

## 4.4 PAGE REPLACEMENT ALGORITHMS

When a page fault occurs, the operating system has to choose a page to remove from memory to make room for the page that has to be brought in. If the page to be removed has been modified while in memory, it must be rewritten to the disk to bring the disk copy up to date. If, however, the page has not been changed (e.g., it contains program text), the disk copy is already up to date, so no rewrite is needed. The page to be read in just overwrites the page being evicted.

While it would be possible to pick a random page to evict at each page fault, system performance is much better if a page that is not heavily used is chosen. If a heavily used page is removed, it will probably have to be brought back in quickly, resulting in extra overhead. Much work has been done on the subject of page replacement algorithms, both theoretical and experimental. Below we will describe some of the most important algorithms.

It is worth noting that the problem of “page replacement” occurs in other areas of computer design as well. For example, most computers have one or more memory caches consisting of recently used 32-byte or 64-byte memory blocks. When the cache is full, some block has to be chosen for removal. This problem is precisely the same as page replacement except on a shorter time scale (it has to be done in a few nanoseconds, not milliseconds as with page replacement). The reason for the shorter time scale is that cache block misses are satisfied from main memory, which has no seek time and no rotational latency.

A second example is in a Web server. The server can keep a certain number of heavily used Web pages in its memory cache. However, when the memory cache is full and a new page is referenced, a decision has to be made which Web page to evict. The considerations are similar to pages of virtual memory, except for the fact that the Web pages are never modified in the cache, so there is always a fresh copy on disk. In a virtual memory system, pages in main memory may be either clean or dirty.

### 4.4.1 The Optimal Page Replacement Algorithm

The best possible page replacement algorithm is easy to describe but impossible to implement. It goes like this. At the moment that a page fault occurs, some set of pages is in memory. One of these pages will be referenced on the very next instruction (the page containing that instruction). Other pages may not be referenced until 10, 100, or perhaps 1000 instructions later. Each page can be labeled with the number of instructions that will be executed before that page is first referenced.

The optimal page algorithm simply says that the page with the highest label should be removed. If one page will not be used for 8 million instructions and another page will not be used for 6 million instructions, removing the former pushes the page fault that will fetch it back as far into the future as possible.

Computers, like people, try to put off unpleasant events for as long as they can.

The only problem with this algorithm is that it is unrealizable. At the time of the page fault, the operating system has no way of knowing when each of the pages will be referenced next. (We saw a similar situation earlier with the shortest job first scheduling algorithm—how can the system tell which job is shortest?) Still, by running a program on a simulator and keeping track of all page references, it is possible to implement optimal page replacement on the *second* run by using the page reference information collected during the *first* run.

In this way it is possible to compare the performance of realizable algorithms with the best possible one. If an operating system achieves a performance of, say, only 1 percent worse than the optimal algorithm, effort spent in looking for a better algorithm will yield at most a 1 percent improvement.

To avoid any possible confusion, it should be made clear that this log of page references refers only to the one program just measured and then with only one specific input. The page replacement algorithm derived from it is thus specific to that one program and input data. Although this method is useful for evaluating page replacement algorithms, it is of no use in practical systems. Below we will study algorithms that *are* useful on real systems.

#### 4.4.2 The Not Recently Used Page Replacement Algorithm

In order to allow the operating system to collect useful statistics about which pages are being used and which ones are not, most computers with virtual memory have two status bits associated with each page. *R* is set whenever the page is referenced (read or written). *M* is set when the page is written to (i.e., modified). The bits are contained in each page table entry, as shown in Fig. 4-0. It is important to realize that these bits must be updated on every memory reference, so it is essential that they be set by the hardware. Once a bit has been set to 1, it stays 1 until the operating system resets it to 0 in software.

If the hardware does not have these bits, they can be simulated as follows. When a process is started up, all of its page table entries are marked as not in memory. As soon as any page is referenced, a page fault will occur. The operating system then sets the *R* bit (in its internal tables), changes the page table entry to point to the correct page, with mode READ ONLY, and restarts the instruction. If the page is subsequently written on, another page fault will occur, allowing the operating system to set the *M* bit and change the page's mode to READ/WRITE.

The *R* and *M* bits can be used to build a simple paging algorithm as follows. When a process is started up, both page bits for all its pages are set to 0 by the operating system. Periodically (e.g., on each clock interrupt), the *R* bit is cleared, to distinguish pages that have not been referenced recently from those that have been.

