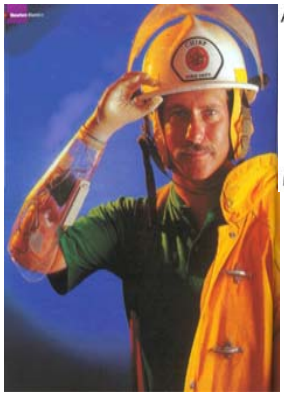
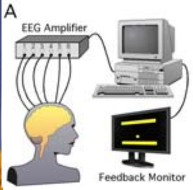
Learning and Remapping in the Sensory Motor System

F.A. Mussa-Ivaldi
Northwestern University

Movement and the brain.

- The anecdote of the sea-squirt (Llinas).

Prologue – A practical Context

" *…movements are possible under conditions of the most accurate and uninterrupted agreement- unforeseen in advance- between the central impulses and the events occurring at the periphery, and are frequently quantitatively less dependent on these central impulses that on the external force field."*

N. Bernstein 1935

The Brain is expert in Dynamics

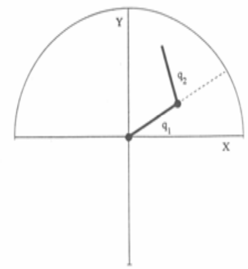
F=m a

$$x(t) = \iint \frac{F}{m} dt dt'$$

$$F(t) = m\ddot{x}(t)$$

Simplified limb dynamics

$$D(q,\dot{q},\ddot{q}) = C(q,t)$$

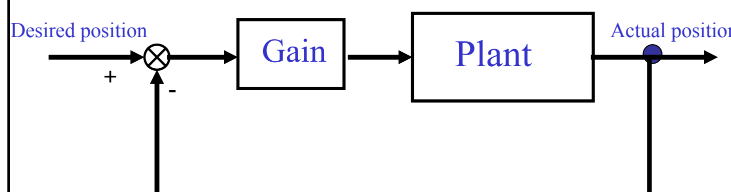


$$\begin{aligned} D_1 = & \left(I_1 + I_2 + m_2 l_1 l_2 cos(q_2) + \frac{m_1 l_1^2 + m_2 l_2^2}{4} + m_2 l_1^2\right)\ddot{q}_1 \\ & + \left(I_2 + \frac{m_2 l_2^2}{4} + \frac{m_2 l_1 l_2}{2} cos(q_2)\right)\ddot{q}_2 \\ & - \left(\frac{m_2 l_1 l_2}{2} sin(q_2)\right)\dot{q}_2^{\,2} - (m_2 l_1 l_2 sin(q_2))\,\dot{q}_1 \dot{q}_2 \\ & + B_1(q_1, q_2, \dot{q}_1, \dot{q}_2), + K_1(q_1, q_2) \end{aligned}$$

$$\begin{aligned} D_2 = & \left(I_2 + \frac{m_2 l_1 l_2}{2} cos(q_2) + \frac{m_2 l_2^2}{4}\right)\ddot{q}_1 + \left(I_2 + \frac{m_2 l_2^2}{4}\right)\ddot{q}_2 \\ & + \left(\frac{m_2 l_1 l_2}{2} sin(q_2)\right)\dot{q}_1^{\,2} \\ & + B_2(q_1, q_2, \dot{q}_1, \dot{q}_2), + K_2(q_1, q_2) \end{aligned}$$

SHORTCUTS

FEEDBACK



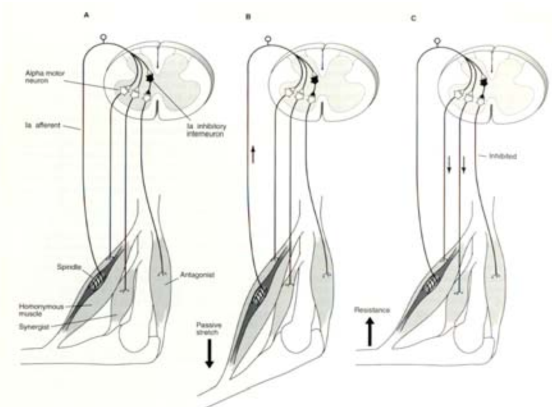


FIGURE 37–11
Excitation of muscle spindles is responsible for the stretch reflex.
A. Ia afferent fibers make monosynaptic excitatory connections to alpha motor neurons innervating the same (homonymous) muscle from which they arise and motor neurons innervating synergist muscles. They also inhibit motor neurons to antagonist muscles through an inhibitory interneuron.
B. When a muscle is stretched the Ia afferents increase their firing rate.
C. This leads to contraction of the same muscle and its synergists and relaxation of the antagonist. The reflex therefore tends to counteract the stretch, enhancing the spring-like properties of the muscles.

