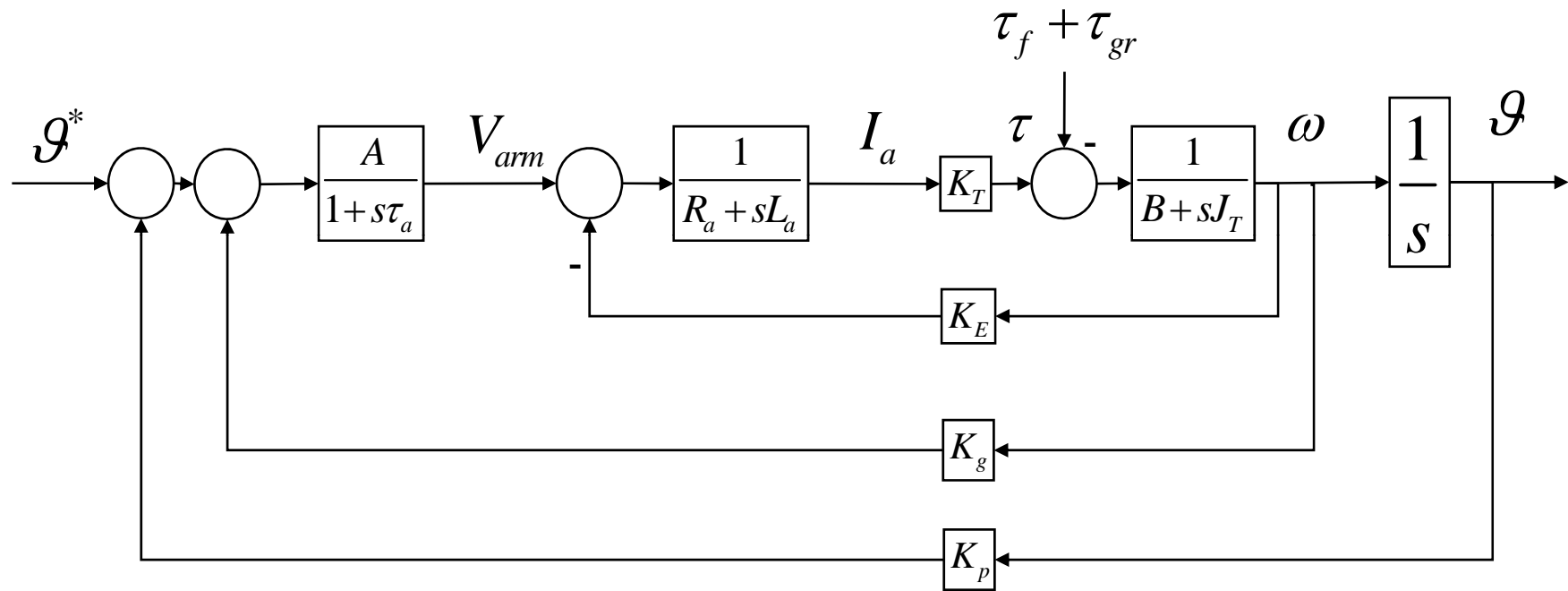
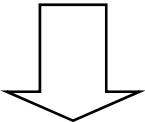


Overall...

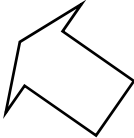


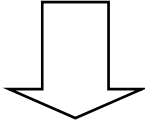
Error and performance

$$\mathcal{G} = \frac{\mathcal{G}_d}{s} \quad M(s) = \frac{K_T}{(R_a + sL_a)(B + sJ_T) + K_E K_T}$$

closed loop (position) 

$$\mathcal{G}(s) = \frac{1}{s} \omega(s)$$



closed loop (velocity) 

$$\omega(s) = \frac{\frac{A}{1 + s\tau_a} M(s)}{1 + \frac{A}{1 + s\tau_a} M(s) K_g}$$

$$\mathcal{G}(s) = \frac{\frac{1}{s} \omega(s)}{1 + \frac{1}{s} \omega(s) K_p}$$

finally

$$\lim_{s \rightarrow 0} sH(s) = \lim_{t \rightarrow \infty} h(t)$$

$$\Rightarrow \lim_{s \rightarrow 0} s \frac{\mathcal{G}_d}{s} \mathcal{G}(s) = \lim_{s \rightarrow 0} \frac{s \frac{1}{s} \frac{\mathcal{G}_d}{s} \omega(s)}{1 + \frac{1}{s} \omega(s) K_p} = \frac{\mathcal{G}_d}{K_p}$$

- For zero error K must be 1 or the control structure must be different

Same line of reasoning

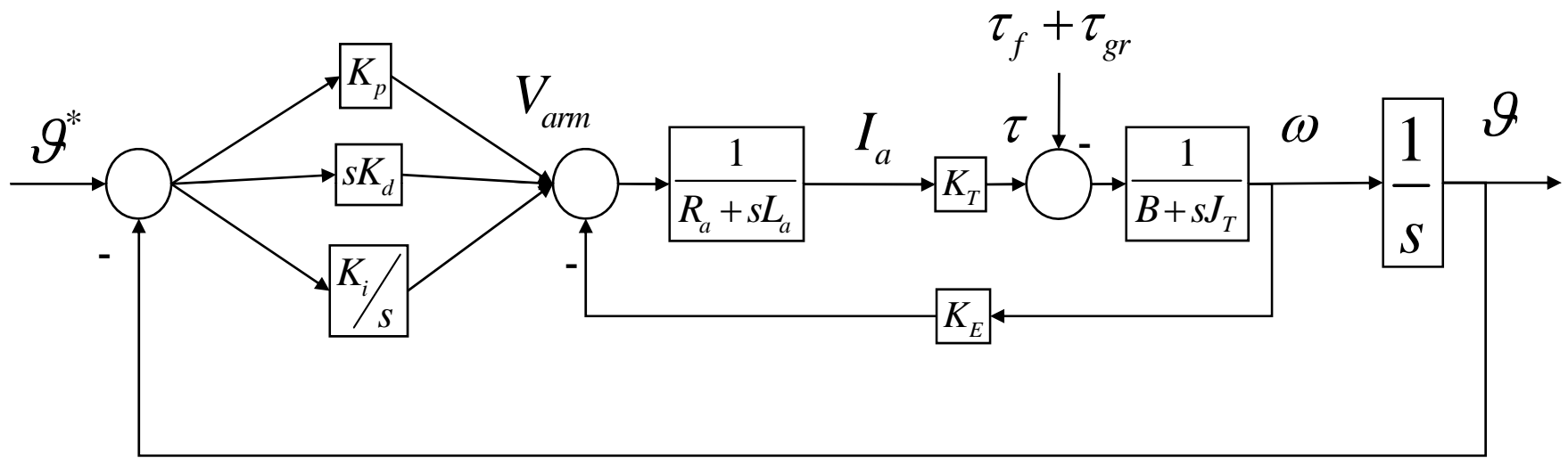
$$\mathcal{J}_{final} = -\frac{\tau_{gr} R_a}{AK_T K_p}$$

- Final value due to friction and gravity

$$\left| \frac{\tau_{gr} R_a}{AK_T K_p} \right| \leq \mathcal{J}_{\max} \Rightarrow K_p \geq \frac{\tau_{gr} R_a}{AK_T \mathcal{J}_{\max}}$$

$$K_{p \min} = \frac{\tau_{gr} R_a}{AK_T \mathcal{J}_{\max}}$$

PID controller



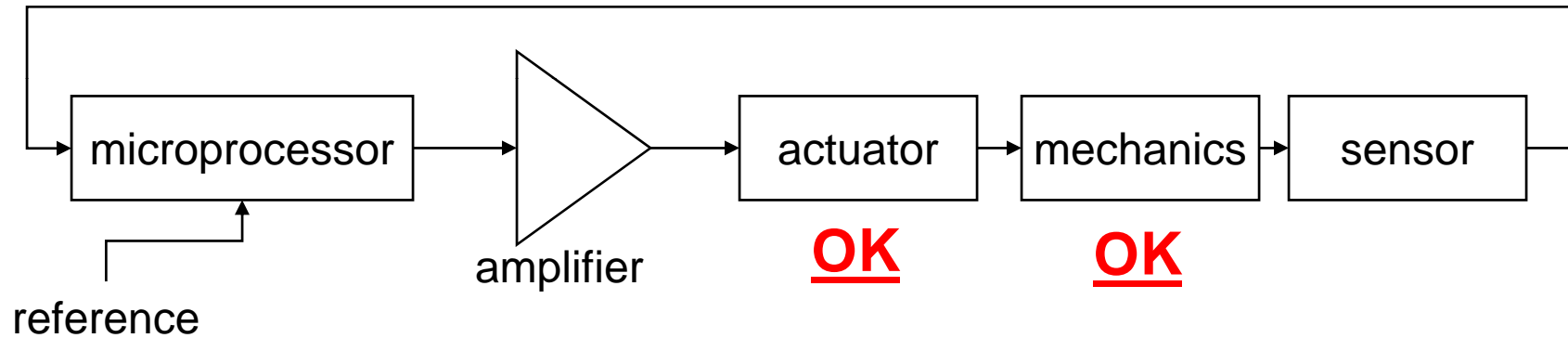
PID controller

- We now know why we need the proportional
- We also know why we need the derivative
- Finally, we add the integral
 - Integrates the error, in practice needs to be limited

Interpreting the PID

- Proportional: to go where required, linked to the steady-state error
- Derivative: damping
- Integral: to reduce the steady-state error