When a page fault occurs, the operating system inspects all the pages and divides them into four categories based on the current values of their *R* and *M*

bits:

- Class 0: not referenced, not modified.
- Class 1: not referenced, modified.
- Class 2: referenced, not modified.
- Class 3: referenced, modified.

Although class 1 pages seem, at first glance, impossible, they occur when a class 3 page has its  $R$  bit cleared by a clock interrupt. Clock interrupts do not clear the  $M$  bit because this information is needed to know whether the page has to be rewritten to disk or not. Clearing  $R$  but not  $M$  leads to a class 1 page.

The **NRU (Not Recently Used)** algorithm removes a page at random from the lowest numbered nonempty class. Implicit in this algorithm is that it is better to remove a modified page that has not been referenced in at least one clock tick (typically 20 msec) than a clean page that is in heavy use. The main attraction of NRU is that it is easy to understand, moderately efficient to implement, and gives a performance that, while certainly not optimal, may be adequate.

#### 4.4.3 The First-In, First-Out (FIFO) Page Replacement Algorithm

Another low-overhead paging algorithm is the **FIFO (First-In, First-Out)** algorithm. To illustrate how this works, consider a supermarket that has enough shelves to display exactly  $k$  different products. One day, some company introduces a new convenience food—instant, freeze-dried, organic yogurt that can be reconstituted in a microwave oven. It is an immediate success, so our finite supermarket has to get rid of one old product in order to stock it.

One possibility is to find the product that the supermarket has been stocking the longest (i.e., something it began selling 120 years ago) and get rid of it on the grounds that no one is interested any more. In effect, the supermarket maintains a linked list of all the products it currently sells in the order they were introduced. The new one goes on the back of the list; the one at the front of the list is dropped.

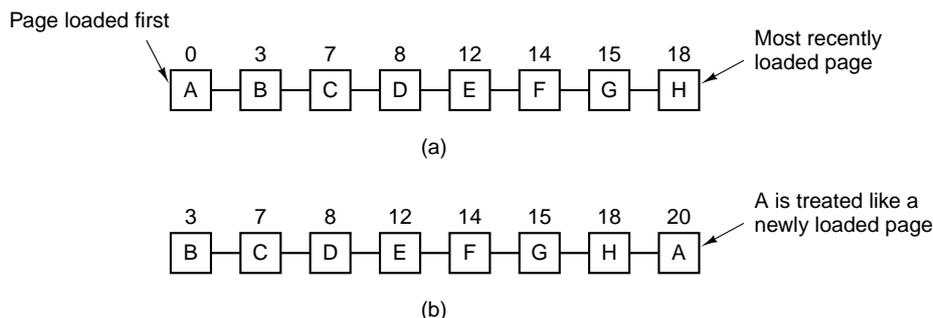
As a page replacement algorithm, the same idea is applicable. The operating system maintains a list of all pages currently in memory, with the page at the head of the list the oldest one and the page at the tail the most recent arrival. On a page fault, the page at the head is removed and the new page added to the tail of the list. When applied to stores, FIFO might remove mustache wax, but it might also remove flour, salt, or butter. When applied to computers the same problem arises. For this reason, FIFO in its pure form is rarely used.

#### 4.4.4 The Second Chance Page Replacement Algorithm

A simple modification to FIFO that avoids the problem of throwing out a heavily used page is to inspect the  $R$  bit of the oldest page. If it is 0, the page is both old and unused, so it is replaced immediately. If the  $R$  bit is 1, the bit is

cleared, the page is put onto the end of the list of pages, and its load time is updated as though it had just arrived in memory. Then the search continues.

The operation of this algorithm, called **second chance**, is shown in Fig. 4-1. In Fig. 4-1(a) we see pages *A* through *H* kept on a linked list and sorted by the time they arrived in memory.



**Figure 4-1.** Operation of second chance. (a) Pages sorted in FIFO order. (b) Page list if a page fault occurs at time 20 and *A* has its *R* bit set. The numbers above the pages are their loading times.

Suppose that a page fault occurs at time 20. The oldest page is *A*, which arrived at time 0, when the process started. If *A* has the *R* bit cleared, it is evicted from memory, either by being written to the disk (if it is dirty), or just abandoned (if it is clean). On the other hand, if the *R* bit is set, *A* is put onto the end of the list and its “load time” is reset to the current time (20). The *R* bit is also cleared. The search for a suitable page continues with *B*.

