Perception and Synthesis of ‘Shape’
GNOSYS ARCHITECTURE: FROM AUTOMATION TO AUTONOMOUS
GNOSYS ARCHITECTURE: - FROM AUTOMATION TO AUTONOMOUS
Perception and Synthesis of ‘Shape’
‘Shape’ our Bodies to the ‘Shape’ of the World.............
What is 'Shape'?  

Vague Concept

It is not easy to give a precise mathematical or quantitative definition of 'shape'.

Or even express it in mensurational quantities like length, angles or topology.
What is ‘Shape’?

Vauge Concept

It is not easy to give a precise mathematical or quantitative definition of ‘shape’

Or even express it in mensurational quantities like length, angles or topology

Shape is the core information in any object/action that survives the effects of

changes in location  🟢 🟡
What is ‘Shape’?

Vague Concept

It is not easy to give a precise mathematical or quantitative definition of ‘shape’

Or even express it in mensurational quantities like length, angles or topology

Shape is the core information in any object/action that survives the effects of

changes in location

scale
What is ‘Shape’?

Vague Concept

It is not easy to give a precise mathematical or quantitative definition of ‘shape’

Or even express it in mensurational quantities like length, angles or topology.

Shape is the core information in any object/action that survives the effects of

- changes in location
- scale
- orientation
What is 'Shape'?

Vauge Concept

It is not easy to give a precise mathematical or quantitative definition of 'shape'

Or even express it in mensurational quantities like length, angles or topology

Shape is the core information in any object/action that survives the effects of

changes in location
scale
orientation
end effectors/bodies used in its creation

Make it on PAPER
Run on a football ground
What is 'Shape'?

Vague Concept

It is not easy to give a precise mathematical or quantitative definition of 'shape'.

Or even express it in mensurational quantities like length, angles or topology.

Shape is the core information in any object/action that survives the effects of:

- changes in location
- scale
- orientation
- end effectors/bodies used in its creation
- noise, and even minor structural injury.
It is this informational invariance that makes 'shape' the seed for any high level sensorimotor interaction.
It is this informational invariance that makes ‘shape’ the seed for any high level sensorimotor interaction.

Shape is where Seeing and Doing MEET
It is this *informational invariance* that makes ‘shape’ the seed for any high level sensorimotor interaction.

Shape is where *Seeing* and *Doing* MEET
Perception and Synthesis of ‘Shape’

An unified treatment to the dual operations of shape perception and synthesis is critical both from the intrinsic viewpoint of better understanding our own perceptions and actions, to creating autonomous robots that can flexibly aid us in our needs and in the environments we inhabit and create.
Shape Perception + Shape Synthesis

Imitation

iCub Write

iCub Draw
Shape Perception + Shape Synthesis

Imitation

iCub Write

iCub Draw

Window to understand Perception-Reason-Action Loop

- Shape Perception + Movement Recognition (+ Goal Perception)
Shape Perception + Shape Synthesis

Window to understand Perception-Reason-Action Loop

• Shape Perception + Movement Recognition (+ Goal Perception)
• Pose estimation, Pose tracking
Shape Perception + Shape Synthesis

Imitation

• Shape Perception + Movement Recognition (+ Goal Perception)
• Pose estimation, Pose tracking
• Body Correspondance

Window to understand Perception-Reason-Action Loop

iCub Write

iCub Draw
Shape Perception + Shape Synthesis

Window to understand Perception-Reason-Action Loop

• Shape Perception + Movement Recognition (+ Goal Perception)
• Pose estimation, Pose tracking
• Body Correspondance
• Coordinate transformation from external to egocentric space

Imitation

iCub Write

iCub Draw
Shape Perception + Shape Synthesis

Imitation

iCub Write

iCub Draw

Window to understand Perception-Reason-Action Loop

- Shape Perception + Movement Recognition (+ Goal Perception)
- Pose estimation, Pose tracking
- Body Correspondance
- Coordinate transformation from external to egocentric space
- Matching ones own movement with Goal

Goal Solutions
Shape Perception + Shape Synthesis

Imitation

iCub Write

iCub Draw

Window to understand Perception-Reason-Action Loop

• Shape Perception + Movement Recognition (+ Goal Perception)
• Pose estimation, Pose tracking
• Body Correspondance
• Coordinate transformation from external to egocentric space
• Matching one's own movement with Goal
• Using previously learnt motor schemas for generalization
Shape Perception + Shape Synthesis