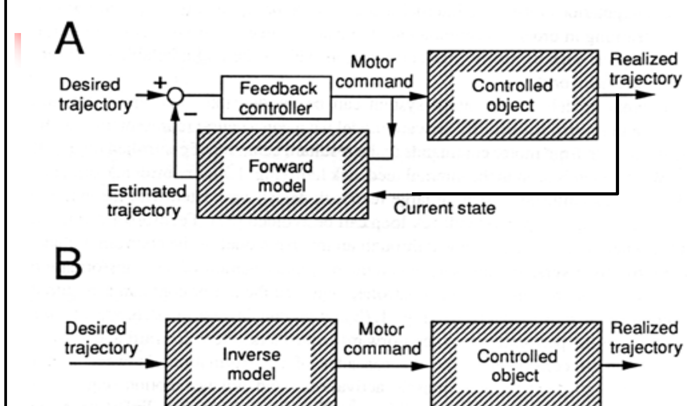
From Kandel, Schwartz and Jessel. *Principles of Neural Science*

•The look-up table approach

- Can motor patterns just be stored and then used later on when necessary?

A MORE PLAUSIBLE APPROACH

Forward and inverse models

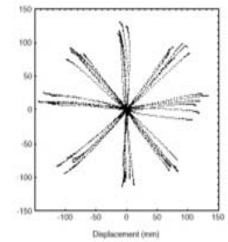


From Kawato et al., 1992

SOME EXPERIMENTAL EVIDENCE

Adaptation of hand movements to perturbing fields

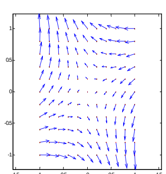
Unperturbed movements

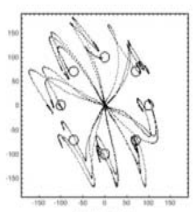


Shadmehr & Mussa-Ivaldi, 1994

Perturbing field

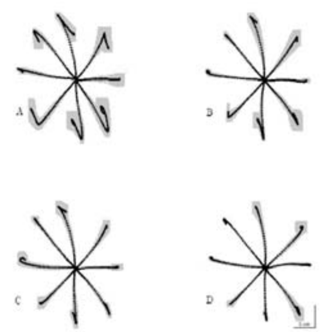
$$\mathbf{F} = \begin{bmatrix} -10.1 & -11.2 \\ -11.2 & 11.1 \end{bmatrix} \dot{\mathbf{r}}$$