What second chance is doing is looking for an old page that has not been referenced in the previous clock interval. If all the pages have been referenced, second chance degenerates into pure FIFO. Specifically, imagine that all the pages in Fig. 4-1(a) have their *R* bits set. One by one, the operating system moves the pages to the end of the list, clearing the *R* bit each time it appends a page to the end of the list. Eventually, it comes back to page *A*, which now has its *R* bit cleared. At this point *A* is evicted. Thus the algorithm always terminates.

#### 4.4.5 The Clock Page Replacement Algorithm

Although second chance is a reasonable algorithm, it is unnecessarily inefficient because it is constantly moving pages around on its list. A better approach is to keep all the page frames on a circular list in the form of a clock, as shown in Fig. 4-2. A hand points to the oldest page.

When a page fault occurs, the page being pointed to by the hand is inspected. If its *R* bit is 0, the page is evicted, the new page is inserted into the clock in its place, and the hand is advanced one position. If *R* is 1, it is cleared and the hand is advanced to the next page. This process is repeated until a page is found with

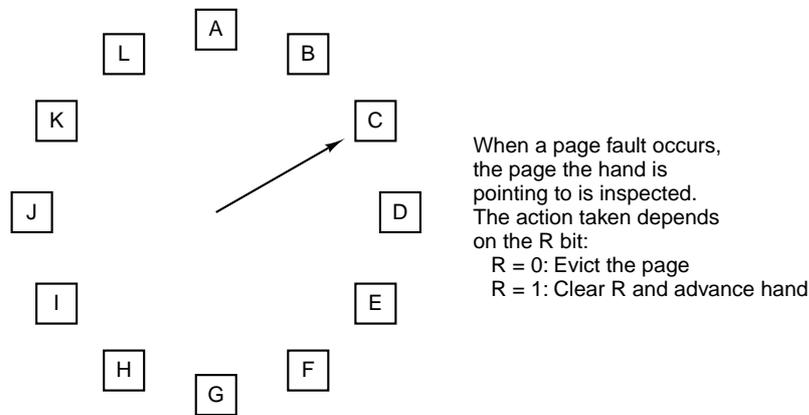


Figure 4-2. The clock page replacement algorithm.

$R = 0$ . Not surprisingly, this algorithm is called **clock**. It differs from second chance only in the implementation.

#### 4.4.6 The Least Recently Used (LRU) Page Replacement Algorithm

A good approximation to the optimal algorithm is based on the observation that pages that have been heavily used in the last few instructions will probably be heavily used again in the next few. Conversely, pages that have not been used for ages will probably remain unused for a long time. This idea suggests a realizable algorithm: when a page fault occurs, throw out the page that has been unused for the longest time. This strategy is called **LRU (Least Recently Used)** paging.

Although LRU is theoretically realizable, it is not cheap. To fully implement LRU, it is necessary to maintain a linked list of all pages in memory, with the most recently used page at the front and the least recently used page at the rear. The difficulty is that the list must be updated on every memory reference. Finding a page in the list, deleting it, and then moving it to the front is a very time consuming operation, even in hardware (assuming that such hardware could be built).

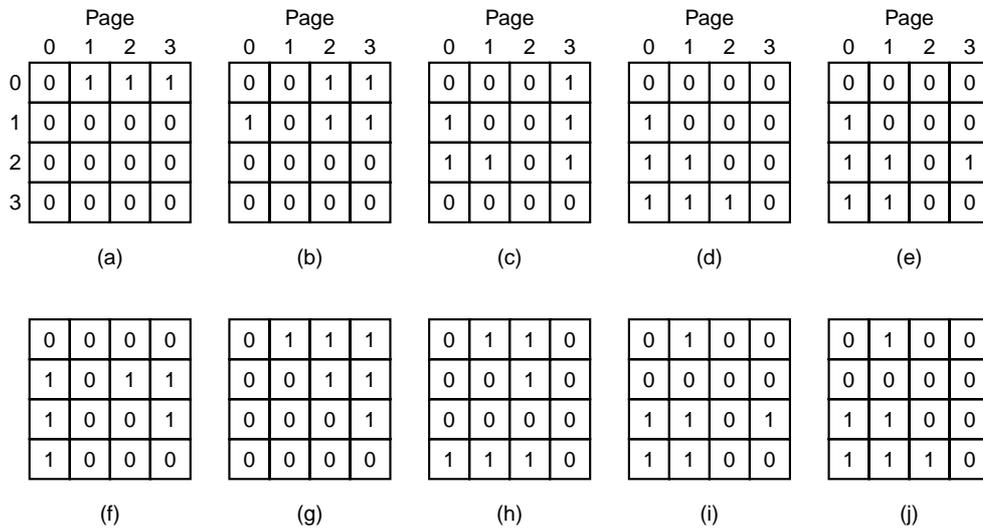
However, there are other ways to implement LRU with special hardware. Let us consider the simplest way first. This method requires equipping the hardware with a 64-bit counter,  $C$ , that is automatically incremented after each instruction. Furthermore, each page table entry must also have a field large enough to contain the counter. After each memory reference, the current value of  $C$  is stored in the page table entry for the page just referenced. When a page fault occurs, the operating system examines all the counters in the page table to find the lowest one. That page is the least recently used.

