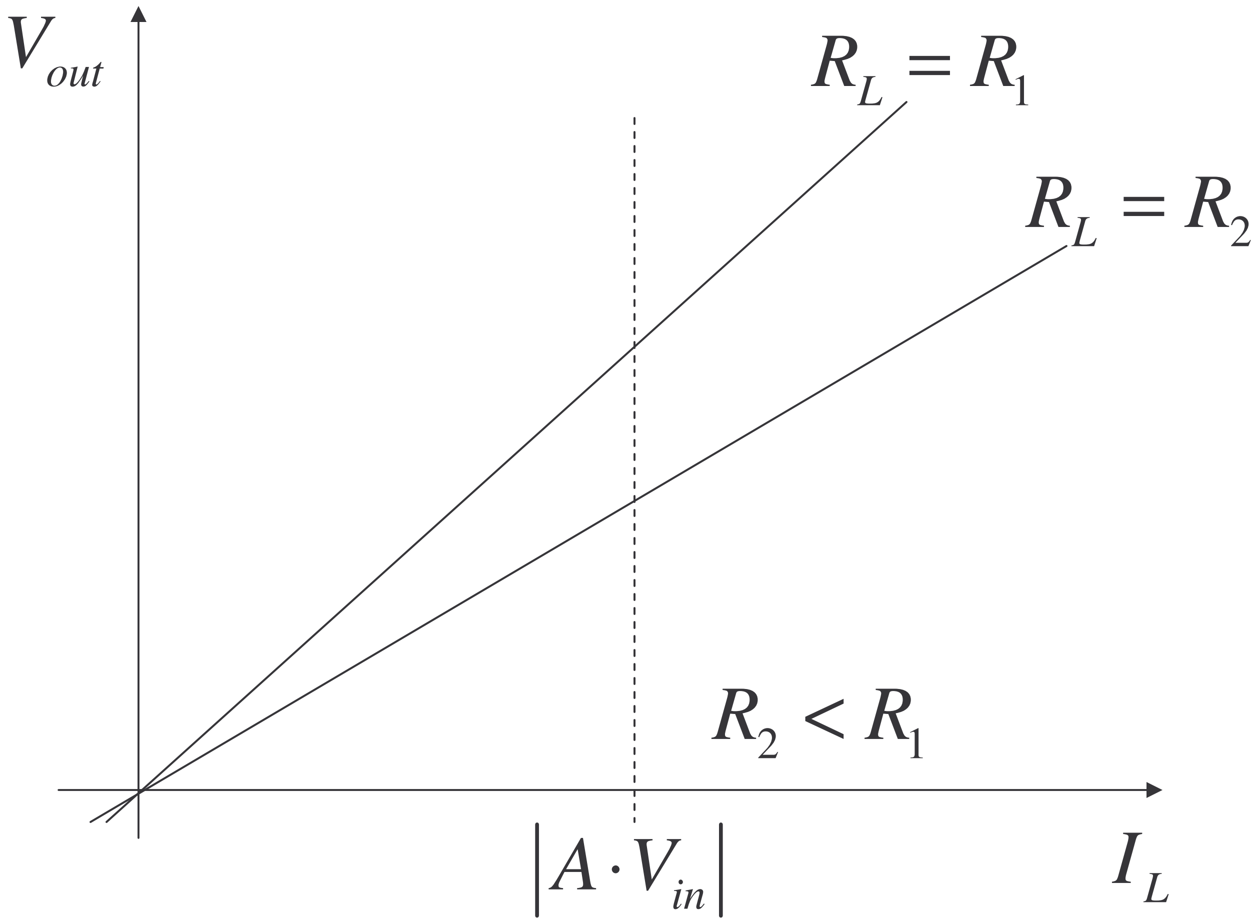


$$\frac{\omega(s)}{V_{in}(s)} = \frac{K_T / L_a J_T}{s^2 + [(R_a J_T + L_a B)/L_a J_T]s + (K_T K_E + R_a B)/L_a J_T} \frac{A_v}{(1+s\tau_a)}$$

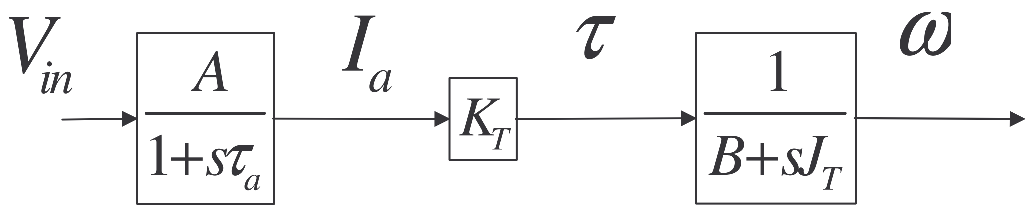
Current feedback



Current feedback



Motor driven by a current amplifier



$$\frac{\omega(s)}{V_{in}(s)} = \frac{K_T A_i}{(sJ_T + B)(1+s\tau_a)}$$