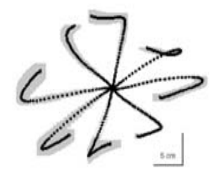


Note: there is no force at rest $(\dot{\mathbf{r}} = \mathbf{0})$

Adaptation

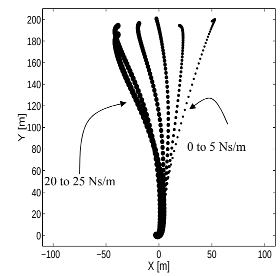


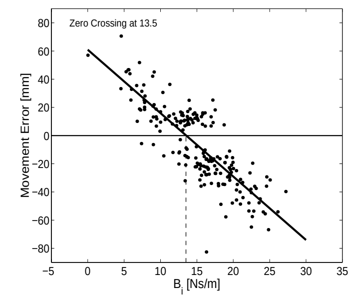
After-effects



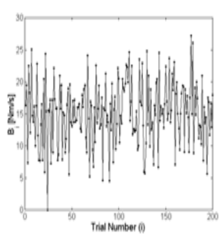
TIME COURSE OF ADAPTATION

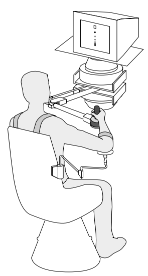
Subjects Adapt to the Mean Field

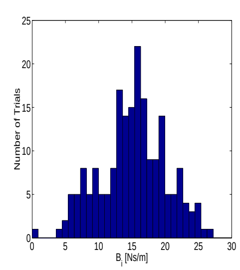




Adaptation to Random Fields

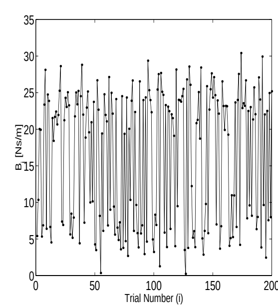


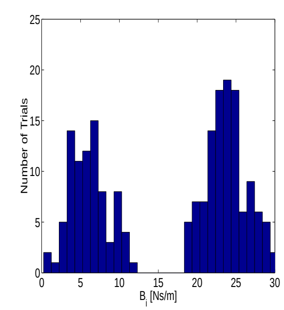




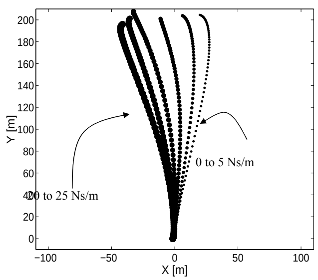
Scheidt, Dingwell & Mussa-Ivaldi

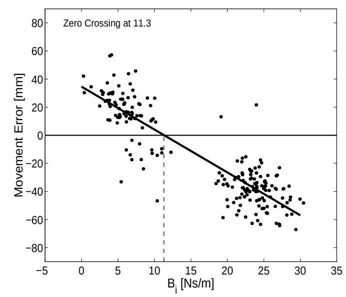
When the Field is Bi-Modal...





...They Still Adapt to the Mean



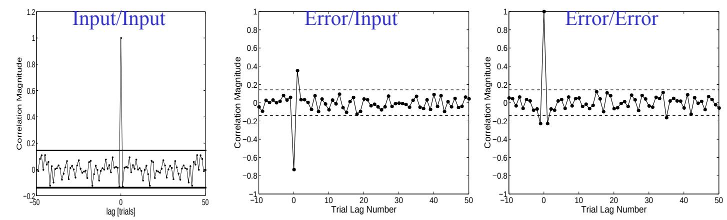


Only Recent Memories Contribute to Adaptation

ARX Model: $\varepsilon_i = \sum_{j=1}^{i-L} a_j \, \varepsilon_{i-j} + \sum_{k=0}^{i-M} b_k \, B_{i-k}$

Error

Input (field strength)



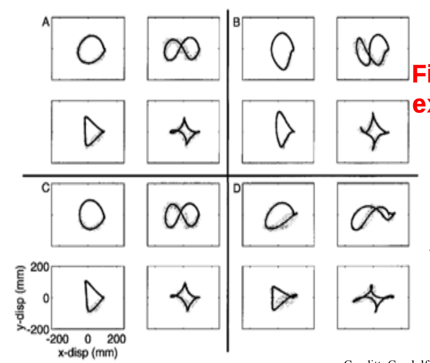
$$\boxed{\varepsilon_i = a_1 \, \varepsilon_{i-1} + b_0 \, B_i + b_1 \, B_{i-1}}$$

MODEL OR MEMORY?

Not only reaching movements....

Baseline

First exposure



Adapted

After-effects

Conditt, Gandolfo and and Mussa-Ivaldi, (1999).

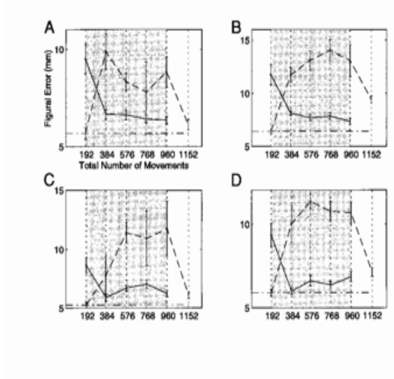
Learning curves for shapes

Circle

Infinity

Triangle

Diamond



Not by simple playback of forces

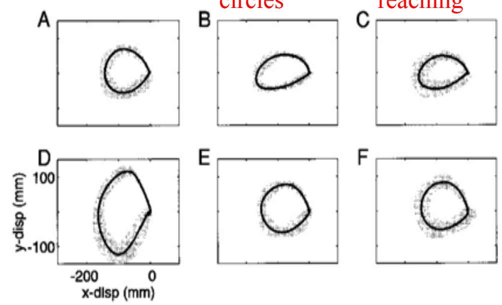
Control

Trained w. circles

Trained w. reaching

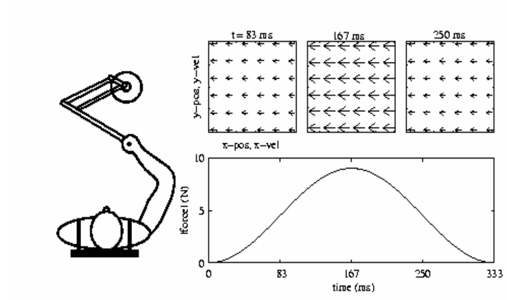
Null field

Viscous field



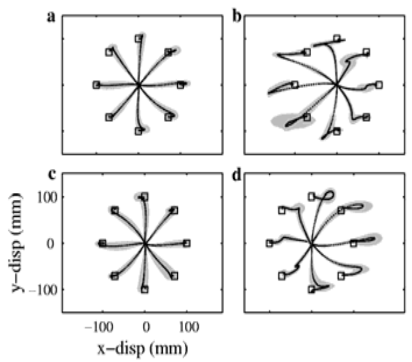
TIME OR STATE?