Now let us look at a second hardware LRU algorithm. For a machine with  $n$  page frames, the LRU hardware can maintain a matrix of  $n \times n$  bits, initially all

zero. Whenever page frame  $k$  is referenced, the hardware first sets all the bits of row  $k$  to 1, then sets all the bits of column  $k$  to 0. At any instant, the row whose binary value is lowest is the least recently used, the row whose value is next lowest is next least recently used, and so forth. The workings of this algorithm are given in Fig. 4-3 for four page frames and page references in the order

0 1 2 3 2 1 0 3 2 3

After page 0 is referenced, we have the situation of Fig. 4-3(a). After page 1 is reference, we have the situation of Fig. 4-3(b), and so forth.



**Figure 4-3.** LRU using a matrix when pages are referenced in the order 0, 1, 2, 3, 2, 1, 0, 3, 2, 3.

### 4.4.7 Simulating LRU in Software

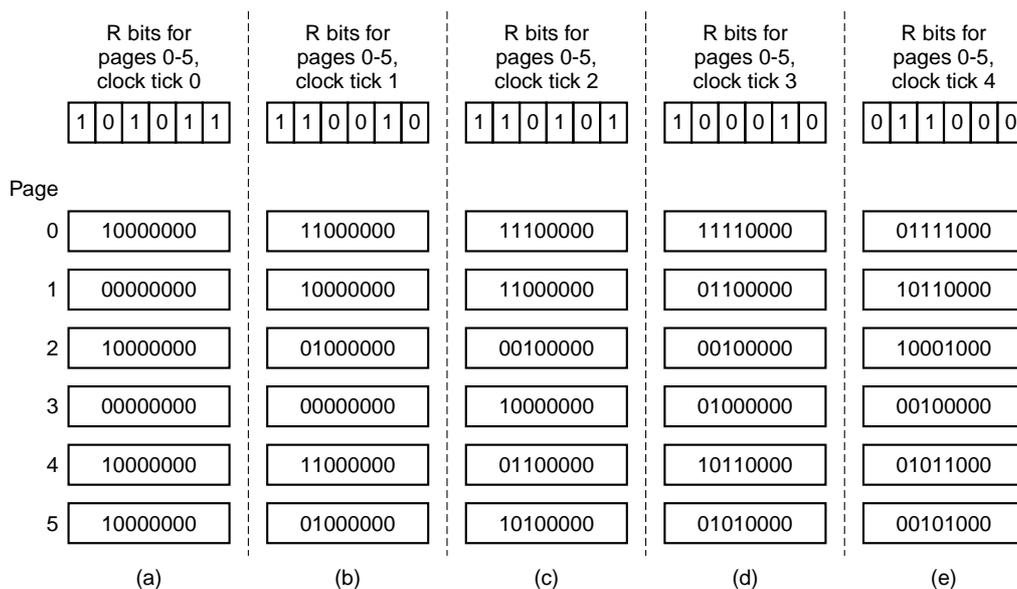
Although both of the previous LRU algorithms are realizable in principle, few, if any, machines have this hardware, so they are of little use to the operating system designer who is making a system for a machine that does not have this hardware. Instead, a solution that can be implemented in software is needed. One possibility is called the **NFU (Not Frequently Used)** algorithm. It requires a software counter associated with each page, initially zero. At each clock interrupt, the operating system scans all the pages in memory. For each page, the  $R$  bit, which is 0 or 1, is added to the counter. In effect, the counters are an attempt to keep track of how often each page has been referenced. When a page fault occurs, the page with the lowest counter is chosen for replacement.