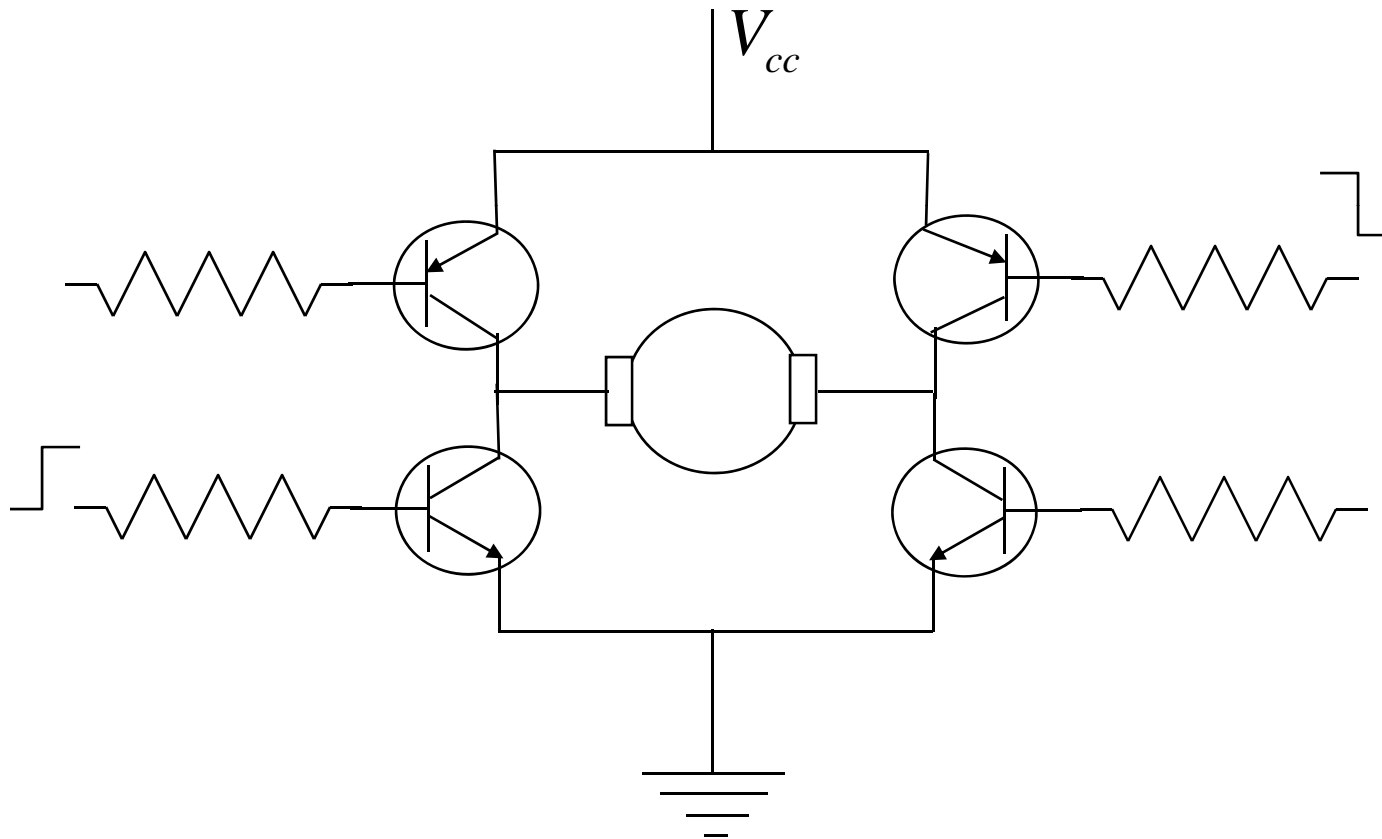
Global view



About the amplifiers

- Linear amplifiers
 - H type
 - T type
- PWM (switching) amplifiers

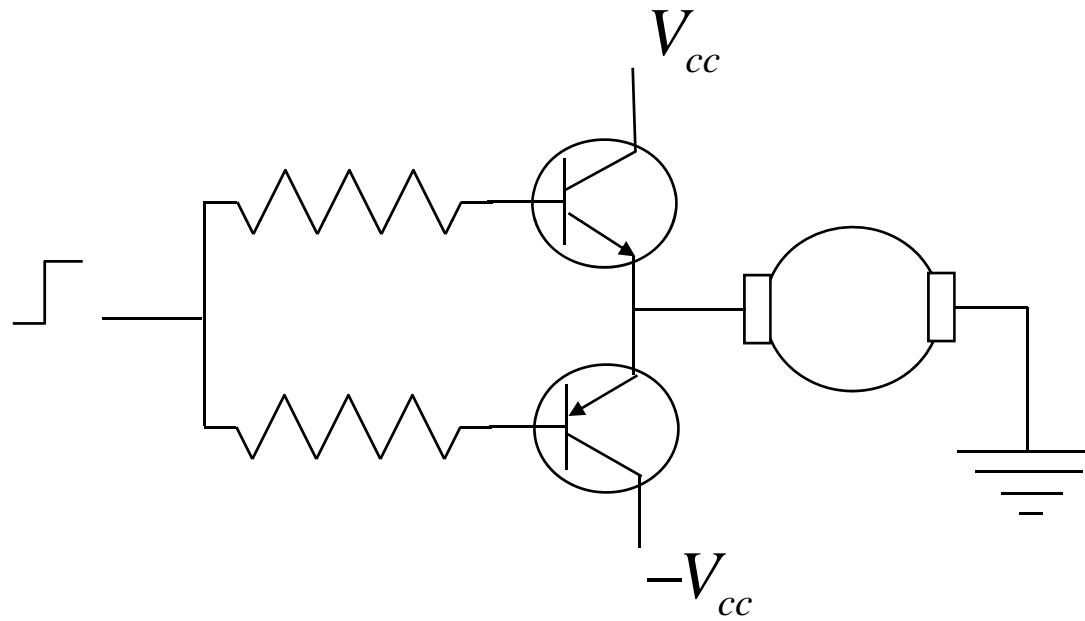
Let's consider the linear as a
starting point



H-type

- The motor doesn't have a reference to ground (floating)
- It's difficult to get feedback signals (e.g. to measure the current flowing through the motor)

T-type



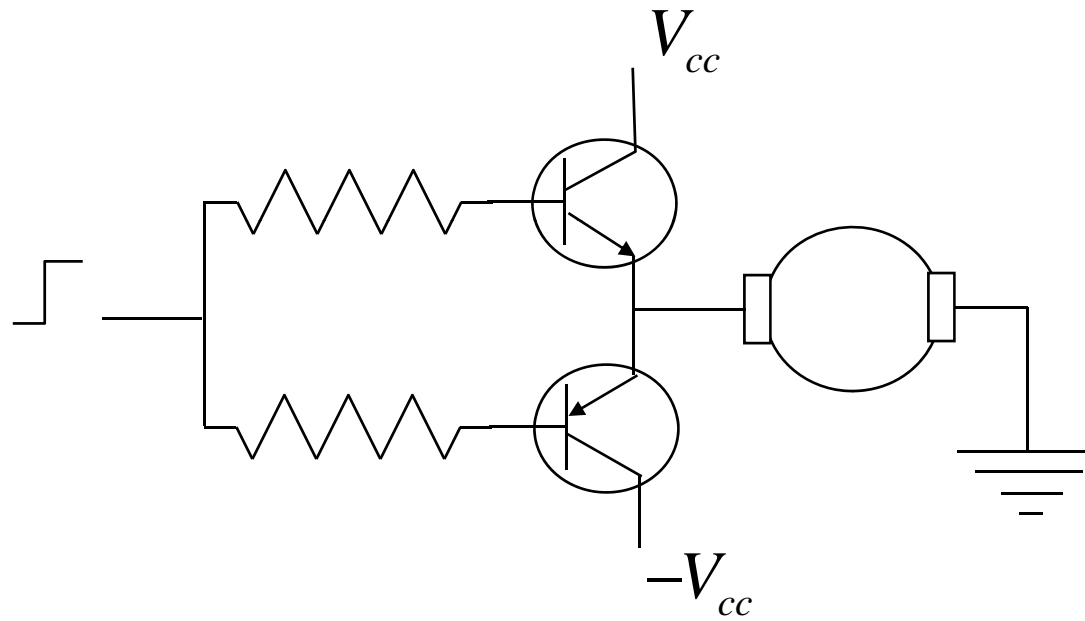
On the T-type

- Bipolar DC supply
- Dead band (around zero)
- Need to avoid simultaneous conduction (short circuit)

Things not shown

- Transistor protection (currents flowing back from the motor)
- Power dissipation and heat sink
 - Cooling
- Sudden stop due to obstacles
 - High currents → current limits and timeouts