Time-varying Field

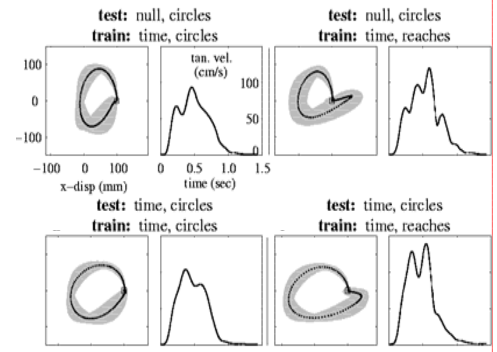


Conditt and Mussa-Ivaldi, (1999).

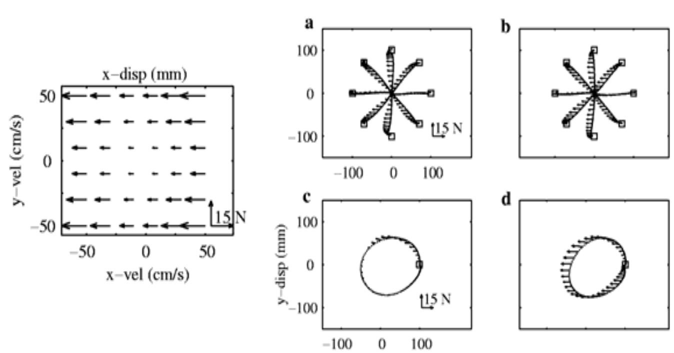
Compensation of Time-varying Forces…



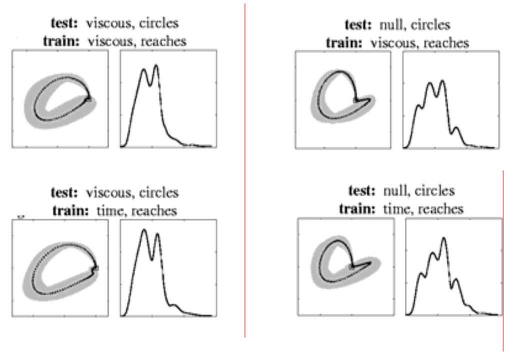
…But No Learning



Equivalent State-dependent Field

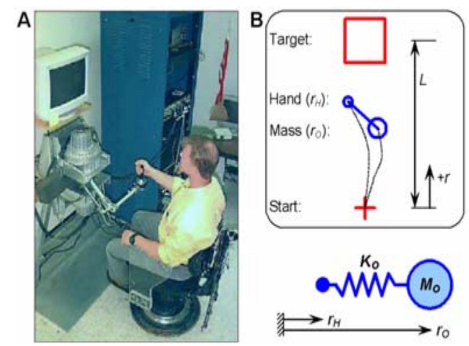


A “Wrong” Generalization?



CONTROLLING OBJECTS WITH INTERNAL DEGREES OF FREEDOM

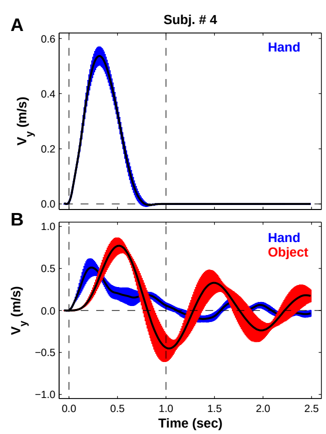
Smooth motion: a general principle for trajectory formation



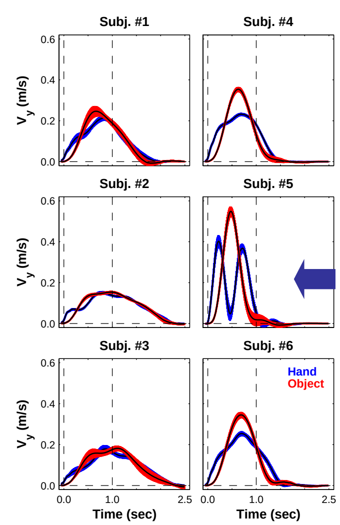
J Dingwell, Mah, C., F.A. Mussa-Ivaldi, 2003

Baseline

Initial response



Post-adaptation kinematics



Minimum-Jerk (Hogan and Flash)

Goal: Generate the smoothest motion which brings a limb from equilibrium at the starting position to equilibrium at the target position in a given time.