The main problem with NFU is that it never forgets anything. For example,

in a multipass compiler, pages that were heavily used during pass 1 may still have a high count well into later passes. In fact, if pass 1 happens to have the longest execution time of all the passes, the pages containing the code for subsequent passes may always have lower counts than the pass 1 pages. Consequently, the operating system will remove useful pages instead of pages no longer in use.

Fortunately, a small modification to NFU makes it able to simulate LRU quite well. The modification has two parts. First, the counters are each shifted right 1 bit before the *R* bit is added in. Second, the *R* bit is added to the leftmost, rather than the rightmost bit.

Figure 4-4 illustrates how the modified algorithm, known as **aging**, works. Suppose that after the first clock tick the *R* bits for pages 0 to 5 have the values 1, 0, 1, 0, 1, and 1, respectively (page 0 is 1, page 1 is 0, page 2 is 1, etc.). In other words, between tick 0 and tick 1, pages 0, 2, 4, and 5 were referenced, setting their *R* bits to 1, while the other ones remain 0. After the six corresponding counters have been shifted and the *R* bit inserted at the left, they have the values shown in Fig. 4-4(a). The four remaining columns show the six counters after the next four clock ticks.



**Figure 4-4.** The aging algorithm simulates LRU in software. Shown are six pages for five clock ticks. The five clock ticks are represented by (a) to (e).

When a page fault occurs, the page whose counter is the lowest is removed. It is clear that a page that has not been referenced for, say, four clock ticks will have four leading zeros in its counter and thus will have a lower value than a counter that has not been referenced for three clock ticks.

This algorithm differs from LRU in two ways. Consider pages 3 and 5 in Fig. 4-4(e). Neither has been referenced for two clock ticks; both were referenced in the tick prior to that. According to LRU, if a page must be replaced, we should choose one of these two. The trouble is, we do not know which of these two was referenced last in the interval between tick 1 and tick 2. By recording only one bit per time interval, we have lost the ability to distinguish references early in the clock interval from those occurring later. All we can do is remove page 3, because page 5 was also referenced two ticks earlier and page 3 was not.

The second difference between LRU and aging is that in aging the counters have a finite number of bits, 8 bits in this example. Suppose that two pages each have a counter value of 0. All we can do is pick one of them at random. In reality, it may well be that one of the pages was last referenced 9 ticks ago and the other was last referenced 1000 ticks ago. We have no way of seeing that. In practice, however, 8 bits is generally enough if a clock tick is around 20 msec. If a page has not been referenced in 160 msec, it probably is not that important.

#### 4.4.8 The Working Set Page Replacement Algorithm

In the purest form of paging, processes are started up with none of their pages in memory. As soon as the CPU tries to fetch the first instruction, it gets a page fault, causing the operating system to bring in the page containing the first instruction. Other page faults for global variables and the stack usually follow quickly. After a while, the process has most of the pages it needs and settles down to run with relatively few page faults. This strategy is called **demand paging** because pages are loaded only on demand, not in advance.

Of course, it is easy enough to write a test program that systematically reads all the pages in a large address space, causing so many page faults that there is not enough memory to hold them all. Fortunately, most processes do not work this way. They exhibit a **locality of reference**, meaning that during any phase of execution, the process references only a relatively small fraction of its pages. Each pass of a multipass compiler, for example, references only a fraction of all the pages, and a different fraction at that.

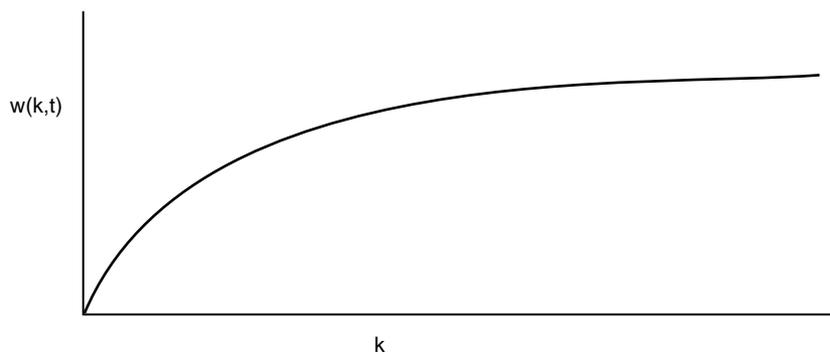
The set of pages that a process is currently using is called its **working set** (Denning, 1968a; Denning, 1980). If the entire working set is in memory, the process will run without causing many faults until it moves into another execution phase (e.g., the next pass of the compiler). If the available memory is too small to hold the entire working set, the process will cause many page faults and run slowly since executing an instruction takes a few nanoseconds and reading in a page from the disk typically takes 10 milliseconds. At a rate of one or two instructions per 10 milliseconds, it will take ages to finish. A program causing page faults every few instructions is said to be **thrashing** (Denning, 1968b).