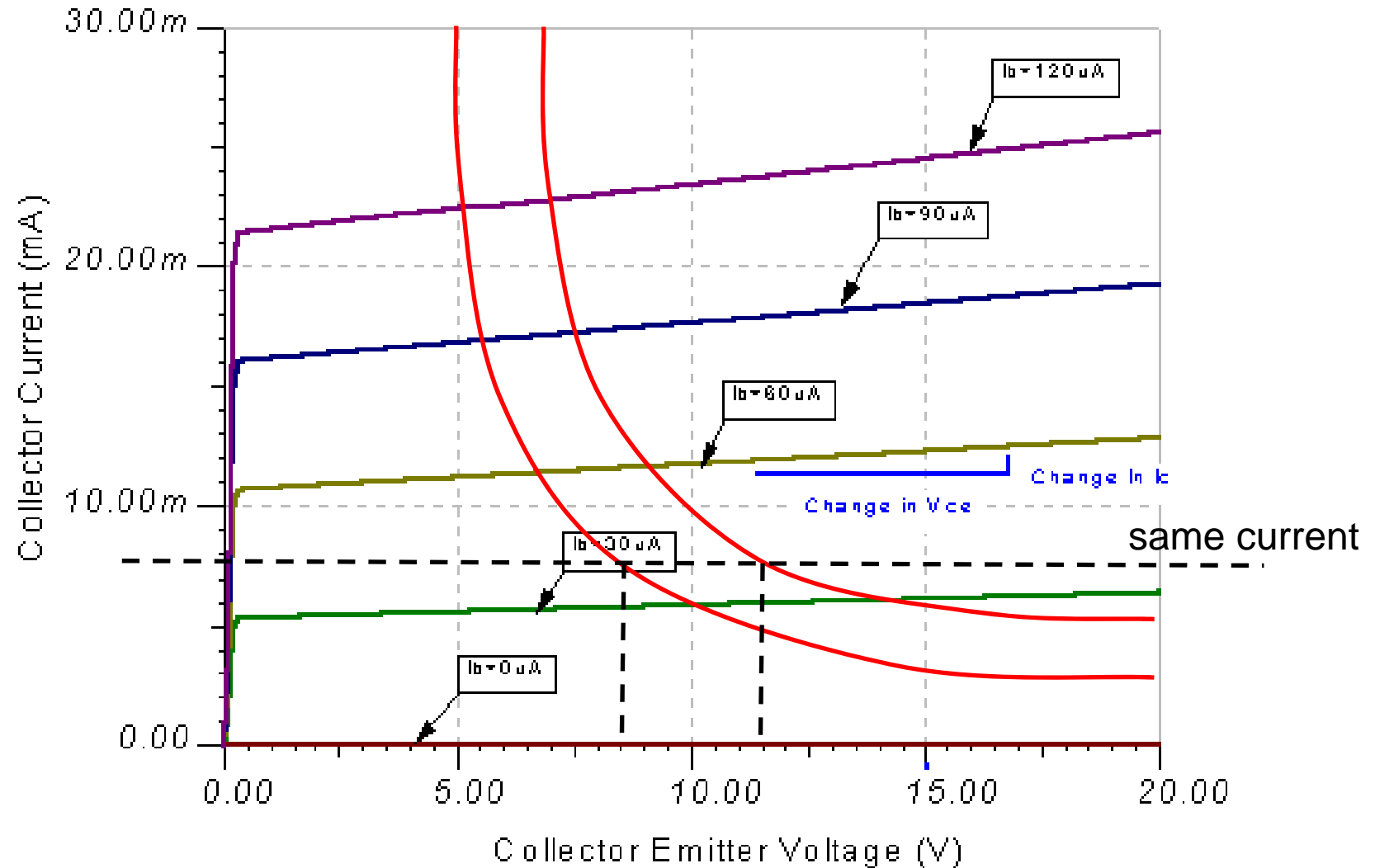
T-type



$$I_c \approx \frac{V_{cc}}{R_{transistor} + R_{motor}}$$

PWM amplifiers

$$P = V_{ce} I_c$$



PWM signal

$$P = V_{ce} I_c$$

- Transistors either “on” or “off”
 - When off, current is very low, little power too
 - When on, V is low, working point close to (or in) saturation, power dissipation is low

Comparison

- 12W for a 6A current using a switching amplifier
- 72W for a corresponding linear amplifier

Why does it work?

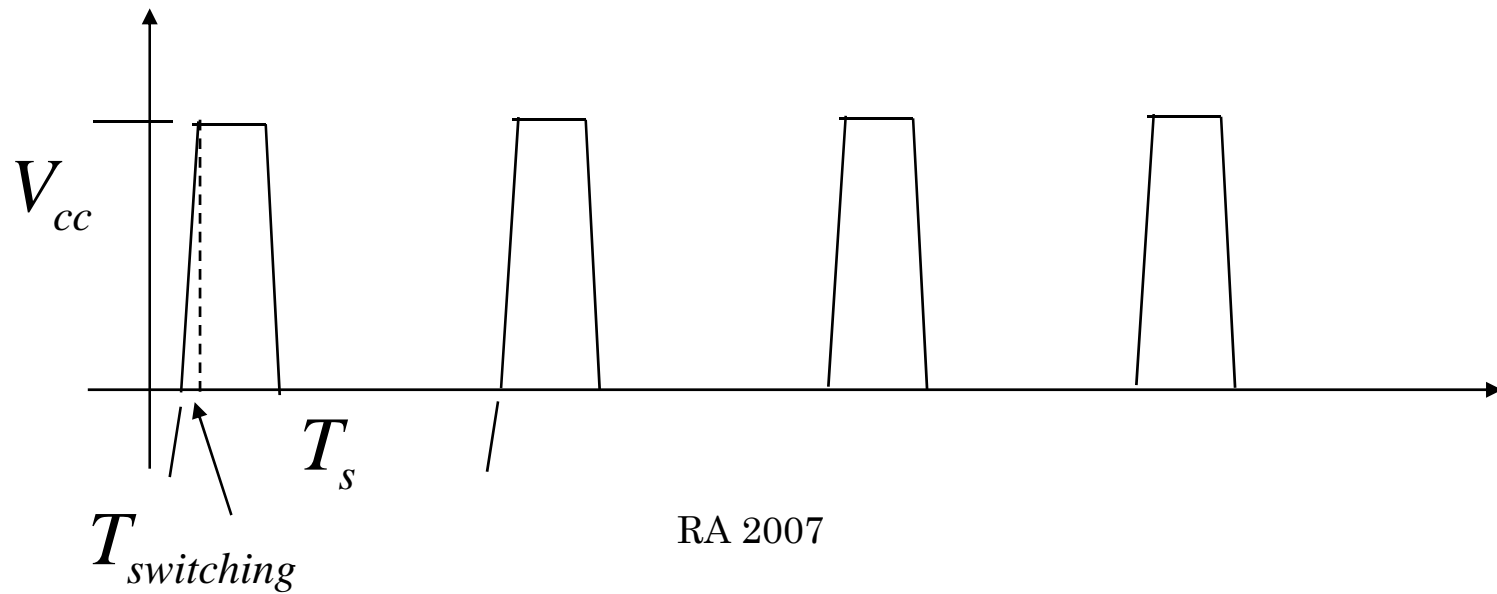
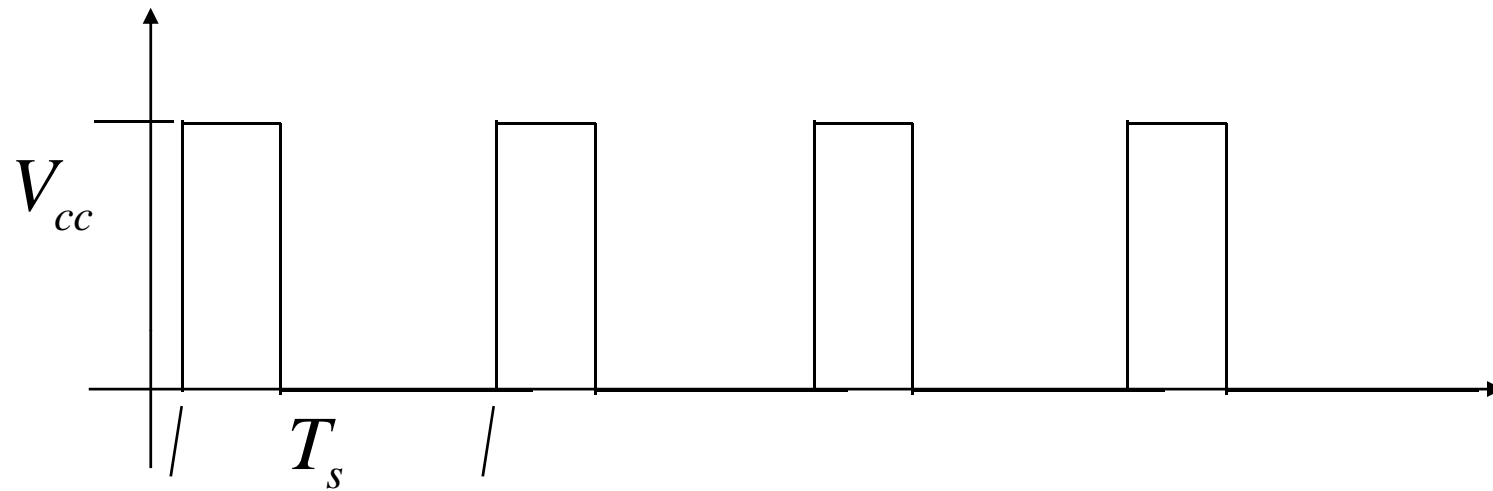
$$\frac{\omega(s)}{V_{arm}(s)} = \frac{K_T / L_a J_T}{s^2 + [(R_a J_T + L_a B) / L_a J_T] s + (K_T K_E + R_a B) / L_a J_T}$$

- In practice the motor transfer function is a low-pass filter

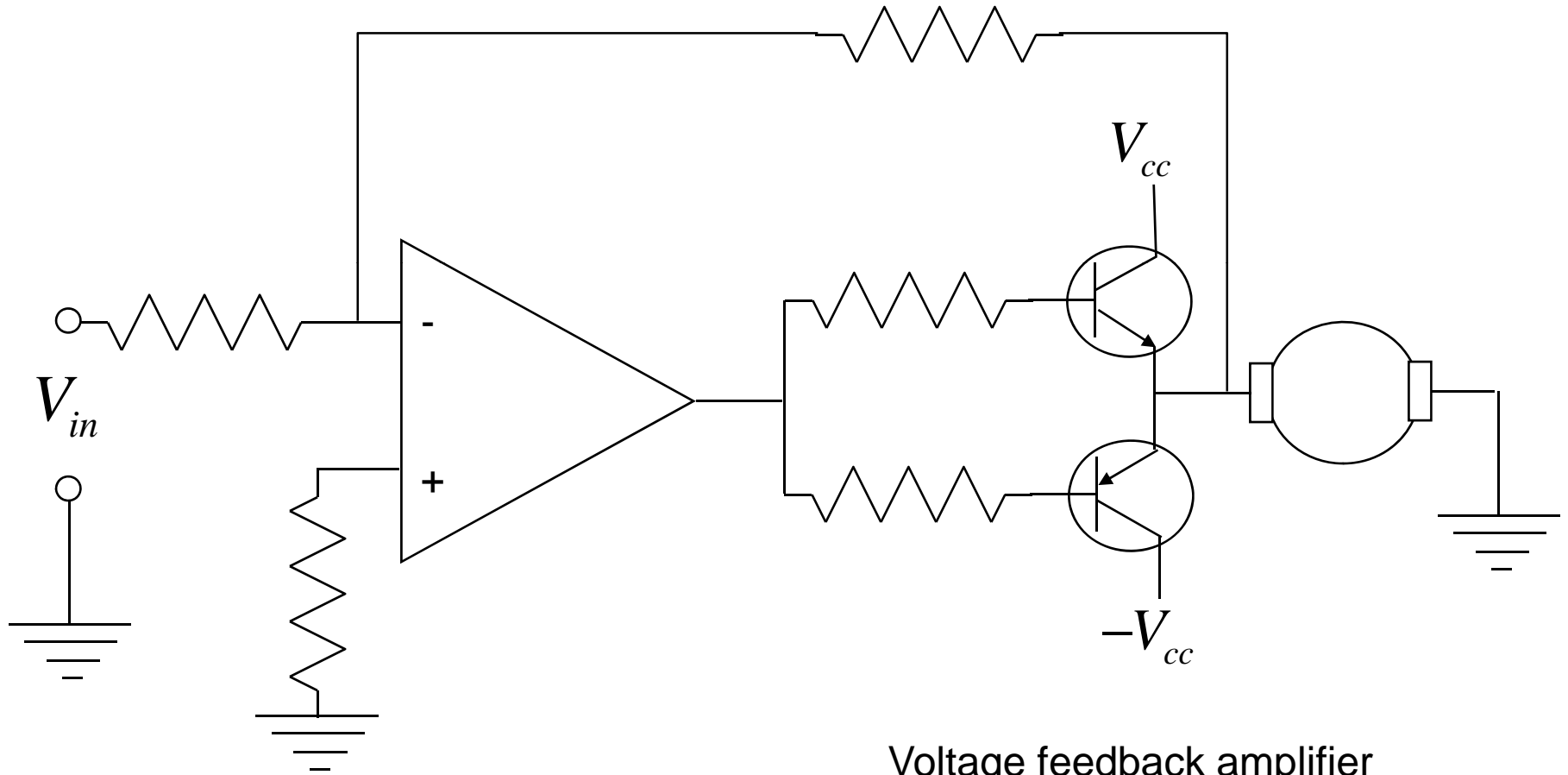
$$T_s \text{ with } f_s \gg f_E (f_s > 100 f_E)$$