Shape Perception + Movement Recognition (+ Goal Perception)
• Pose estimation, Pose tracking
• Body Correspondance
• Coordinate transformation from external to egocentric space
• Matching ones own movement with Goal
• Using previoulsy learnt motor schemas for generalization
• Redundancy resolution

Window to understand Perception-Reason-Action Loop

Imitation

iCub Write

iCub Draw
Shape Perception + Shape Synthesis

Imitation

Window to understand Perception-Reason-Action Loop

iCub Write

• Minimal/compact Representation, memory
• Ease in learning, robustness, reuse, categorization
• Actions driven by thoughts, reasoning + Mental states/concepts: grounding, meaning ..................
• Communication, Language, Self Consciouness ++ ........

iCub Draw
Shapes and Shapeing: Information flows

- Demonstration to iCub
- Analysis of Critical Points in the perceived shape using Catastrophe theory
- Abstract Visual Program

Concrete Motor Goal (3D Reconstruction, Task specific constraints, Scale, and effectors)

- Motor Action Generation
- Abstract Motor Program

- Virtual Trajectory Synthesis
- Passive Motion Paradigm (iCub Upper body internal model)

- Analysis of Critical Points in the self generated movement (or Forward model output) using Catastrophe theory

Reward signal / Score of performance

V. Stiffness Timing of the Time base generators Learning loop
From Teachers demonstration to Abstract visual program

Catastrophe Theory + Morphogenesis

The Chemical Basis of Morphogenesis

A. M. Turing

Atoms of SHAPE - The Catastrophe theory
Atoms of SHAPE - The Catastrophe theory

• C.T: Dwells into the origin of forms in nature (Morphogenesis)
Atoms of SHAPE - The Catastrophe theory

• C.T :- Dwells into the origin of forms in nature (Morphogenesis)

• A system/agent is ‘visually’ tuned to recognize 12 universal primitives, each having unique local features or critical points (like peaks, valleys etc)
Atoms of SHAPE - The Catastrophe theory

• C.T :- Dwells into the origin of forms in nature (Morphogenesis)
• A system/agent is 'visually' tuned to recognize 12 universal primitives, each having unique local features or critical points (like peaks, valleys etc)

Shape Atoms

- Interior Point 'I'
- End Point 'E' Bump 'B'
- Dot 'D'
- Cross 'X'
- Cusp 'C'
- The T 'T'
- Wiggle 'W'
- Star 'S'
- Contact 'Co'
- Peck 'P'
- Angle 'A'
Atoms of SHAPE - The Catastrophe theory

Shape Atoms

Interior Point 'I'
End Point 'E'
Bump 'B'
Dot 'D'
Cross 'X'
Cusp 'C'
The T 'T'
Wiggle 'W'
Star 'S'
Contact 'Co'
Peck 'P'
Angle 'A'

Complex/Global shapes are 'weighted' combinations of local shape features

Shape to Abstract visual programs
Atoms of SHAPE - The Catastrophe theory

Shape Atoms

<table>
<thead>
<tr>
<th>Interior Point 'I'</th>
<th>End Point 'E' Bump 'B'</th>
<th>Dot 'D'</th>
<th>Cross 'X'</th>
<th>Cusp 'C'</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram of Interior Point]</td>
<td>![Diagram of End Point Bump]</td>
<td>![Diagram of Dot]</td>
<td>![Diagram of Cross]</td>
<td>![Diagram of Cusp]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The T 'T'</th>
<th>Wiggle 'W'</th>
<th>Star 'S'</th>
<th>Contact 'Co' Peck 'P'</th>
<th>Angle 'A'</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram of The T]</td>
<td>![Diagram of Wiggle]</td>
<td>![Diagram of Star]</td>
<td>![Diagram of Contact Peck]</td>
<td>![Diagram of Angle]</td>
</tr>
</tbody>
</table>

- Critical points in a complex shape, are formally characterized using four measures:

1) Stability
2) Codimension
3) Composition
4) Valency
Valency: If enclosed by an infinitely small circle, no of lines that intersect it is the valency

Simple CP

B + B

Interior Point 'I'

End Point 'E' Bump 'B'

Dot 'D'

Cross 'X'

Cusp 'C'

The T 'T'

Wiggle 'W'

Star 'S'

Contact 'Co'

Peck 'P'

Angle 'A'
Valency: If enclosed by an infinitely small circle, no of lines that intersect it is the valency.

