Anthropomorphic robotics

WHAT A SINGLE JOINT IS MADE OF
Notation

\[ F = \frac{d}{dt} (mv) = m\ddot{x} \]

Since links are physical objects with mass

\[ \tau = J\ddot{\theta} \]

\( J \) = moment of inertia

\[ \tau = F \times r \]
Moment of inertia

Around an axis

\[ J = \sum_{i=1}^{N} m_i r_i^2 \]

\[ J = \int_{volume} \rho r^2 dV \]
Parallel axis theorem

\[ J = J_c + Mr^2 \]

Through the center of gravity
Example

Mass = M, \( \rho = \frac{M}{l} \)

\( J_x = 0 \)

\[
J_y = \rho \int r^2 dV = \rho \int_{-l/2}^{l/2} x^2 dx = \rho \left( \frac{1}{3} x^3 \right)_{-l/2}^{l/2} = \frac{Ml^2}{12}
\]

\[
J_{y=-l/2} = \frac{Ml^2}{12} + M \frac{l^2}{4} = M \frac{l^2}{3}
\]
Sum of $J$

\[ J_x = \frac{M}{12}(a^2 + b^2) \]

\[ J_y = \frac{M}{12}(a^2 + c^2) \]

\[ J_z = \frac{M}{12}(b^2 + c^2) \]

E.g. \( J_{\text{top-}x} = \frac{M_{\text{top}}}{12}(a^2 + c^2) + M_{\text{top}}(\frac{a}{2} + L)^2 \) \( J_{\text{hand-}x} = J_{\text{top-}x} + J_{\text{side-}x} + J_{\text{bottom-}x} \)
Experimental estimation of $J$

Use a photodiode and a computer to measure the frequency

$\frac{f}{2\pi} = \sqrt[2]{\frac{K}{J}}$

Requires calibration from known $J$
Experimental estimation of $J$

$$f \approx \frac{1}{2\pi} \sqrt{\frac{Mgh}{J}}$$
Work and power

\[ E = \text{const} \quad \text{if} \quad \sum F_{ext} = 0 \]

\[ W = \int_{s_1}^{s_2} Fds \quad W = \Delta E, E = \text{energy} \]

\[ K = \frac{1}{2}mv^2 \quad \text{kinetic energy} \]

\[ P = \frac{dW}{dt} \quad \text{Power} \rightarrow \quad P = Fv \]
Rotational case

\[ E = \text{const} \quad \text{if} \quad \sum \tau_{\text{ext}} = 0 \]

\[ W = \int_{\theta_1}^{\theta_2} \tau d\theta \]

\[ W = \Delta E, \ E = \text{energy} \]

\[ K = \frac{1}{2} J \omega^2 \]

\[ P = \frac{dW}{dt} \quad \text{Power} \rightarrow \quad P = \tau \omega \]
Single joint model

- Motor
- Reduction gears
- Link/load
- End-point
Motor

- Let’s imagine for now that it is something that generates a given torque
Mechanical transmission

• Gears
• Belts
• Lead screws
• Cables
• Cams
• etc.
Distance traveled is the same:

\[ r_1 \theta_1 = r_2 \theta_2 \]

Because the size of teeth is the same:

\[ \frac{N_1}{r_1} = \frac{N_2}{r_2} \]
Furthermore...

$$ r_1 \mathcal{G}_1 = r_2 \mathcal{G}_2 $$

$$ \frac{N_1}{r_1} = \frac{N_2}{r_2} $$

- No loss of energy
  $$ \tau_1 \mathcal{G}_1 = \tau_2 \mathcal{G}_2 $$
Combining...

\[
\frac{N_1}{N_2} = \frac{r_1}{r_2} = \frac{\vartheta_2}{\vartheta_1} = \frac{\tau_1}{\tau_2} = \frac{\omega_2}{\omega_1} = \frac{\alpha_2}{\alpha_1}
\]