In a multiprogramming system, processes are frequently moved to disk (i.e., all their pages are removed from memory) to let other processes have a turn at the

CPU. The question arises of what to do when a process is brought back in again. Technically, nothing need be done. The process will just cause page faults until its working set has been loaded. The problem is that having 20, 100, or even 1000 page faults every time a process is loaded is slow, and it also wastes considerable CPU time, since it takes the operating system a few milliseconds of CPU time to process a page fault.

Therefore, many paging systems try to keep track of each process' working set and make sure that it is in memory before letting the process run. This approach is called the **working set model** (Denning, 1970). It is designed to greatly reduce the page fault rate. Loading the pages *before* letting processes run is also called **prepaging**. Note that the working set changes over time.

It has been long known that most programs do not reference their address space uniformly, but that the references tend to cluster on a small number of pages. A memory reference may fetch an instruction, it may fetch data, or it may store data. At any instant of time,  $t$ , there exists a set consisting of all the pages used by the  $k$  most recent memory references. This set,  $w(k, t)$ , is the working set. Because the  $k = 1$  most recent references must have used all the pages used by the  $k > 1$  most recent references, and possibly others,  $w(k, t)$  is a monotonically nondecreasing function of  $k$ . The limit of  $w(k, t)$  as  $k$  becomes large is finite because a program cannot reference more pages than its address space contains, and few programs will use every single page. Figure 4-5 depicts the size of the working set as a function of  $k$ .



**Figure 4-5.** The working set is the set of pages used by the  $k$  most recent memory references. The function  $w(k, t)$  is the size of the working set at time  $t$ .

The fact that most programs randomly access a small number of pages, but that this set changes slowly in time explains the initial rapid rise of the curve and then the slow rise for large  $k$ . For example, a program that is executing a loop occupying two pages using data on four pages, may reference all six pages every 1000 instructions, but the most recent reference to some other page may be a million instructions earlier, during the initialization phase. Due to this asymptotic behavior, the contents of the working set is not sensitive to the value of  $k$  chosen.

To put it differently, there exists a wide range of  $k$  values for which the working set is unchanged. Because the working set varies slowly with time, it is possible to make a reasonable guess as to which pages will be needed when the program is restarted on the basis of its working set when it was last stopped. Prepaging consists of loading these pages before the process is allowed to run again.

To implement the working set model, it is necessary for the operating system to keep track of which pages are in the working set. Having this information also immediately leads to a possible page replacement algorithm: when a page fault occurs, find a page not in the working set and evict it. To implement such an algorithm, we need a precise way of determining which pages are in the working set and which are not at any given moment in time.

As we mentioned above, the working set is the set of pages used in the  $k$  most recent memory references (some authors use the  $k$  most recent page references, but the choice is arbitrary). To implement any working set algorithm, some value of  $k$  must be chosen in advance. Once some value has been selected, after every memory reference, the set of pages used by the previous  $k$  memory references is uniquely determined.

Of course, having an operational definition of the working set does not mean that there is an efficient way to monitor it in real time, during program execution. One could imagine a shift register of length  $k$ , with every memory reference shifting the register left one position and inserting the most recently referenced page number on the right. The set of all  $k$  page numbers in the shift register would be the working set. In theory, at a page fault, the contents of the shift register could be read out and sorted. Duplicate pages could then be removed. The result would be the working set. However, maintaining the shift register and processing it at a page fault would both be prohibitively expensive, so this technique is never used.

Instead, various approximations are used. One commonly used approximation is to drop the idea of counting back  $k$  memory references and use execution time instead. For example, instead of defining the working set as those pages used during the previous 10 million memory references, we can define it as the set of pages used during the past 100 msec of execution time. In practice, such a definition is just as good and much easier to use. Note that for each process, only its own execution time counts. Thus if a process starts running at time  $T$  and has had 40 msec of CPU time at real time  $T + 100$  msec, for working set purposes, its time is 40 msec. The amount of CPU time a process has actually used has since it started is often called its **current virtual time**. With this approximation, the working set of a process is the set of pages it has referenced during the past  $\tau$  seconds of virtual time.