- Switching frequency must be high enough

PWM signal

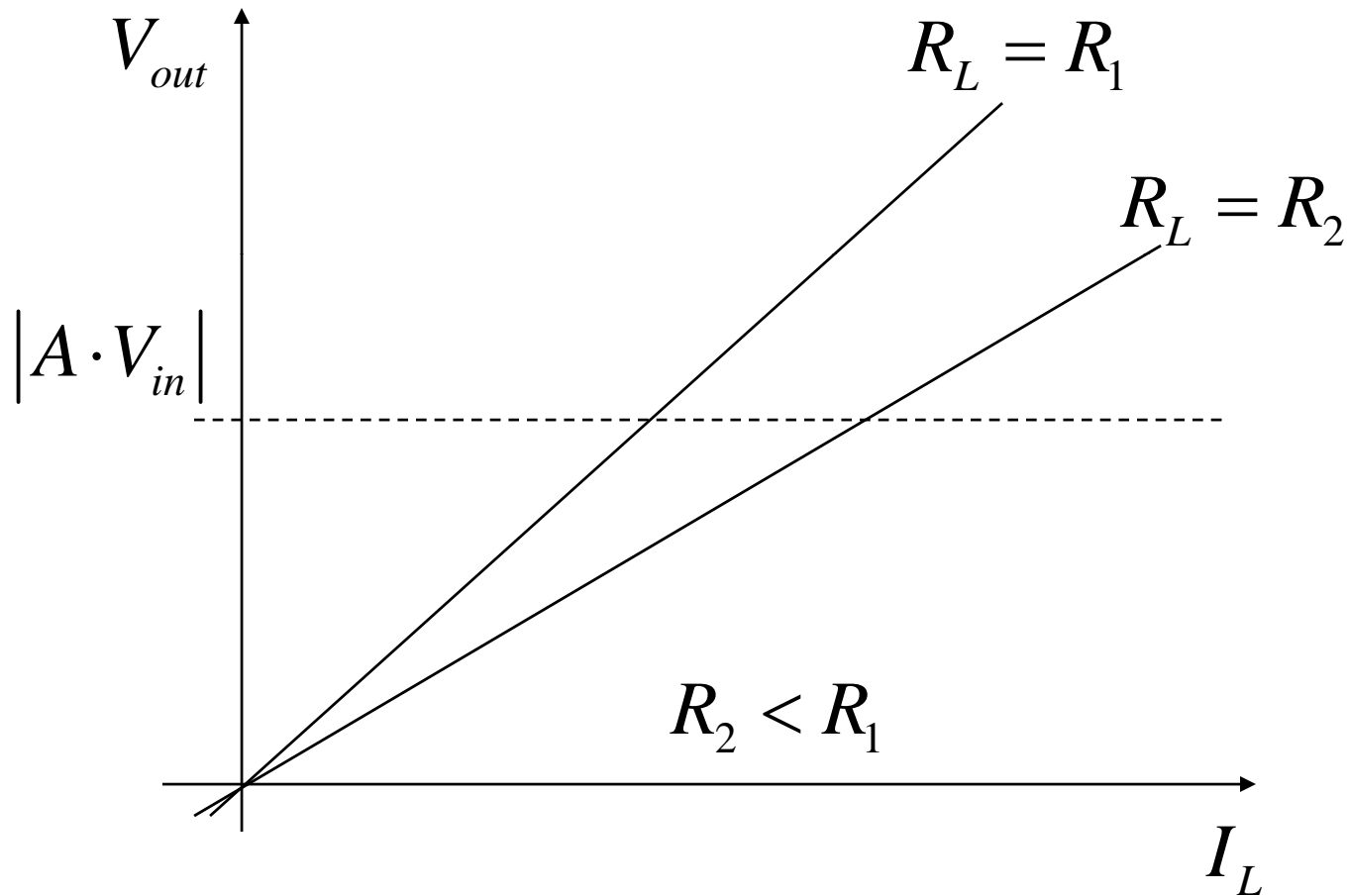


Feedback in servo amplifiers

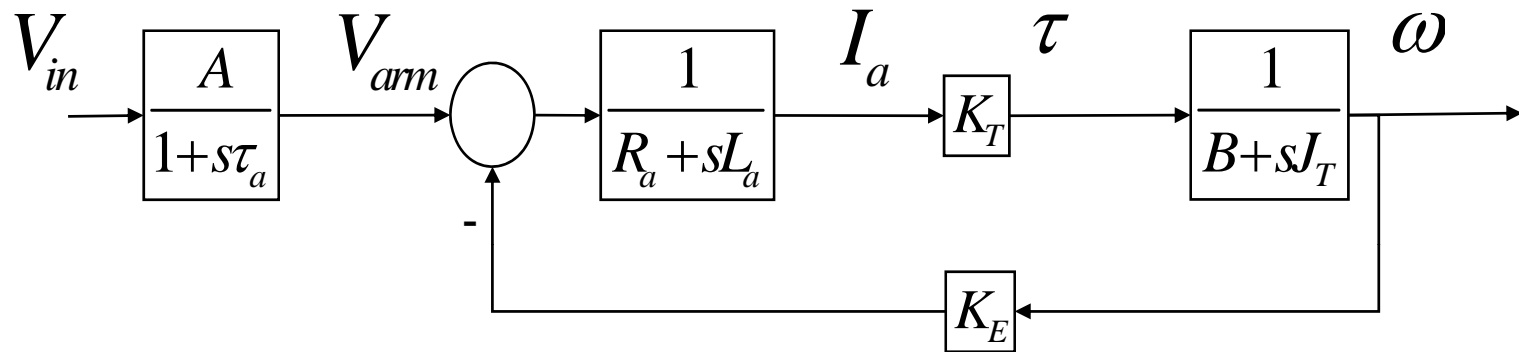


Voltage feedback amplifier

Operating characteristic

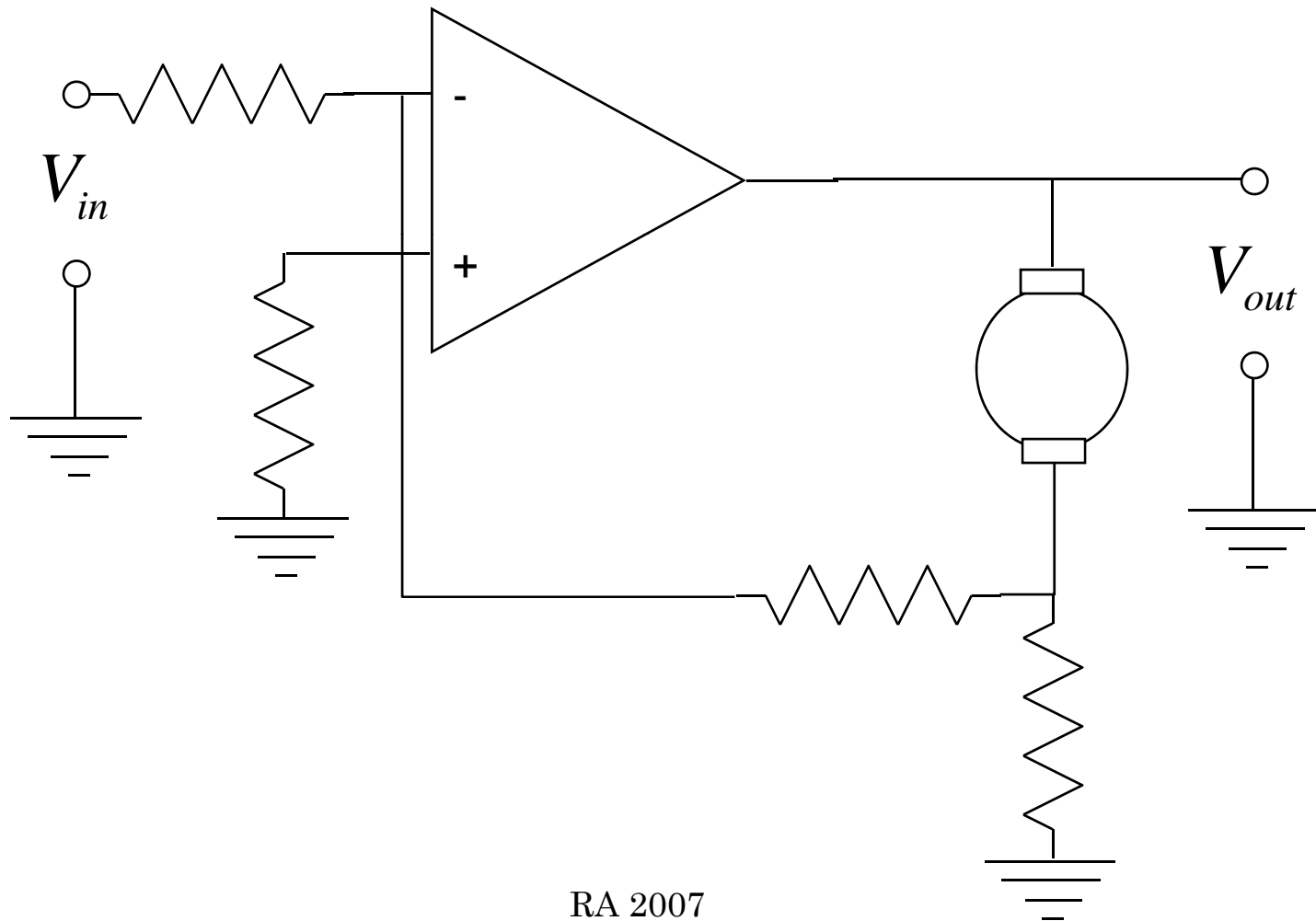


We've already seen this

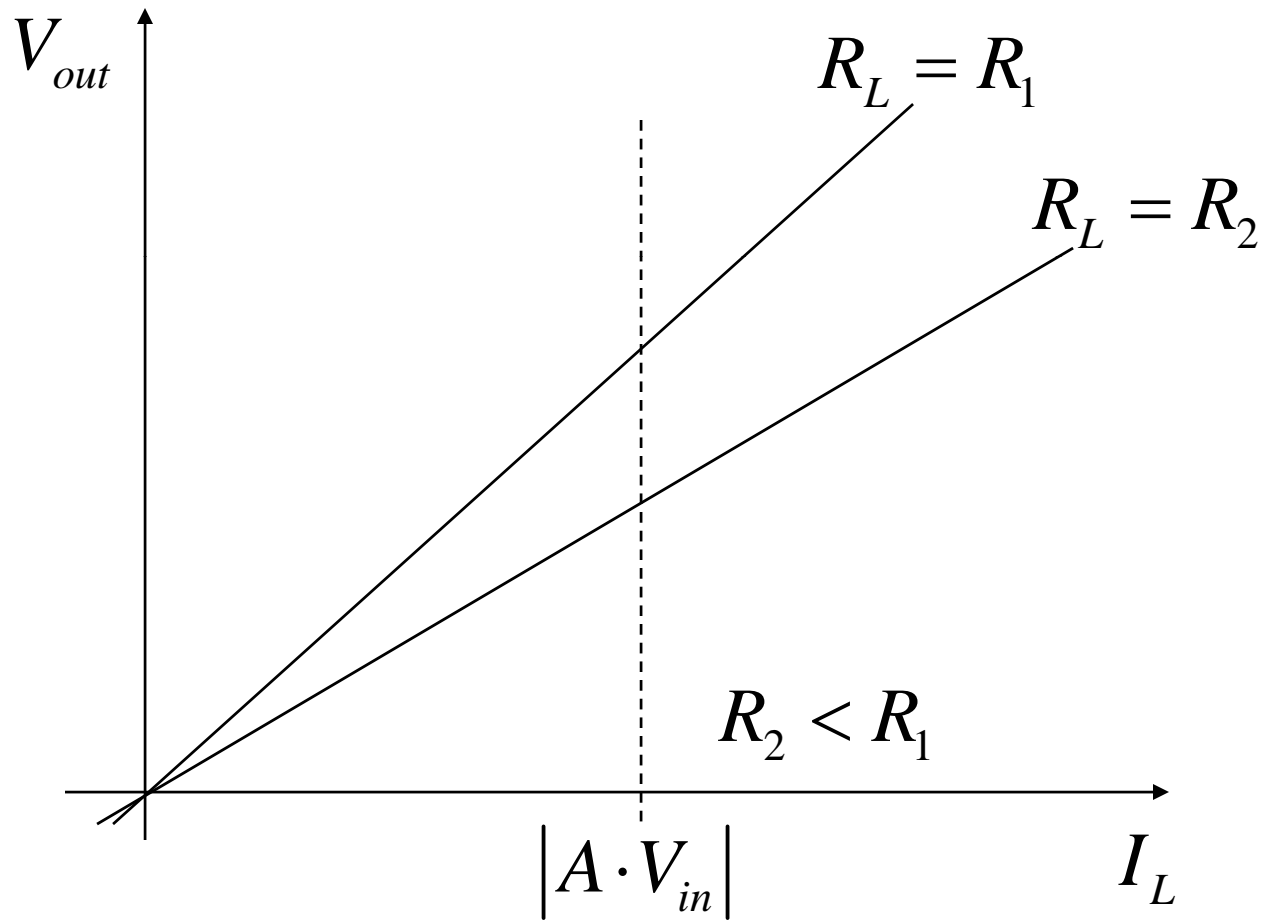


$$\frac{\omega(s)}{V_{in}(s)} = \frac{K_T / L_a J_T}{s^2 + [(R_a J_T + L_a B) / L_a J_T] s + (K_T K_E + R_a B) / L_a J_T} \frac{A_v}{(1 + s\tau_a)}$$