Simple CP

\( B + B \)

Complex CP

\[ f(x) = x^3 \quad f_p(x) = x^3 + \varepsilon x, \ (\varepsilon < 0) \quad f_p(x) = x^3 + \varepsilon x, \ (\varepsilon > 0) \]

Codimention: Min number of parameters necessary to bring the function back from its perturbed state.

Codim=0

Cross

Codim=1

Dot point

Codim=1

T point

Codim=1
**Bump (B):** A Bump is an interior point where the derivative of either \( U(t) \) or \( V(t) \) is zero. A bump simply is the minimum or a maximum of a one dimensional smooth function.

\[
U'(t_B) = \frac{dU}{dt}_{|_{t=t_B}} = 0; \quad d^2U \neq 0; \quad V'(t_B) \neq 0;
\]

\[
V'(t_B) = \frac{dV}{dt}_{|_{t=t_B}} = 0; \quad d^2V \neq 0; \quad U'(t_B) \neq 0;
\]

**Wiggle (W):** Wiggle is a complex CP. At a wiggle both the first and second derivative along \( U \) or \( V \) dimensions vanish.

\[
\frac{dU}{dt} = \frac{d^2U}{dt^2} = 0; \quad \frac{dV}{dt} = \frac{d^2V}{dt^2} = 0;
\]

Wiggle is unstable under perturbation and either breaks up into two bumps or vanishes completely.
Dot 'D'  Cross 'X'  Cusp 'C'  The T 'T'  Star 'S'  Contact 'Co'  Peck 'P'  Angle 'A'

4b. Under Perturbation

4c. Under Composition

\[
\begin{align*}
    D &= E + E \\
    X &= I + I \\
    C &= B_x + B_y \\
    T &= I + E \\
    T &= I + I + I \\
    Co &= X + X \\
    P &= C + I \\
    A &= E + E
\end{align*}
\]
From demonstration to Abstract visual program
From demonstration to Abstract visual program
Shapes and Shaping: Information flows

1. Demonstration to iCub
   - Concrete Motor Goal (3D Reconstruction, Task specific constraints, Scale, and effectors)
2. Analysis of Critical Points in the perceived Shape using Catastrophe theory
   - Motor Action Generation
   - Passive Motion Paradigm (iCub Upper body internal model)
3. Abstract Visual Program
   - Abstract Motor Program
   - Analysis of Critical Points in the self generated movement (or Forward model output) using Catastrophe theory

Reward signal / Score of performance

Learning loop

V. Stiffness Timing of the Time base generators
From Abstract visual program to Concrete Motor Goal
\[
\begin{align*}
    u &= \frac{L_1 x + L_2 y + L_3 z + L_4}{L_9 x + L_{10} y + L_{11} z + 1} \\
    v &= \frac{L_5 x + L_6 y + L_7 z + L_8}{L_9 x + L_{10} y + L_{11} z + 1}
\end{align*}
\]
\[
\begin{bmatrix}
    u_1 \\
    v_1 \\
    \vdots \\
    u_N \\
    v_N
\end{bmatrix} = 
\begin{bmatrix}
    x_1 & y_1 & z_1 & 1 & 0 & 0 & 0 & 0 & -u_1 x_1 & -u_1 y_1 & -u_1 z_1 \\
    0 & 0 & 0 & 0 & x_1 & y_1 & z_1 & 1 & -v_1 x_1 & -v_1 y_1 & -v_1 z_1 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    x_N & y_N & z_N & 1 & 0 & 0 & 0 & 0 & -u_N x_N & -u_N y_N & -u_N z_N \\
    0 & 0 & 0 & 0 & x_N & y_N & z_N & 1 & -v_N x_N & -v_N y_N & -v_N z_N
\end{bmatrix} \begin{bmatrix}
    L_1 \\
    L_2 \\
    \vdots \\
    L_N
\end{bmatrix}
\]

\[U = A \cdot L \quad \text{or} \quad L = [A^T A]^{-1} A^T \cdot U\]
Virtual Trajectory Synthesis

Discrete set of critical points
To a
Continuous trajectory
From Concrete Motor Goal to Virtual Trajectory