- # of teeth
- Inverse relationship between speed and torque
- Input
- Output
- \( \tau_2 = \tau_1 \frac{N_2}{N_1} \)
- \( TR = \frac{N_1}{N_2} \)
- Mechanical parameter
Equivalent $J$

\[ \ddot{g}_1 J_1 \leftrightarrow \tau_1 = \tau_2 \frac{N_1}{N_2} = \ddot{g}_2 J_2 \frac{N_1}{N_2} \]

\[ J_1 = \frac{\ddot{g}_2}{\ddot{g}_1} \frac{N_1}{N_2} \Rightarrow \left( \frac{N_1}{N_2} \right)^2 J_2 \]

\[ J_1 = TR^2 J_2 \]

- $J$ as seen from the motor
In reality

\[ \tau_2 = \tau_1 \frac{1}{TR} \eta \]

- Where \( \eta \) is the efficiency of the mechanism (from 0 to 1)
- \( \eta \) is related to power, speed ratio doesn’t change
- \( \eta \) is also the ratio of input power vs. power at the output
For example

\[ \eta = 20\% \]

\[ \eta = 1 \]
### Specifications

<table>
<thead>
<tr>
<th>Reduction ratio (nominal)</th>
<th>Weight without motor</th>
<th>Length without motor</th>
<th>Length with motor</th>
<th>Output continuous operation</th>
<th>Torque intermittent operation</th>
<th>Direction of rotation (reversible)</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA 2009-10</td>
<td></td>
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<tr>
<td>3.71:1</td>
<td>g</td>
<td>L2 mm</td>
<td>L1 mm</td>
<td>1319 T</td>
<td>1331 T</td>
<td>1336 U</td>
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<td>14:1</td>
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<td>300 mNm</td>
<td>450 mNm</td>
</tr>
</tbody>
</table>
Motion conversion

• Start with

\[ \tau_2 = \frac{N_2}{N_1} \tau_1 \]

• Design \( TR \), more torque (usually)

\[ TR < 1 \]

\[ N_2 > N_1 \]

\[ J_1 < J_2 \iff \omega_2 < \omega_1 \]
Viscous friction

• Easy:

\[ \tau_{viscous} = B_2 \dot{\theta}_2 \]

\[ \tau_{eq\_viscous} = TR \cdot \tau_{viscous} = TR \cdot B_2 \dot{\theta}_2 \]

\[ B_{eq} \dot{\theta}_1 = TR \cdot B_2 \dot{\theta}_2 \Rightarrow B_{eq} = TR^2 B_2 \]

• Coulomb friction:

\[ \tau_{eq} = TR \cdot F_c \text{ sgn}(\dot{\theta}_2) \]
Lead screw

- Rotary to linear motion conversion
  (P=pitch in # of turns/mm or inches)

\[
\theta[\text{rad}] = 2\pi Px \\
\dot{\theta} = 2\pi P\dot{x}
\]

\[
E_{\text{rot}} = E_{\text{lin}} \Rightarrow \frac{1}{2} M_{\text{load}} v^2 = \frac{1}{2} J \omega^2 \Rightarrow
\]

\[\Rightarrow J = \frac{M_{\text{load}}}{(2\pi P)^2}\]
Harmonic drives

From the harmonic drive website
http://www.harmonicdrive.de
Gearhead (for real)

Standard (serial)

Planetary
Example

• Designing the single joint
  ○ Given:

  \[ \ddot{\theta}_{\text{max}} \Rightarrow \tau = J_{eq} \ddot{\theta} \Rightarrow \tau_{\text{max}} = J_{eq} \ddot{\theta}_{\text{max}} = J_{\text{load}} TR^2 \ddot{\theta}_{\text{max}} \]

  • Then taking into account some more realistic components:

  \[ \tau_{\text{max}} = J_{\text{load}} \frac{TR^2}{\eta} \ddot{\theta}_{\text{max}} \]
\[
\tau_{\text{max}} = J_{\text{load}} \frac{TR^2}{\eta} \dot{\theta}_{\text{max}}
\]

\[
P = \tau_{\text{max}} \dot{\theta} \Rightarrow \text{given } \dot{\theta}_{\text{max}} \Rightarrow \text{get } P
\]

This guarantees that the motor can still deliver maximum torque at maximum speed.

motor power, from catalog
More on real world components

- Efficiency
- Eccentricity
- Backlash
- Vibrations

- To get better results during design mechanical systems can be simulated
Control of a single joint