Now let us look at a page replacement algorithm based on the working set. The basic idea is to find a page that is not in the working set and evict it. In Fig. 4-6 we see a portion of a page table for some machine. Because only pages that are in memory are considered as candidates for eviction, pages that are absent from memory are ignored by this algorithm. Each entry contains (at least) two



age is evicted. In the worst case, all pages have been referenced during the current clock tick (and thus all have  $R = 1$ ), so one is chosen at random for removal, preferably a clean page, if one exists.

#### 4.4.9 The WSClock Page Replacement Algorithm

The basic working set algorithm is cumbersome since the entire page table has to be scanned at each page fault until a suitable candidate is located. An improved algorithm, that is based on the clock algorithm but also uses the working set information is called **WSClock** (Carr and Hennessey, 1981). Due to its simplicity of implementation and good performance, it is widely used in practice.

The data structure needed is a circular list of page frames, as in the clock algorithm, and as shown in Fig. 4-7(a). Initially, this list is empty. When the first page is loaded, it is added to the list. As more pages are added, they go into the list to form a ring. Each entry contains the *Time of last use* field from the basic working set algorithm, as well as the  $R$  bit (shown) and the  $M$  bit (not shown).

As with the clock algorithm, at each page fault the page pointed to by the hand is examined first. If the  $R$  bit is set to 1, the page has been used during the current tick so it is not an ideal candidate to remove. The  $R$  bit is then set to 0, the hand advanced to the next page, and the algorithm repeated for that page. The state after this sequence of events is shown in Fig. 4-7(b).

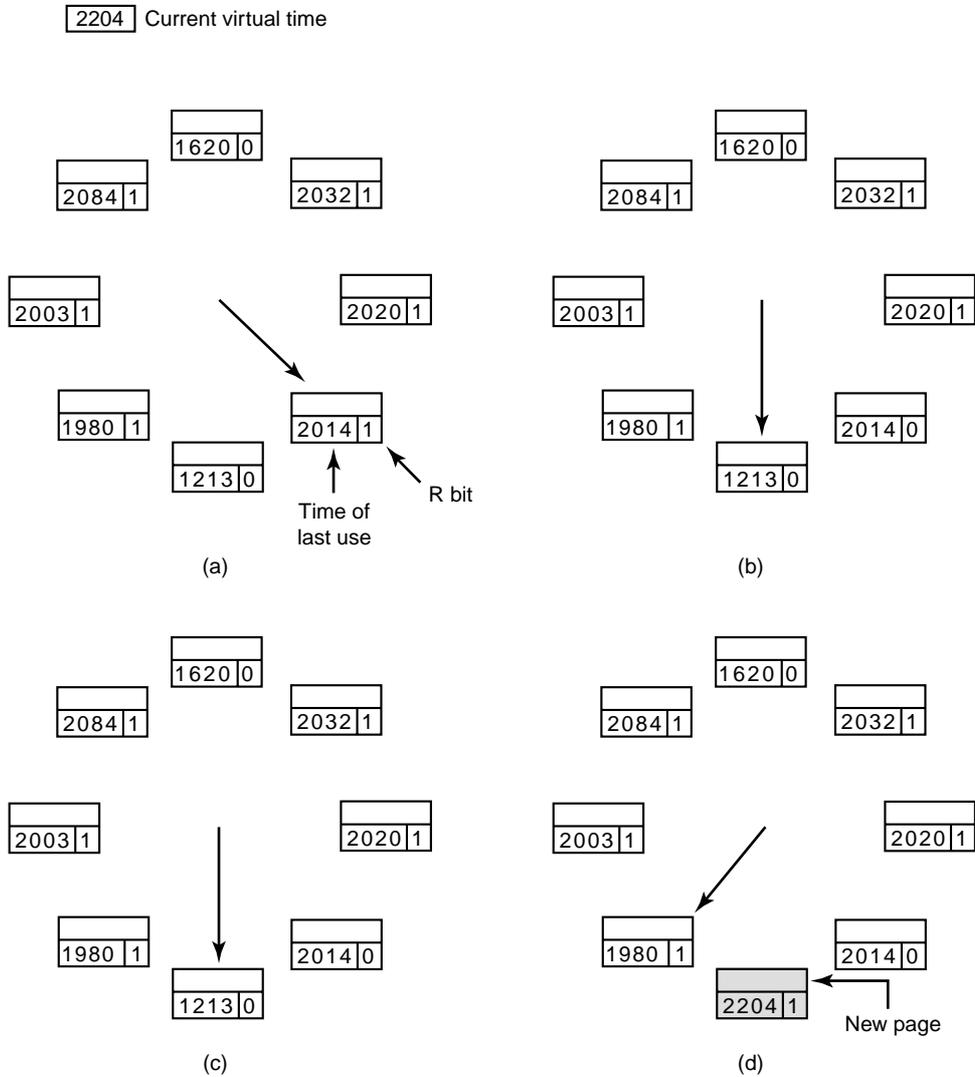
Now consider what happens if the page pointed to has  $R = 0$ , as shown in Fig. 4-7(c). If the age is greater than  $\tau$  and the page is clean, it is not in the working set and a valid copy exists on the disk. The page frame is simply claimed and the new page put there, as shown in Fig. 4-7(d). On the other hand, if the page is dirty, it cannot be claimed immediately since no valid copy is present on disk. To avoid a process switch, the write to disk is scheduled, but the hand is advanced and the algorithm continues with the next page. After all, there might be an old, clean page further down the line that can be used immediately.