Approach: quantify smoothness by high temporal derivative of motion

$$C = \int \sum_{n=1}^{\infty} a_n \left\| \frac{d^n x}{dt^n} \right\|^2 dt$$

Simpler (but sufficient) form

$$C = \int_0^T \frac{1}{2} \left(\frac{d^3 x}{dt^3} \right)^2 dt$$

"Jerk"

Euler-Poisson-Lagrange

Find the trajectory x(t) that minimizes the functional

$$C[x(t)] = \int_0^T L(t, x(t), \dot{x}, \ddot{x} \cdots, x^{(n)}) dt$$

This reduces to solving the Euler-Poisson equation:

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} + \cdots + (-1)^n \frac{d^n}{dt^n}\frac{\partial L}{\partial x^{(n)}} = 0$$

For reaching movements

$$L = \frac{1}{2}(\dddot{x})^2$$

Euler equation becomes simply

$$-\frac{d^3}{dt^3}\frac{\partial L}{\partial \dddot{x}} = -\frac{d^3}{dt^3}\dddot{x} = -\frac{d^6 x}{dt^6} = 0$$

$$\frac{d^6 x}{dt^6} = 0$$

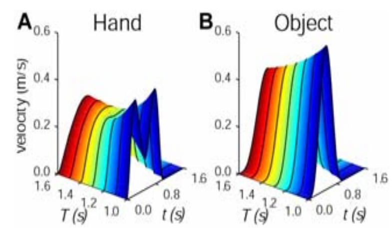
The most general solution is the 5th degree polynomial

$$x(t) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5$$

The six coefficients are determined by the boundary conditions:

- Initial/final position
- Initial/final velocity (0)
- Initial/final acceleration (0)

Optimally Smooth Transport (OST)



Both hand and object must start and end at rest this gives 6x2= 12 Boundary conditions.

$$M_O \ddot{r}_O = K_O(r_H - r_O)$$

Hand and object movements are not independent (two BC on the object acceleration BC on object and hand position)

$\Downarrow$

10 Boundary Conditions (12-2)

$\Downarrow$

Minimum "crackle" $$C = \int_0^T \frac{1}{2}\left(\frac{d^5 r_O}{dt^5}\right)^2 dt$$

Summing up

- **The motor system adapts to novel dynamical conditions by forming internal representations of the functional relationship between forces and motions**
- **Internal models of limb dynamics are constructed based on recent memories**
- **Internal models of limb dynamics are are based on representations of states of motion but (probably) not of time.**
- **Adaptation of reaching movements aims at preserving the smoothness and linearity of the endpoint motion**
- **Planning and control of movements are separate processes**

Glove Talk

Sidney Fels and Geoffrey Hinton

Vocabulary

Sam-I-am

The representation of space.

What is space ?

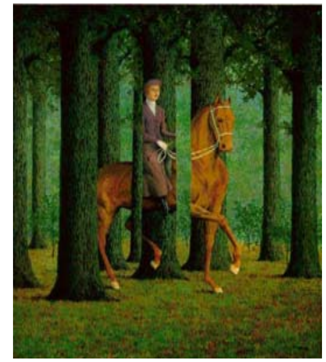
- Intuitive Concept

Geometry, from a physical standpoint is the totality of the laws according to which rigid bodies mutually at rest can be placed with respect to each other… "Space" in this interpretation is in principle an infinite rigid body – or skeleton - to which the position of all other bodies is related."

A. Einstein, 1949

- Less Intuitive Concepts

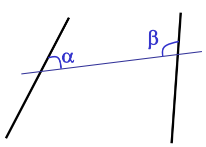
Signals spaces, configuration spaces, state spaces, feature spaces, etc...



Ordinary Space

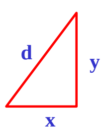
Two equivalent statements

- Euclid



$$\alpha+\beta=180^o \quad ?$$

- Pythagoras



$$d^2 = x^2 + y^2$$

Euclidean Symmetry

Among all possible norm only the L_2 (Euclidean) norm is invariant under rigid transformations (e.g. rotations, translations).