- Microprocessor
- Amplifier
- Actuator
- Mechanics
- Sensor

Reference
Components

- **Digital microprocessor:**
  - Microcontroller, processor + special interfaces
- **Amplifier (drives the motor):**
  - Turns control signals into power signals
- **Actuator**
  - E.g. electric motor
- **Mechanics/load**
  - The robot!
- **Sensors**
  - For intelligence
Actuators

- Various types:
  - AC, DC, stepper, etc.
  - DC
    - Brushless
    - With brushes
- We’ll have a look at the DC with brushes, simple to control, widely used in robotics
DC-brushless
DC with brushes
Modeling the DC motor

- Speed-torque and torque-current relationships are linear
In particular

\[ \tau \]

\[ \omega \]

No-load speed

current-torque

speed-torque

stall torque
## Real numbers!

### http://www.minimotor.ch

### Series 1331 ... SR

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1331 T</th>
<th>006 SR</th>
<th>012 SR</th>
<th>024 SR</th>
<th>Unit</th>
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<tr>
<td>Nominal voltage</td>
<td>11V</td>
<td>6V</td>
<td>12V</td>
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<td>Volt</td>
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<td>Terminal resistance</td>
<td>R</td>
<td>2.8Ω</td>
<td>13.7Ω</td>
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<td>Output power</td>
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<td>3.11W</td>
<td>2.57W</td>
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<td>Efficiency</td>
<td>ηmax</td>
<td>81%</td>
<td>80%</td>
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<td>No-load speed</td>
<td>n0</td>
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<td>9500</td>
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<td>No-load current</td>
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<td>Stall torque</td>
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<td>Friction torque</td>
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<td>kτ</td>
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<td>0.087</td>
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<td>Slope of n-M curve</td>
<td>Δn/ΔM</td>
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<td>K/W</td>
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<td></td>
<td></td>
<td>s</td>
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<td>Operating temperature range:</td>
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<td></td>
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<td>°C</td>
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<tr>
<td>- Motor</td>
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<td>-30...+85 (optional 55...+125)</td>
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<td></td>
<td>°C</td>
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<tr>
<td>- Rotor, max. permissible</td>
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<td>+125</td>
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<td></td>
<td>°C</td>
</tr>
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</table>

RA 2009-10
$E_g = \omega(t) K_E$
Meaning of components

- $R_a$: Armature resistance (including brushes)
- $V_{arm}$: Armature voltage
- $R_l$: Losses due to magnetic field
- $E_g$: Back EMF produced by the rotation of the armature in the field
- $L_a$: Coil inductance
We can write...

\[ V_{arm} = R_a I_a + L_a \dot{I}_a + \omega(t) K_E \]

for \( R_l << R_a \)

which is the case at the frequency of interest, and we also have...

\[ \tau = K_T I_a \]
On torque and current

\[ F = iL \times \vec{B} \]

\[ F_{\text{lorentz}} = q\vec{v} \times \vec{B} \]
Thus for many coils...

Torque is related to the total current.

Torque from each loop added.
Back to motor modeling...

\[ \tau = (J_M + J_L) \dot{\omega}(t) + B\omega(t) + \tau_f + \tau_{gr} \]

- \( \tau \): Torque generated
- \( J_M \): Inertia of the motor
- \( J_L \): Inertia of the load
- \( \tau_f \): Friction
- \( \tau_{gr} \): Gravity
Furthermore...

\[ V_{arm} = R_a I_a + L_a \dot{I}_a + \omega(t) K_E \]

\[ \tau = K_T I_a \]

\[ \tau = (J_M + J_L) \ddot{\omega}(t) + B \omega(t) + \tau_f + \tau_{gr} \]
Consequently

\[
\begin{bmatrix}
\dot{I}_a \\
\dot{\omega}
\end{bmatrix} = \begin{bmatrix}
\frac{R_a}{L_a} & \frac{K_E}{L_a} \\
\frac{K_T}{J_M + J_L} & \frac{B}{J_M + J_L}
\end{bmatrix} \cdot \begin{bmatrix}
I_a \\
\omega
\end{bmatrix} + \begin{bmatrix}
\frac{-V_{arm}}{L_a} \\
\frac{\tau_f + \tau_{gr}}{J_M + J_L}
\end{bmatrix}
\]