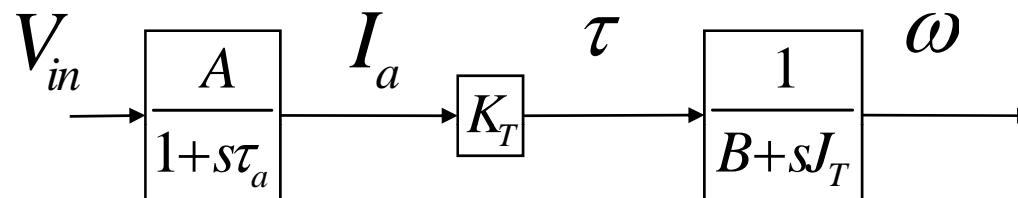
Current feedback



Current feedback

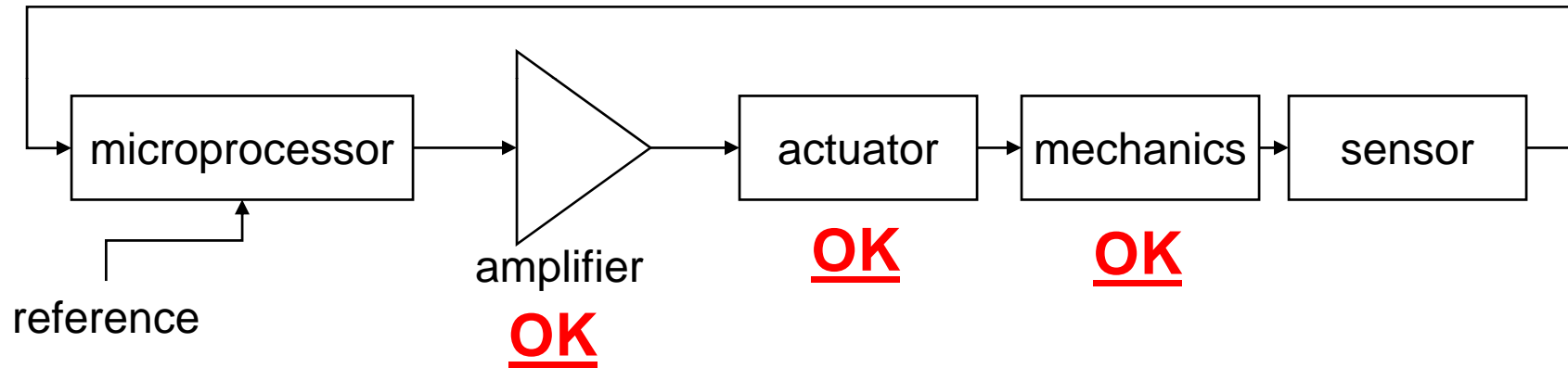


Motor driven by a current amplifier



$$\frac{\omega(s)}{V_{in}(s)} = \frac{K_T A_i}{(sJ_T + B)(1 + s\tau_a)}$$

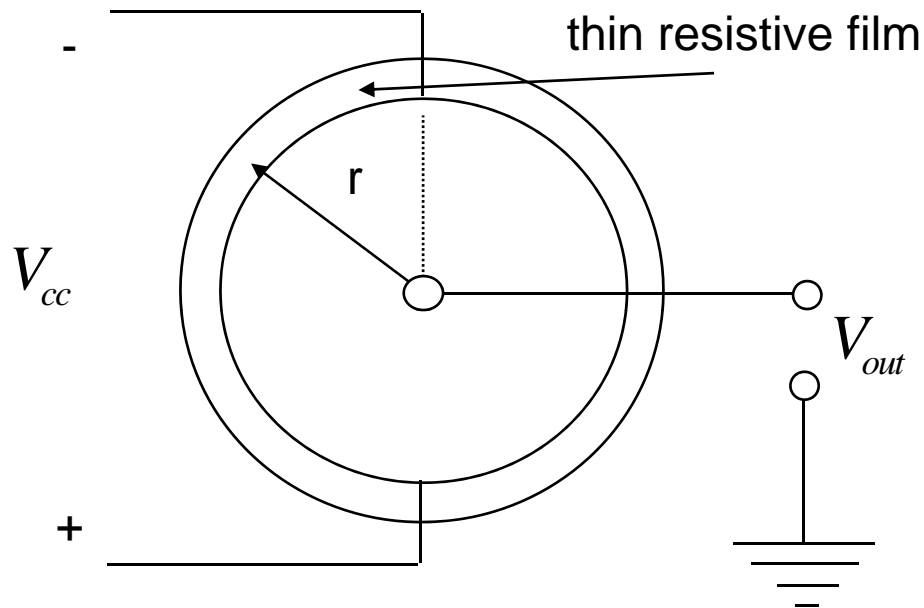
Back to the global view



Sensors

- Potentiometers
- Encoders
- Tachometers
- Inertial sensors
- Strain gauges
- Hall-effect sensors
- and many more...