Size does not depend upon position and orientation.

$$\|x\|^2 = x^T \cdot x \qquad \text{Euclidean } (L_2) \text{ Norm}$$

$$y = Ax$$
$$A^T = A^{-1} \qquad \text{Orthogonal transformation}$$

$$\|y\|^2 = y^T y = x^T A^T A x$$
$$= x^T A^{-1} A x = x^T x = \|x\|^2$$

Signal spaces

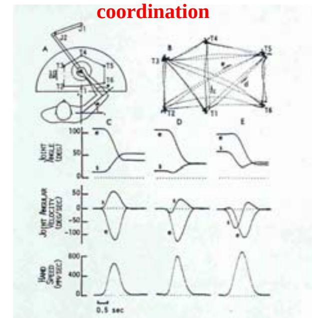
- Neither the visual nor the motor signals spaces are Euclidean.

Yet…

•We perceive Euclidean symmetries

Francesco Borromini. Palazzo Spada. Rome

•Euclidean kinematics of the endpoint guides complex coordination

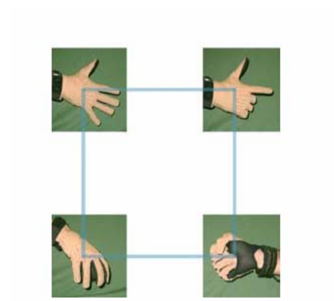


Morasso (1981)

Is the bias toward Euclidean symmetry a guiding principle for the emergence of new motor programs?

Motor Program: Open loop rapid movement

– Not under visual guidance



19 SIGNALS
(Hand Configuration)

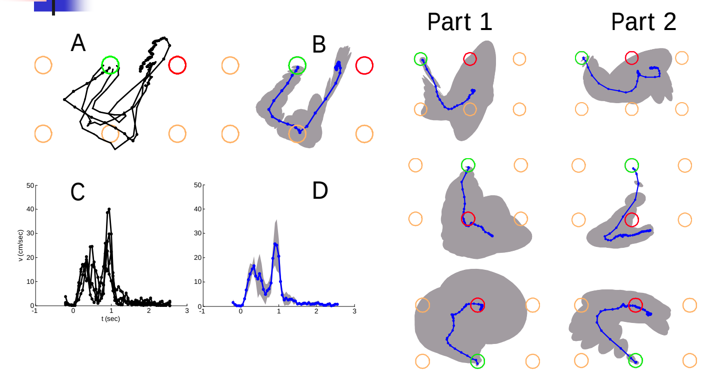
2 Screen Coordinates

F.A. Mussa-Ivaldi, S. Acosta, R. Scheidt, K.M. Mosier

The task

- The subject must place the cursor corresponding to the hand inside a circular target
- Experiment begins with the cursor inside a target.
- New target is presented – Cursor is suppressed.
- The subject must try to make a single rapid movement of the fingers so as to place the (invisible) cursor inside the new target and stop.
- As the hand is at rest after this initial movement, the cursor reappears
- The subject corrects by moving the cursor under visual guidance inside the target
- Only the initial open loop movement is

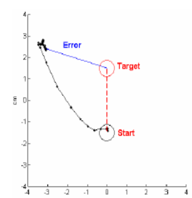
Some examples

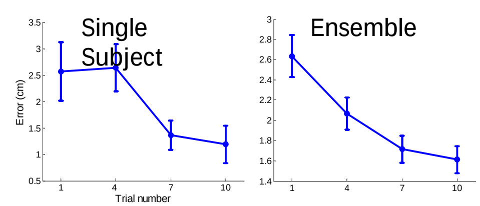


Some observations

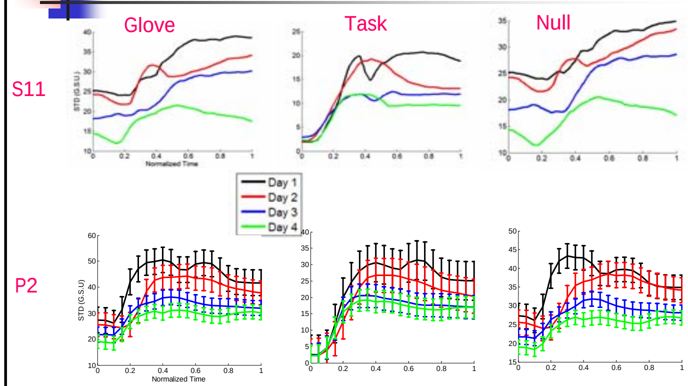
- Weird task. Cannot be represented before the experiment
- Body and task endpoint are physically uncoupled
- Only feedback pathway is vision
- Dimensional imbalance
- Metric imbalance