A

Concrete Motor Goal

Working memory
Critical point buffer

CP1
CP2

B

VT_1 := (k_1, k_2, TBG_1, TBG_2)_1
VT_2 := (k_1, k_2, TBG_1, TBG_2)_2

X_{ini}

k_1
k_2

F_1
F_2

1/S

Virtual stiffness

Set of Attractive force fields to CP's (virtual targets)

Planned Trajectory
To iCub action generation system (PMP)

Learning

TBG_1
TBG_2

Terminal Attractor Dynamics

Time base generators
Virtual Trajectory Synthesis

Changing **Stiffness** and **Timing** to generate a range of **Shapes**
From Virtual Trajectory to Motor Action

[Diagram showing the process from virtual trajectory to motor action, with various mathematical expressions and symbols indicating the transition from end effector space to arm/joint space and finally to waist space.]
Comparing abstract Motor and Visual representations to evaluate performance

Do a Catastrophe theory analysis on SELF GENERATED Movement
Shapes and Shapeing: Information flows

Demonstration to iCub → Analysis of Critical Points in the perceived Shape using Catastrophe theory → Abstract Visual Program

Concrete Motor Goal (3D Reconstruction, Task specific constraints, Scale, and effectors) → Virtual Trajectory Synthesis

Motor Action Generation

Abstract Motor Program → Analysis of Critical Points in the self generated movement (or Forward model output) using Catastrophe theory

Reward signal / Score of performance

V. Stiffness Timing of the Time base generators

Learning loop
Comparing abstract Motor and Visual representations to evaluate performance

\[ S = \frac{1}{p} \left( \sum_{CP=1}^{p} \Psi(CP_{S1}, CP_{S2}) \cdot \text{dist}(CP_{S1}, CP_{S2}) \right) - q \]
Learning to Shape

Concrete Motor Goal

Working memory
Critical point buffer

CP1  CP2

Virtual stiffness

F1  F2

Set of Attractive force fields to CP's (virtual targets)

VT1 := \((k1, k2, TBG1, TBG2)_1\)

VT2 := \((k1, k2, TBG1, TBG2)_2\)

X_{ini}

X

TBG1  TBG2

Time base generators

Terminal Attractor Dynamics

Planned Trajectory
To iCub action generation system (PMP)
Learning to Shape: Trajectories between 2 Points in space
Examples of shaping 'C'
Learning to Shape: Underlying Principle ?????
Why some value of stiffness gives rise to some Shape

CP₁: (Kₓₓ=10, Kzz=10)
CP₂: (Kₓₓ=10, Kzz=10)
CP₁: (Kₓₓ=10, Kzz=1)
CP₂: (Kₓₓ=1, Kzz=10)
CP₁: (Kₓₓ=1, Kzz=10)
CP₂: (Kₓₓ=10, Kzz=1)
iCubArt: First few steps
iCubArt: First drawings of iCub
Generalization during Compositional Synthesis
Generalization during Compositional Synthesis
Generalization during Compositional Synthesis

Since complex shapes can be 'decomposed' into combinations of primitive shape CP's using CT, inversely can the motor actions needed to create them be 'composed' using combinations of the corresponding 'learnt' primitive actions........
The ‘Shape’ of Gandhi
The 'Shape' of Gandhi
Shapes and Shapeing: Information flows

- Demonstration to iCub
  - Concrete Motor Goal (3D Reconstruction, Task specific constraints, Scale, and effectors)
    - Virtual Trajectory Synthesis
      - $K$, $TBG_n$
  - Motor Action Generation
    - Passive Motion Paradigm (iCub Upper body internal model)

- Analysis of Critical Points in the perceived Shape using Catastrophe theory
  - Abstract Motor Program
    - Analysis of Critical Points in the self generated movement (or Forward model output) using Catastrophe theory

- Abstract Visual Program
  - Reward signal / Score of performance

Learning loop
V. Stiffness Timing of the Time base generators
Open Questions?

Are actions ‘represented’?

Computing through stiffness (physical/mental)?

Time, timing, sync and temporal order

Motor Control, Motor learning and Mental simulation of action

The ‘Motivation -Exploration- Imagination-Introspection’ Loop revisited

Socially Cognitive Humanoids: A minimal (brain based) neural architecture for imitation

Neuromotor rehabilitation: Robots teaching/assisting to draw?

Shapes of ‘Signals’, Multimodal sensory resonance

Reaction diffusion systems and Valued goals?

iCubArt driven by iCubthought?
Thank You for your attention
(vishwanathan.mohan@iit.it)
Skype: Vishuu