- A linear system of two equations (differential)
- Q: can you write a transfer function from these equations?
- Q: can you transform the equations into a block diagram?
By Laplace-transforming

\[ V_{\text{arm}}(s) = R_a I_a(s) + L_a I_a(s)s + \omega(s)K_E \Rightarrow I_a(s) = \frac{V_{\text{arm}}(s) - \omega(s)K_E}{R_a + L_a s} \]

\[ \tau = K_T I_a \]

\[ K_T \frac{V_{\text{arm}}(s) - \omega(s)K_E}{R_a + L_a s} = (J_M + J_L)\omega(s)s + B\omega(s) + \tau_f + \tau_{gr} \]
and finally

\[
\frac{\omega(s)}{V_{\text{arm}}(s)} = \frac{K_T / L_a J_T}{s^2 + [(R_a J_T + L_a B) / L_a J_T] s + (K_T K_E + R_a B) / L_a J_T}
\]

- Considering gravity and friction as additional inputs
Block diagram

\[ V_{arm} \xrightarrow{1} I_a \xrightarrow{K_T} \tau \xrightarrow{1} \omega \]

\[ \tau_f + \tau_{gr} \]

\[ \frac{1}{R_a + sL_a} \]

\[ \frac{1}{B + sJ_T} \]

\[ K_E \]
• Control: determine $V_a$ so to move the motor as desired

• Root locus
• Pole placement
• Frequency response
• Etc.
First block diagram

\[ H_{\text{open\_loop}} = \frac{A}{1 + s\tau_a} \frac{K_m}{1 + s\tau_m} \frac{K_p}{s} \]
Root locus

\[ H_{\text{open\_loop}} = \frac{A}{1 + s\tau_a} \frac{K_m}{1 + s\tau_m} \frac{K_p}{s} \]

\[ K = AK_mK_p \]
Changing $K$

Diagram showing the transition of $\mathcal{G}$ over time ($t$) for small and higher values of $K$. The graph illustrates the behavior of $\mathcal{G}$ as $K$ changes, with different patterns for small and higher $K$. The y-axis represents $\mathcal{G}$, and the x-axis represents time ($t$).
Let’s add something, second diagram

\[ H_{\text{open\_loop}} = \frac{AK_m (K_p + sK_g)}{(1 + s\tau_a)(1 + s\tau_m)s} \]
Analysis

\[ H_{\text{open\_loop}} = \frac{AK_m K_p (1 + s \frac{K_g}{K_p})}{(1 + s \tau_a)(1 + s \tau_m)s} \]

\[ K = AK_m K_p \]

\[ Z_{\text{feedback}} = \frac{K_g}{K_p} \]
Root locus (case 1)

\[ j \omega \]

\[ \sigma \]

\[ -\frac{1}{\tau_a} \]

\[ -\frac{1}{\tau_m} \]

\[ -\frac{K_p}{K_g} \]

\[ \frac{K_p}{K_g} < \frac{1}{\tau_m} \]
Root locus (case 2)

\[ -\frac{1}{\tau_a}, \quad -\frac{K_p}{K_g}, \quad -\frac{1}{\tau_m} \]

\[ \frac{K_p}{K_g} > \frac{1}{\tau_m} \]
Overall...
Error and performance

\[ \mathcal{J} = \frac{\mathcal{J_d}}{s} \]

\[ M(s) = \frac{K_T}{(R_a + sL_a)(B + sJ_T) + K_E K_T} \]

\[ \mathcal{J}(s) = \frac{1}{s} \omega(s) \]

\[ \omega(s) = \frac{A}{1 + s\tau_a} M(s) \]

\[ \mathcal{J}(s) = \frac{1}{s} \omega(s) \]

\[ \mathcal{J}(s) = \frac{1}{s} \omega(s) K_p \]
\[
\lim_{s \to 0} sH(s) = \lim_{t \to \infty} h(t)
\]

\[
\Rightarrow \lim_{s \to 0} s \frac{\mathcal{G}_d \mathcal{G}(s)}{s} = \lim_{s \to 0} \frac{s \frac{1}{s} \omega(s)}{s + \frac{1}{s} \omega(s) K_p} = \frac{\mathcal{G}_d}{K_p}
\]