Endpoint error



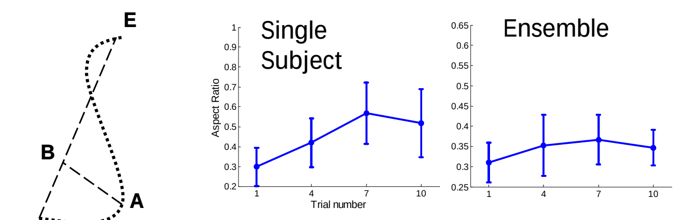


Analysis of variability: Glove



Linearity

Aspect-Ratio: AB/SE



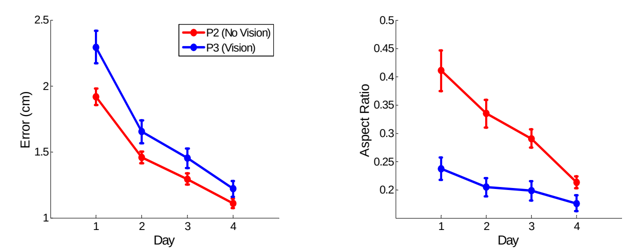
•Cursor movements DO NOT become straighter !

Extended training

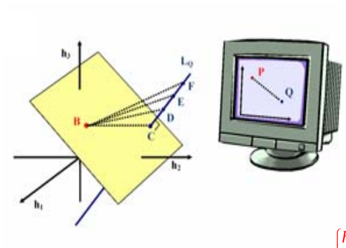
- 4 consecutive days
- 2 groups of subjects
- No-Vision subjects (P2) just engage in the task for 4 days
- Vision subjects (P3) train with continuous visual feedback trials. <u>But they are tested in the same no-vision condition as the other group.</u>
- The two groups receive the same amount of training

Extended Training Results

- Both group have the same learning trend.
- Continuous visual feedback of endpoint movements facilitates the formation of straighter open-loop trajectories



Decomposition of Glove Signals

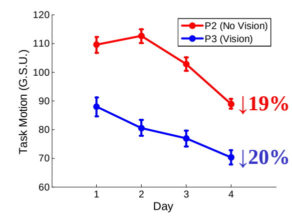


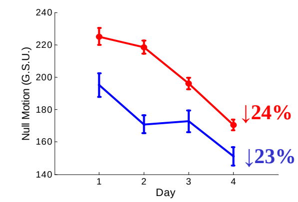
$$A^+ \equiv A^T(A\cdot A^T)^{-1}$$
$$T(A) \equiv A^+ \cdot A$$
$$N(A) \equiv I_n - T(A)$$

$$BF \xrightarrow{A} PQ$$
$$BF = BC + CF$$

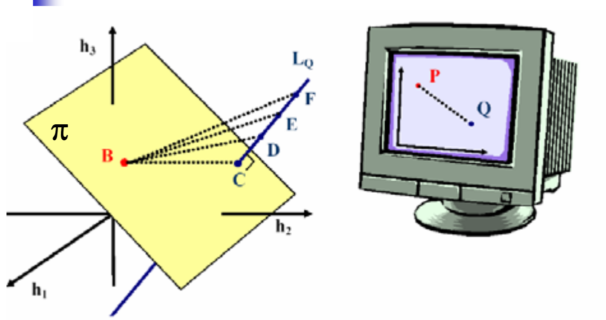
$$\begin{cases} h_N = N(A)\cdot h & \text{"Null-Space Component"} \\ h_T = T(A)\cdot h & \text{"Task-Space Component"} \\ h = h_N + h_T & \\ h^T{}_N \cdot h_T = 0 & \end{cases}$$