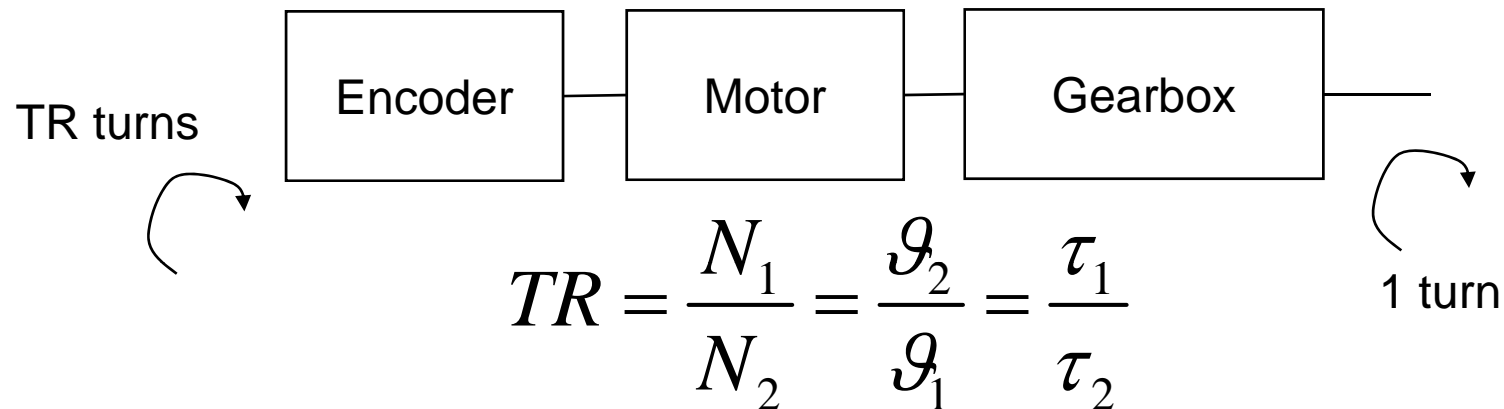
Potentiometer



$$V_{out} = \frac{r}{R} V_{cc}$$

- Simple but noisy
- Requires A/D conversion
- Absolute position (good!)

Note



$$\tau_2 = \frac{N_2}{N_1} \tau_1 \Rightarrow (\text{most of the time}) N_2 > N_1$$

$$g_2 = \frac{N_1}{N_2} g_1$$

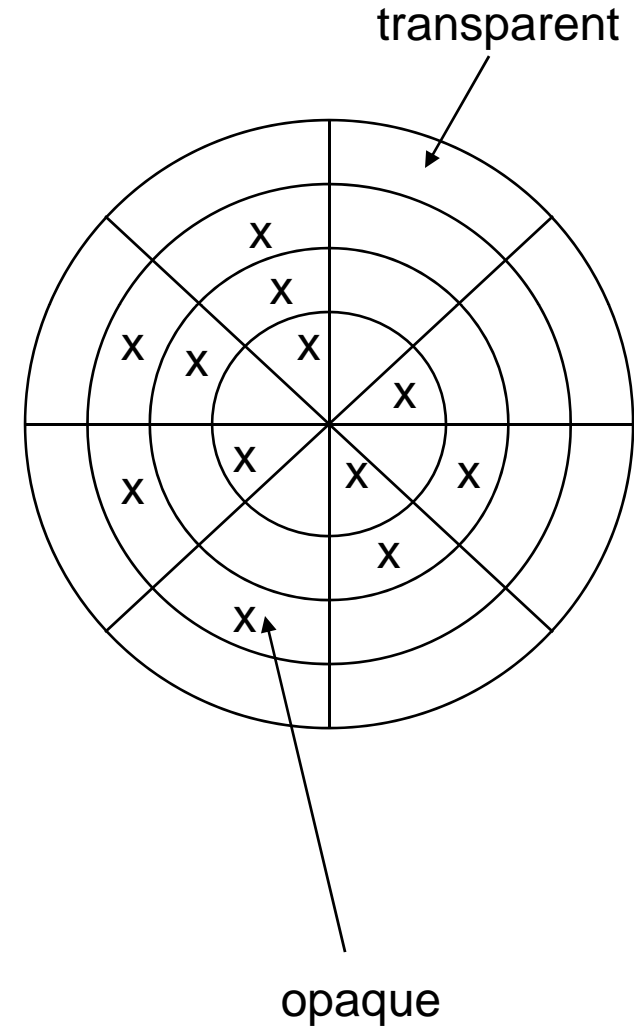
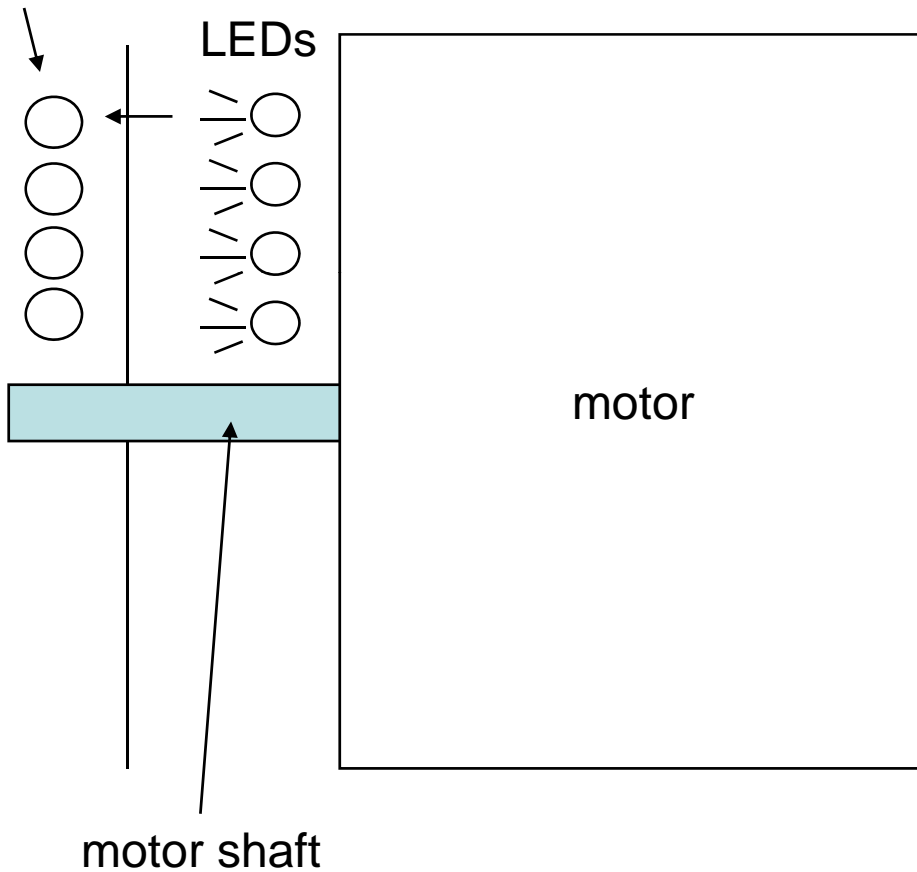
- The resolution of the sensor multiplied by TR

Encoder

- Absolute
- Incremental

Absolute encoder

phototransistors



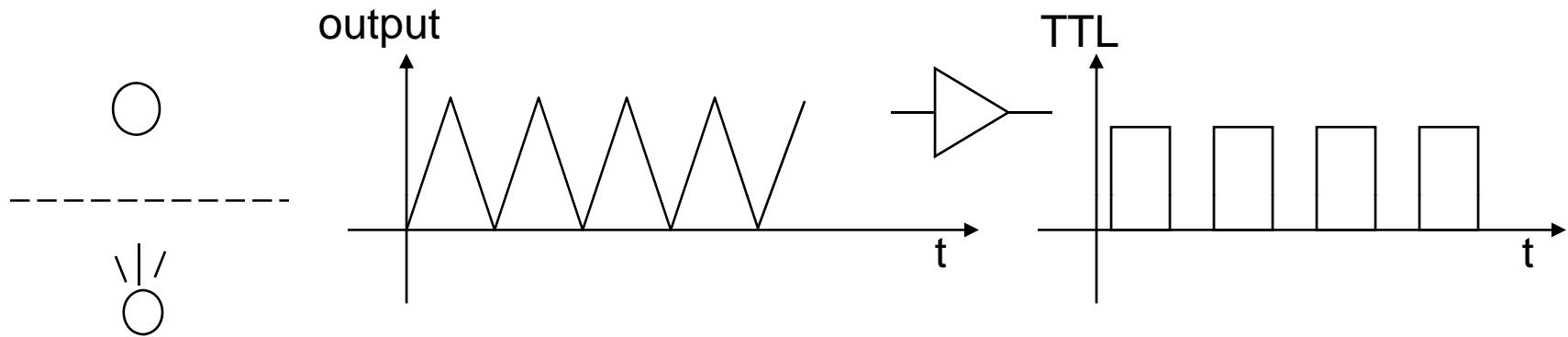
13 bits required for 0.044 degrees

RA 2007

Incremental encoder

- Disk single track instead of multiple
- No absolute position
- Usually an index marks the beginning of a turn

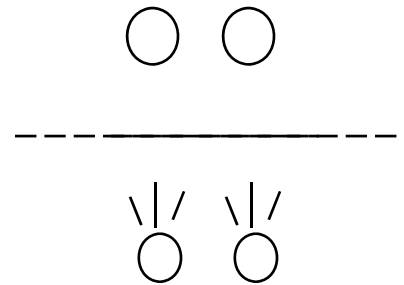
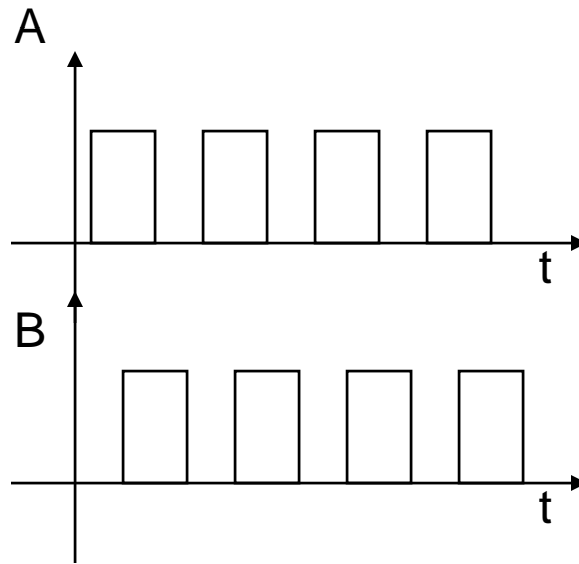
Incremental encoder