- For zero error \( K \) must be 1 or the control structure must be different
Same line of reasoning

\[ \mathcal{G}_{\text{final}} = -\frac{\tau_{\text{gr}} R_a}{A K_T K_p} \]

- Final value due to friction and gravity

\[ \left| \frac{\tau_{\text{gr}} R_a}{A K_T K_p} \right| \leq \mathcal{G}_{\text{max}} \Rightarrow K_p \geq \frac{\tau_{\text{gr}} R_a}{A K_T \mathcal{G}_{\text{max}}} \]

\[ K_{p\text{min}} = \frac{\tau_{\text{gr}} R_a}{A K_T \mathcal{G}_{\text{max}}} \]
PID controller

- We now know why we need the proportional
- We also know why we need the derivative
- Finally, we add the integral
  - Integrates the error, in practice needs to be limited
Interpreting the PID

- Proportional: to go where required, linked to the steady-state error
- Derivative: damping
- Integral: to reduce the steady-state error
Global view

- microprocessor
- actuator
- mechanics
- sensor

OK

reference

amplifier

OK

OK
About the amplifiers

- Linear amplifiers
  - H type
  - T type
- PWM (switching) amplifiers
Let's consider the linear as a starting point
H-type

- The motor doesn’t have a reference to ground (floating)
- It’s difficult to get feedback signals (e.g. to measure the current flowing through the motor)
T-type

\[ V_{cc} \]

\[ -V_{cc} \]
On the T-type

- Bipolar DC supply
- Dead band (around zero)
- Need to avoid simultaneous conduction (short circuit)
Things not shown

- Transistor protection (currents flowing back from the motor)
- Power dissipation and heat sink
  - Cooling
- Sudden stop due to obstacles
  - High currents → current limits and timeouts
\[ I_c \approx \frac{V_{cc}}{R_{transistor} + R_{motor}} \]
PWM amplifiers

\[ P = V_{ce} I_c \]
PWM signal

\[ P = V_{ce}I_c \]

- Transistors either “on” or “off”
  - When off, current is very low, little power too
  - When on, \( V \) is low, working point close to (or in) saturation, power dissipation is low
Comparison

- 12W for a 6A current using a switching amplifier
- 72W for a corresponding linear amplifier
Why does it work?

\[
\frac{\omega(s)}{V_{arm}(s)} = \frac{K_T/L_a J_T}{s^2 + [(R_a J_T + L_a B)/L_a J_T]s + (K_T K_E + R_a B)/L_a J_T}
\]

- In practice the motor transfer function is a low-pass filter

\[T_s \text{ with } f_s \gg f_e \ (f_s > 100 f_e)\]

- Switching frequency must be high enough (s=switching, e=electric pole)
PWM signal

$V_{cc}$

$T_s$

$V_{cc}$

$T_{switching}$

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Feedback in servo amplifiers

Voltage feedback amplifier

\[ V_{in} \]

\[ V_{cc} \]

\[ -V_{cc} \]
Operating characteristic

\[ V_{out} = A \cdot |V_{in}| \]

- \( R_L = R_1 \)
- \( R_L = R_2 \)
- \( R_2 < R_1 \)
We’ve already seen this

\[
\frac{\omega(s)}{V_{in}(s)} = \frac{K_T / L_a J_T}{s^2 + [(R_a J_T + L_a B) / L_a J_T] s + (K_T K_E + R_a B) / L_a J_T} \frac{A_v}{(1 + s \tau_a)}
\]
Current feedback

\[ V_{in} \]

\[ V_{out} \]
Current feedback

\[ V_{\text{out}} = I_L R_L \]

- \( R_L = R_1 \)
- \( R_L = R_2 \)

\[ R_2 < R_1 \]

\[ |A \cdot V_{\text{in}}| \]

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Motor driven by a current amplifier

\[ \omega(s) = \frac{K_T A_i}{V_{in}(s)} \cdot \frac{1}{(sJ_T + B)(1 + s\tau_a)} \]
Sensors

- Potentiometers
- Encoders
- Tachometers
- Inertial sensors
- Strain gauges
- Hall-effect sensors
- and many more...
Potentiometer

\[ V_{out} = \frac{r}{R} V_{cc} \]

- Simple but noisy
- Requires A/D conversion
- Absolute position (good!)