Task-space and Null-space Motion





Importing a metric structure

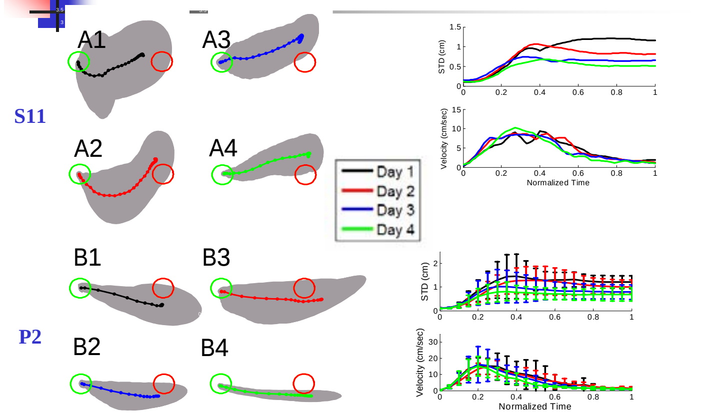


A Different View

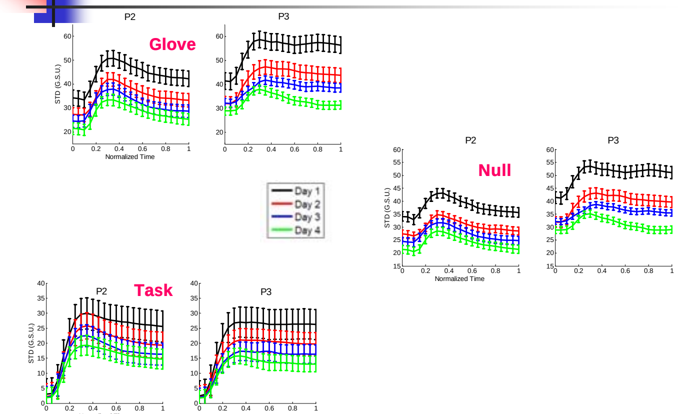
Optimal feedback Control (Todorov, Jordan 2002):

- There is no explicit control of trajectories
- The control system minimizes the variance of the endpoint by allowing variability in redundant dimensions

Analysis of variability: Cursor



Analysis of variability: All Movements

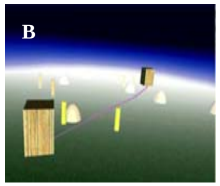


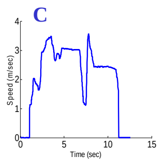
Conclusions

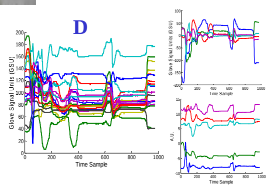
- Practice with and without feedback leads to improved mapping of target positions into finger postures (an ill-posed problem).
- This is not sufficient for trajectory learning (Euclidean metric)
- Extended practice of movement facilitates the generation of rectilinear cursor motion. This trend is stronger in the presence of continuous feedback of endpoint motion
- Final error improvement is independent of trajectory learning
- Practice of movement leads to a general and uniform decrease of movement variability, including null-space variability, NOT an increase.
- Whole movement trajectories (not just final positions) appears to be explicitly regulated to conform with the Euclidean metric of the controlled endpoint

Epilogue 1 – Relearning space

Human / Machine Learning







Tognetti et al 2005

The representation of time. When is "now" ?

Open questions

•How do we perceive simultaneous events ? (How "thick" is now?)

•Can we adapt to perceptual delays ?

Physician  x_C F_C

$x_B(t)=x_C(t-\Delta)$

$F_C(t)=F_B(t-\Delta)$

Patient  x_B F_B

? → Learn time (t-Δ) $F_C=K\cdot x_C(t-2\Delta)$

? → Learn impedance $F_C=K\cdot x_C(t-2\Delta)=K\cdot(x_C(t)-2\Delta\cdot\dot{x}_C(t)+\Delta^2\cdot\ddot{x}_C)$

Libet's Experiment

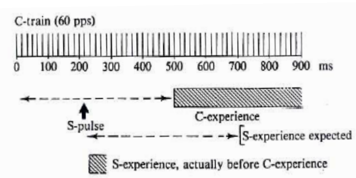


FIG. 1. Diagram of experiment on subjective time order of two sensory experiences, one elicited by a stimulus train to SI cortex (C) and one by a threshold pulse to skin (S). C consisted of repetitive pulses (at 60 pps) applied to postcentral gyrus, at the lowest (liminal) peak current sufficient to elicit any reportable conscious sensory experience. The sensory experience for C ('C-experience') would not be initiated before the end of the utilization-train duration (U-TD, average about 500 ms), but then proceeds without change in its weak *subjective* intensity for the remainder of the applied liminal C train (*see* Libet *et al.*, 1964; Libet, 1966, 1973). The S-pulse, at just above threshold strength for eliciting conscious sensory experience, is here shown delivered when the initial 200 ms of the C train have elapsed. (In other experiments, it was applied at other relative times, earlier and later.) If S were followed by a roughly similar delay of 500 ms of cortical activity before 'neuronal adequacy' is achieved, initiation of S-experience might have also been expected to be delayed until 700 ms of C had elapsed. In fact, S-experience was reported to appear subjectively before C-experience (*see* text).

Epilogue 2. Aristotelian science

- A legend about gravity.