- Sensitive to the amount of light collected
- The direction of motion is not measured

Two-channel encoder

- 2 channels 90 degrees apart (quadrature signals) allow measuring the direction of motion



Moreover

- There are “differential” encoders
 - Taking the difference of two sensors 180 degrees apart
- Typically
 - A, B, Index channel
 - A, B, Index (differential)
- A “counter” is used to compute the position from an incremental encoder

Increasing resolution

- Counting UP and DOWN edges
 - X2 or X4 circuits

Absolute position

- A potentiometer and incremental encoder can be used simultaneously: the pot for the “absolute” reference, and the encoder because of good resolution and robustness to noise

Analog locking

- Use digital encoder as much as possible
 - Get to zero error or so using the digital signal
- When close to zeroing the error:
 - Switch to analog: use the analog signal coming from the photodetector (roughly sinusoidal/triangular)
 - Much higher resolution, precise positioning

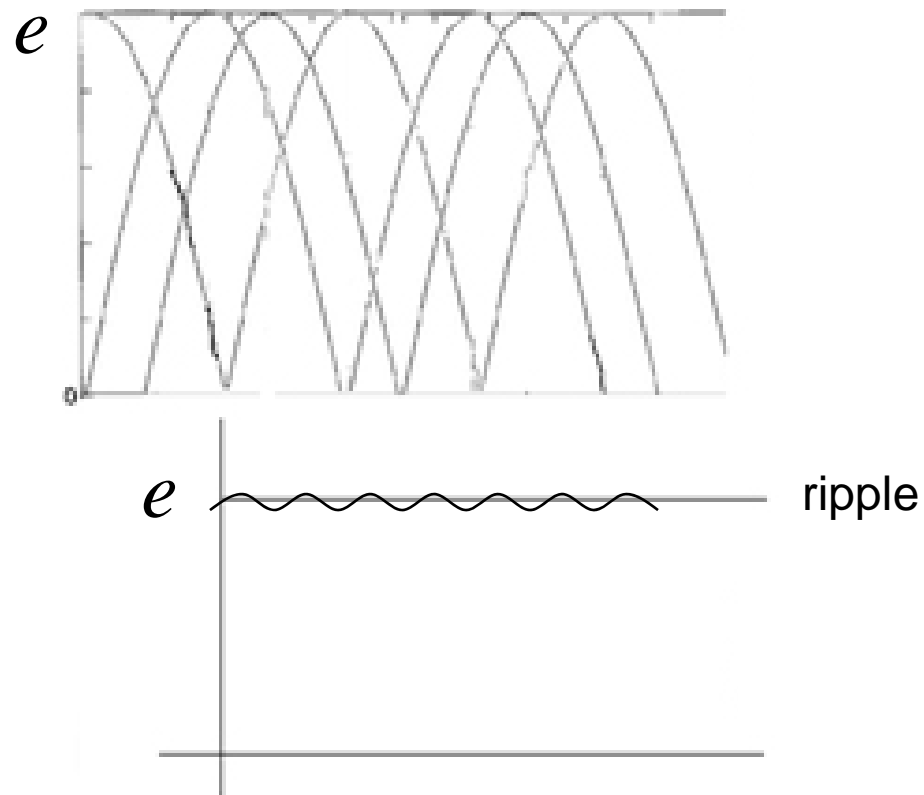
Tachometer

- Use a DC motor
 - The moving coils in the magnetic field will get an induced EMF

$$c \oint_{\delta s} \vec{E} \cdot d\vec{l} = \frac{d}{dt} \iint_s \vec{B} \cdot d\vec{S}$$

- In practice is better to design a special purpose “DC motor” for measuring velocity
- Ripple: typ. 3%

As already seen...



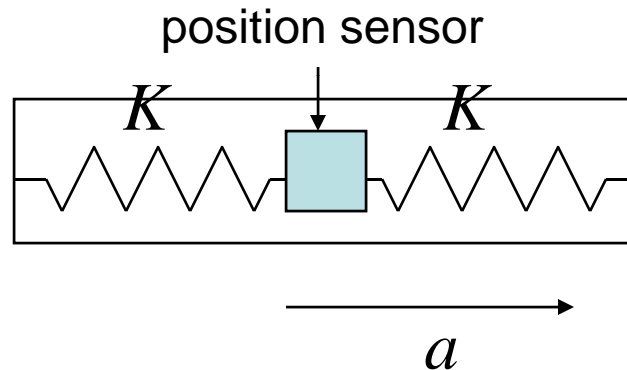
Measuring speed with digital encoders

- Frequency to voltage converters
 - Costly (additional electronics)
- Much better: in software
 - Take the derivative (for free!)

$$v(kT) = \frac{p(kT) - p((k-1)T)}{T}$$

Inertial sensors

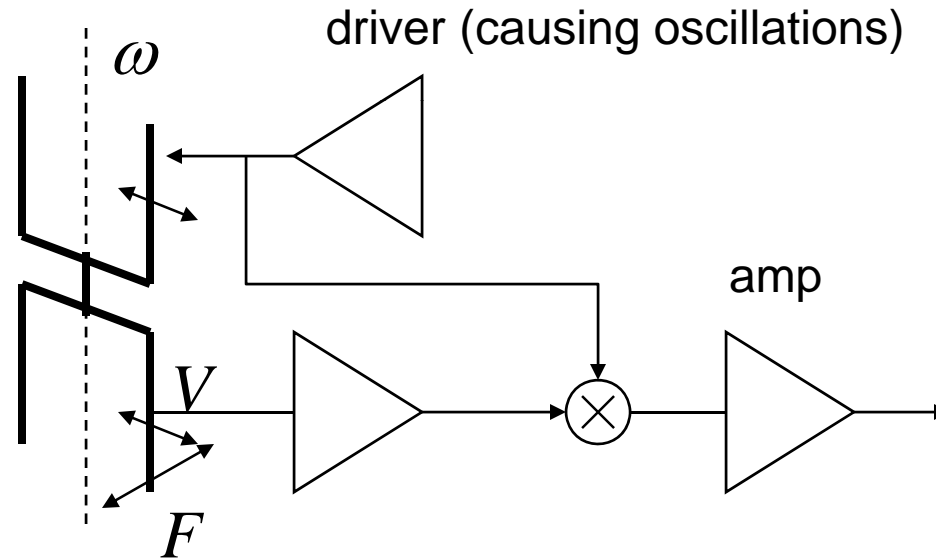
- Accelerometers:



$$Ma = 2Kx \Rightarrow a = \frac{2Kx}{M}$$

Gyroscopes

- Quartz forks



$$F = 2m\omega \times V$$

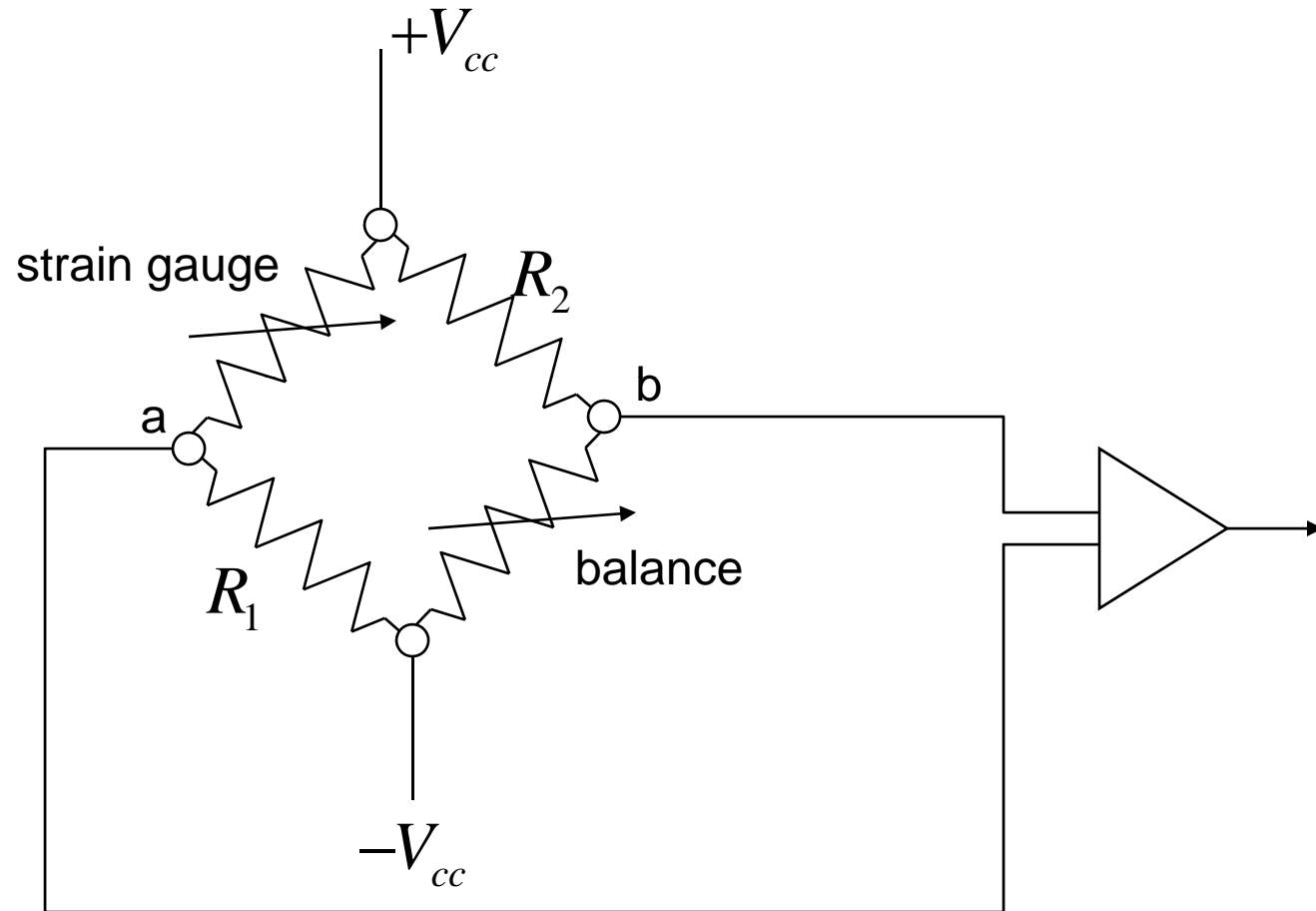
Strain gauges

- Principle: deformation $\rightarrow \Delta R$ (resistance)
 - Example: conductive paint (Al, Cu)
 - The paint covers a deformable non-conducting substrate