Thin resistive film
Note

\[ TR = \frac{N_1}{N_2} = \frac{G_2}{G_1} = \frac{\tau_1}{\tau_2} \]

\[ \tau_2 = \frac{N_2}{N_1} \tau_1 \implies (\text{most of the time}) N_2 > N_1 \]

\[ G_2 = \frac{N_1}{N_2} G_1 \]

- The resolution of the sensor multiplied by TR
Encoder

- Absolute
- Incremental
Absolute encoder

phototransistors

LEDs

motor

motor shaft

13 bits required for 0.044 degrees
Incremental encoder

- Disk single track instead of multiple
- No absolute position
- Usually an index marks the beginning of a turn
Incremental encoder

- Sensitive to the amount of light collected
- The direction of motion is not measured
Two-channel encoder

- 2 channels 90 degrees apart (quadrature signals) allow measuring the direction of motion
Moreover

- There are “differential” encoders
  - Taking the difference of two sensors 180 degrees apart

- Typically
  - A, B, Index channel
  - A, B, Index (differential)

- A “counter” is used to compute the position from an incremental encoder
Increasing resolution

- Counting UP and DOWN edges
  - $X_2$ or $X_4$ circuits
A potentiometer and incremental encoder can be used simultaneously: the pot for the “absolute” reference, and the encoder because of good resolution and robustness to noise.
Analog locking

- Use digital encoder as much as possible
  - Get to zero error or so using the digital signal
- When close to zeroing the error:
  - Switch to analog: use the analog signal coming from the photodetector (roughly sinusoidal/triangular)
  - Much higher resolution, precise positioning
Tachometer

- Use a DC motor
  - The moving coils in the magnetic field will get an induced EMF
- In practice is better to design a special purpose “DC motor” for measuring velocity
- Ripple: typ. 3%
As already seen...
Measuring speed with digital encoders

- Frequency to voltage converters
  - Costly (additional electronics)
- Much better: in software
  - Take the derivative (for free!)

\[ v(kT) = \frac{p(kT) - p((k-1)T)}{T} \]
Inertial sensors

- **Accelerometers:**

\[ Ma = 2Kx \Rightarrow a = \frac{2Kx}{M} \]
Gyroscopes

- Quartz forks

\[ F = 2m\omega \times V \]
Strain gauges

- Principle: deformation $\rightarrow \Delta R$ (resistance)
  - Example: conductive paint (Al, Cu)
  - The paint covers a deformable non-conducting substrate

\[ R = \frac{L}{\sigma A} \Rightarrow \Delta L, \ A = \text{const} \Rightarrow \Delta R \]
Reading from a strain gauge

\[ R_1 R_2 = R_g R_b \implies V_{ab} = 0 \]
\[ \Delta V_{ab} = f(\Delta R_g) \]
Hall-effect sensors

\[ F_{\text{lorentz}} = q\vec{v} \times \vec{B} \]
Example

- Measuring angles (magnetic encoders)

FIGURE 1--Basic Configuration of magnet and hall effect sensor. Edge views are shown.
Back to the global view

- microprocessor
- amplifier
- actuator
- mechanics
- sensor

Reference: OK
Amplifier: OK
Actuator: OK
Mechanics: OK
Sensor: OK
Microprocessors

- Special DSPs for motion control
  - Some are barely programmable (the control law is fixed)
  - Others are general purpose and they are mixed mode (analog and digital in a single chip)
Example

- DSP 16 bit ALU and instruction set
- PWM generator (simply attach this to either T or H amplifier)
- A/D conversion
- CAN bus, Serial ports, digital I/O
- Encoder counters
- Flash memory and RAM on-board
- Enough of all these to control two motors (either brush- or brushless)