In principle, all pages might be scheduled for disk I/O on one cycle around the clock. To reduce disk traffic, a limit might be set, allowing a maximum of  $n$  pages to be written back. Once this limit has been reached, no new writes are scheduled.

What happens if the hand comes all the way around to its starting point? There are two cases to distinguish:

1. At least one write has been scheduled.
2. No writes have been scheduled.

In the former case, the hand just keeps moving, looking for a clean page. Since one or more writes have been scheduled, eventually some write will complete and its page will be marked as clean. The first clean page encountered is evicted. This page is not necessarily the first write scheduled because the disk driver may



**Figure 4-7.** Operation of the WSClock algorithm. (a) and (b) give an example of what happens when  $R = 1$ . (c) and (d) give an example of  $R = 0$ .

reorder writes in order to optimize disk performance.

In the latter case, all pages are in the working set, otherwise at least one write would have been scheduled. Lacking additional information, the simplest thing to do is claim any clean page and use it. The location of a clean page could be kept track of during the sweep. If no clean pages exist, then the current page is chosen and written back to disk.

#### 4.4.10 Summary of Page Replacement Algorithms

We have now looked at a variety of page replacement algorithms. In this section we will briefly summarize them. The list of algorithms discussed is given in Fig. 4-8.

Algorithm	Comment
Optimal	Not implementable, but useful as a benchmark
NRU (Not Recently Used)	Very crude
FIFO (First-In, First-Out)	Might throw out important pages
Second chance	Big improvement over FIFO
Clock	Realistic
LRU (Least Recently Used)	Excellent, but difficult to implement exactly
NFU (Not Frequently Used)	Fairly crude approximation to LRU
Aging	Efficient algorithm that approximates LRU well
Working set	Somewhat expensive to implement
WSClock	Good efficient algorithm

**Figure 4-8.** Page replacement algorithms discussed in the text.

The optimal algorithm replaces the page referenced last among the current pages. Unfortunately, there is no way to determine which page will be last, so in practice this algorithm cannot be used. It is useful as a benchmark against which other algorithms can be measured, however.

The NRU algorithm divides pages into four classes depending on the state of the  $R$  and  $M$  bits. A random page from the lowest numbered class is chosen. This algorithm is easy to implement, but it is very crude. Better ones exist.

FIFO keeps track of the order pages were loaded into memory by keeping them in a linked list. Removing the oldest page then becomes trivial, but that page might still be in use, so FIFO is a bad choice.

Second chance is a modification to FIFO that checks if a page is in use before removing it. If it is, the page is spared. This modification greatly improves the performance. Clock is simply a different implementation of second chance. It has the same performance properties, but takes a little less time to execute the algorithm.

LRU is an excellent algorithm, but it cannot be implemented without special hardware. If this hardware is not available, it cannot be used. NFU is a crude attempt to approximate LRU. It is not very good. However, aging is a much better approximation to LRU and can be implemented efficiently. It is a good choice.

The last two algorithms use the working set. The working set algorithm is reasonable performance, but it is somewhat expensive to implement. WSClock is

a variant that not only gives good performance but is also efficient to implement.

All in all, the two best algorithms are aging and WSClock. They are based on LRU and the working set, respectively. Both give good paging performance and can be implemented efficiently. A few other algorithms exist, but these two are probably the most important in practice.