$$R = \frac{L}{\sigma A} \Rightarrow \Delta L, A = \text{const} \Rightarrow \Delta R$$

conductivity \nearrow

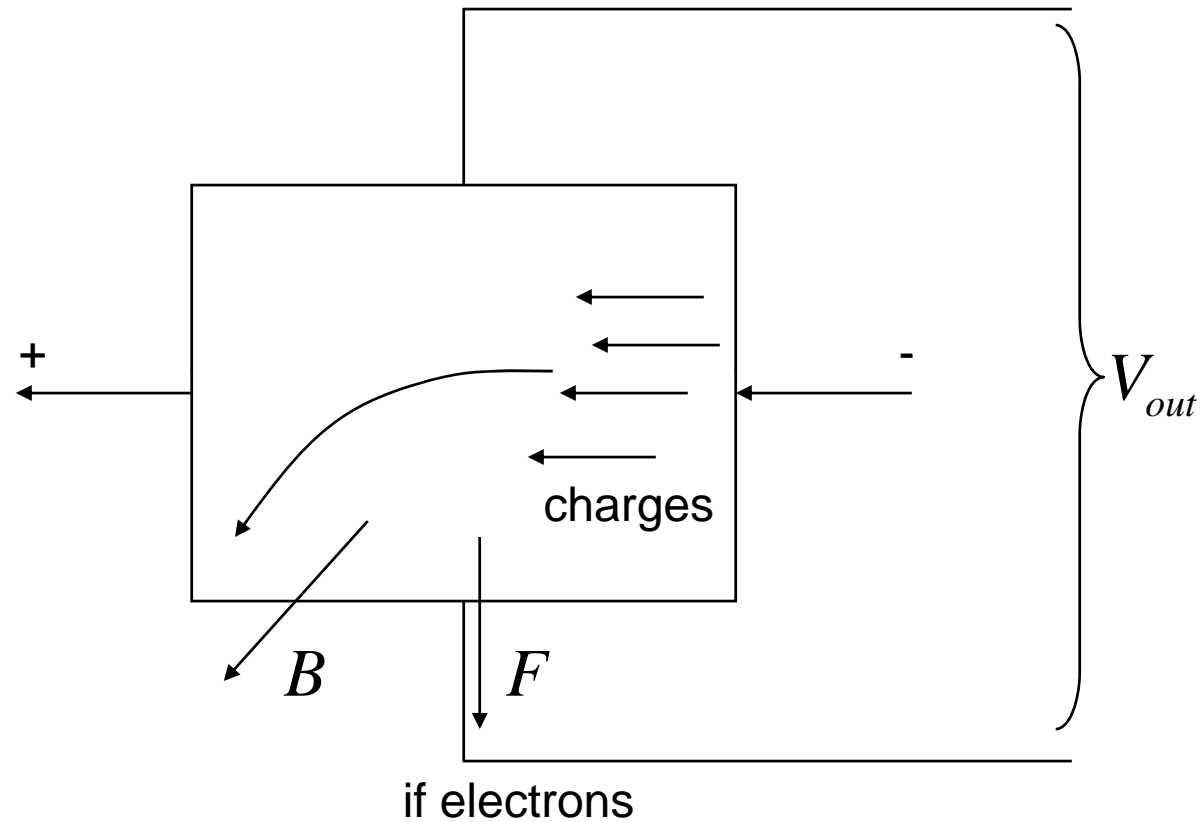
Reading from a strain gauge



$$R_1 R_2 = R_g R_b \Rightarrow V_{ab} = 0$$

$$\Delta V_{ab} = f(\Delta R_g)$$

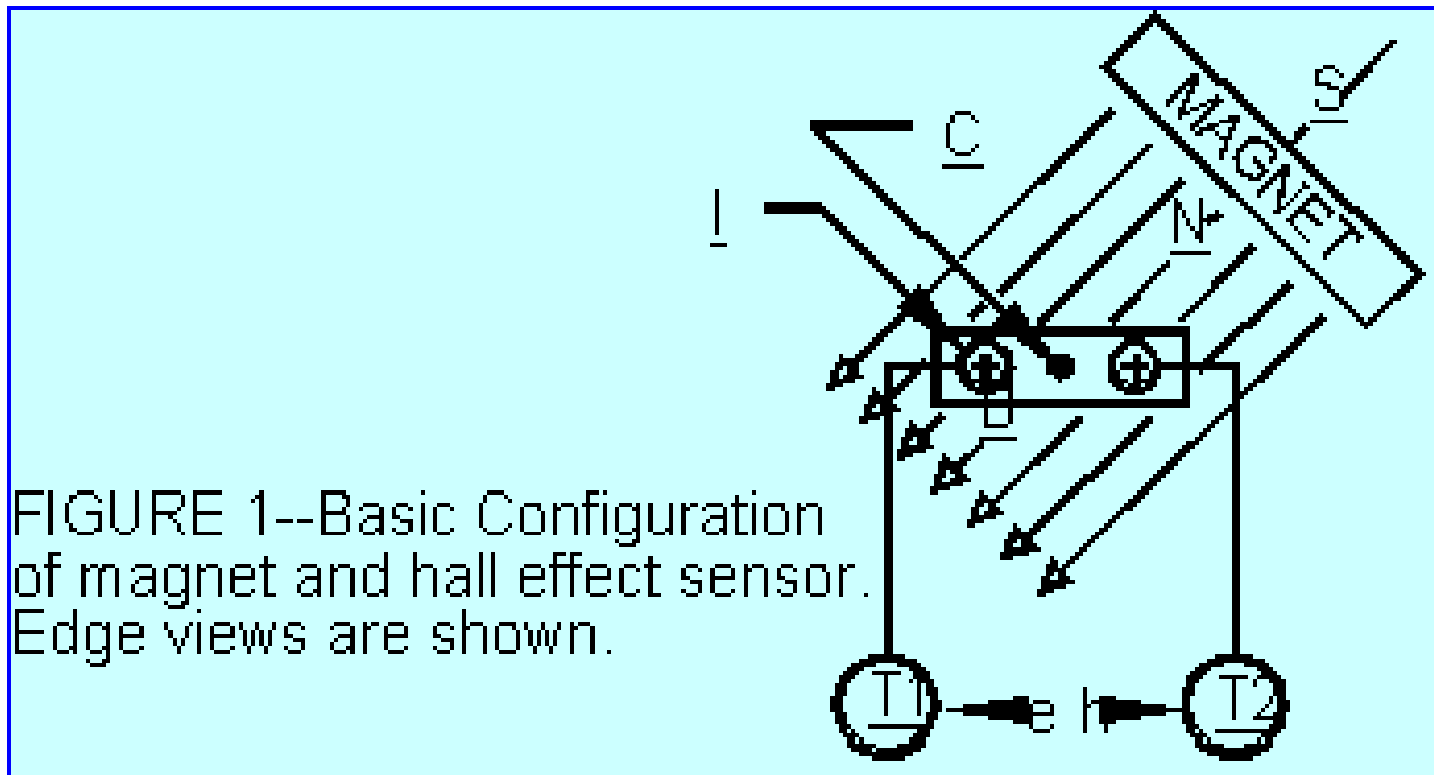
Hall-effect sensors



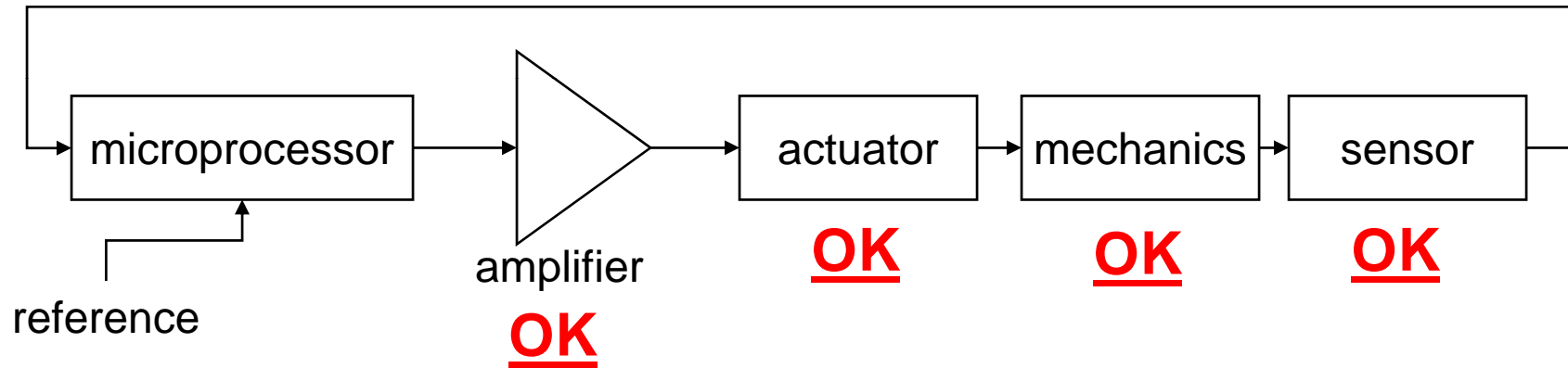
$$F_{\text{lorentz}} = q\vec{v} \times \vec{B}$$

Example

- Measuring angles (magnetic encoders)



Back to the global view



Microprocessors

- Special DSPs for motion control
 - Some are barely programmable (the control law is fixed)
 - Others are general purpose and they are mixed mode (analog and digital in a single chip)

Example

- DSP 16 bit ALU and instruction set
- PWM generator (simply attach this to either T or H amplifier)
- A/D conversion
- CAN bus, Serial ports, digital I/O
- Encoder counters
- Flash memory and RAM on-board
- Enough of all these to control two motors (either brush- or brushless)