MATLAB

Mass in a potential field:

\[ \tan(y)^2 \]

To move the ball from \( x_0 \) to \( x_f \) we have to force with a constant \( F \) if \( T \) isn't enough we can move toward \( x_f \) directly; but if \( T \) is enough we use the fact that the ball goes down and then, with more Energy goes to \( x_f \).

The problem is that there is a local minima. The problem was

\[
\begin{align*}
\min_{p_0} & \quad \| x(t; p_0) - x_f \| \\
\text{s.t.} & \quad \begin{cases}
\dot{x} = \frac{\partial L}{\partial p}(x(t), u(x(t), p(t)), p(t), t) \\
\dot{p} = -\frac{\partial L}{\partial x}(x(t), u(x(t), r(t)), r(t), t) \\
\end{cases}
\end{align*}
\]

This dynamical system has not a unique solution, we can specify the \( p_0 \) initial
\[ C^* = \text{cost optimal} \]

Example: muscles seem to be springs

Stochastic initialization of the initial conditions

The problem is:
\[
\min_{u(.)} \int_{t_0}^{t_f} \frac{1}{2} (u'(x))^2 u(x) \, dx \\
\downarrow \text{Trajectory}
\]

\[ w x + b x + \frac{2 \text{atan}(x)}{1 + x^2} = u \]

Solve this problem corresponds to solve the dynamical system and the force that moves the mass.

The initial conditions transform the trajectory in the seven of two points

\[ \text{The Hamilton - Jacobi} \]

\[ \min_{u(.)} \int_{t_0}^{t_f} g(x(t), u(t), t) \, dt + R(x(tf), tf) \quad \text{s.t.} \quad \begin{cases} x = f(x,u) \\ x(t_0) = x_0 \end{cases} \]
In a general framework, we can think that at each time instant \( t \) there is an optimal trajectory to be followed from the current state to the target state \( x_f \).

At time \( t \), the cost to be optimized is:

\[
J(\bar{x}, t, u(\tau)) = \int_{t}^{t_f} g(x(\tau), u(\tau), \tau) \, d\tau + R(x(t_f), t_f)
\]

s.t. \[
\begin{align*}
\dot{x} &= f(x, u) \\
x(t) &= \bar{x}
\end{align*}
\]

\( J(\bar{x}, t, u(\tau)) \) is called "cost to go".

The optimal control strategy should minimize the "cost to go" and therefore we can define the "optimal cost to go":

\[
\mathcal{J}^*(\bar{x}, t) = \min_{u(\tau)} J(\bar{x}, t, u(\tau))
\]

s.t. \[
\begin{align*}
\dot{x} &= f(x, u) \\
x(t) &= \bar{x}
\end{align*}
\]
Conditions of $J^*(\bar{x}, t)$:

$$J^*(\bar{x}, t) = \min_{u(\cdot)} \left\{ \int_{t}^{t+\Delta} g(x(\cdot), u(\cdot), \tau) \, d\tau + J^*(x(t+\Delta), t+\Delta) \right\}$$

where $x(t+\Delta)$ is the state reached at time $t+\Delta$

starting from the initial condition $\bar{x}$ at time $t$ and applying

the control strategy $u(t), t \in [t, t+\Delta]$.

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**Ex:**

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$J^*(x, t_f) = h(x, t_f)$
It can be shown (taking the derivative with respect to time of [\( H \)]) that:

\[
\frac{\partial J}{\partial t}(t, x) = \min_{u} \left\{ \dot{J}(x, u, t) + \left[ \frac{\partial J}{\partial x}(x, t) \right]^T f(x, u) \right\}
\]

s.t. \( J^*(x, t_f) = \mathcal{L}(x, t_f) \)

→ if you solve this it's guarantee that is an optimal solution.

**Dynamic Programming:**

\( x \in \{x_1, x_2, \ldots, x_n\} \) \quad \text{set of states}

\( u \in \{u_1, u_2, \ldots, u_n\} \) \quad \text{control inputs}

\[ J^*(x, t_f) = \mathcal{L}(x, t_f) \]
\( t \in [t_0, t_0 + \Delta \ldots t_f - \Delta, t_f] \)

\((*)\)

**MATLAB:**

Discretization of the space of variables

- \( N_v = 30 \;
- \( N_x = 50 \;
- \( \text{Vmin} \;
- \( \text{x grid} \;
- \( \text{f final} = \text{fit to grid} \)
- \( \text{V} \rightarrow J^* \times \text{Function 'cost to go'} \)
- \( \text{compute} \; \text{V} \rightarrow J^*(x(t+\Delta), t+\Delta) \)
- \( \text{u star}(t, T, x) \rightarrow \text{it's yet written as a feedback, because you have the solution for each state} \)
- \( \text{images (x grid, dx grid, V(:,:,))} \rightarrow \text{visualization of the cost} \)
- \( \text{color bar} \)

\((*)\) Then at time \( t_f - \Delta \), for a given value \( x \) of the current state, each discretized control strategy \( u \) is applied in the interval \([t_f - \Delta, t_f]\) will bring the system to \( x(t + \Delta) \) and the associated cost will be:

\[
\int_t^{t+\Delta} g(x(\tau), u(\tau), x(\tau)) \, d\tau + J^*(x(t + \Delta), t_f)
\]
or, if we make explicit the fact that \( x ( \omega ) \) depends on the choice of \( \mu \), we have \( x ( \omega, \mu ) \) and therefore:

\[
\int_t^{t+\Delta} g \left( x ( \sigma, \mu_k ), \mu_k, \sigma \right) + J^* \left( x ( t+\Delta; \mu_k ), t, f \right)
\]

so that we have:

\[
J^* \left( x, tf-\Delta \right) = \min \left\{ \int_{tf-\Delta}^{tf} g \left( x ( \sigma; \mu_k ), \mu_k, \sigma \right) d\sigma + \sum_{\mu_k \in \{ \mu_i \ldots \mu_n \}} J^* \left( x ( tf+\Delta; \mu_k ), tf \right) \right\}
\]

where \( x ( \cdot ; \mu_k ) \) is the solution of \( \dot{x} = f ( x, \mu_k ) \)

subject to \( x ( tf-\Delta ) = x \)