On the Stability of Manipulators Performing Contact Tasks

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Abstract—**By any reasonable definition, manipulation requires contact with the object being manipulated, and the full potential of robots can only be realized when they are applied to contact tasks. One of the difficulties engendered by contact tasks is that they require intimate dynamic interaction between the robot and its environment. That interaction changes the performance of the robot and can jeopardize the stability of its control system.**

This paper will discuss the problem of preserving the stability of a manipulator's control system during contact tasks. It will be shown that contact stability may be guaranteed if the control system provides the manipulator with an appropriately structured dynamic response to environmental inputs. Two aspects of one implementation of such a controller will be considered. Robustness to large errors in the manipulator kinematic equations and to unmodeled interface dynamics will be shown.

Nomenclature

$\partial(\cdot)/\partial(\cdot)$	Gradient operator.
$E_k(\cdot)$	Kinetic energy.
$E_k{}^*(\cdot)$	Kinetic co-energy.
$E_p(\cdot)$	Potential energy.
$G(\cdot)$	Transfer function.
$H(\cdot)$	Hamiltonian.
$L(\cdot)$	Lagrangian.
s	Laplace variable.
u	Input variable.
y	Output variable.
$\boldsymbol{B}(\cdot)$	Force/velocity relation.
$\boldsymbol{D}(\cdot)$	Generalized nonconservative internal forces.
$\boldsymbol{F}$	Interaction port force.
$\boldsymbol{I}(\cdot)$	Manipulator inertia tensor.
$\boldsymbol{J}(\cdot)$	Jacobian of a kinematic transformation.
$\boldsymbol{K}(\cdot)$	Force/displacement relation.
$\boldsymbol{L}(\cdot)$	Kinematic transformation equations.
$\boldsymbol{M}$	Rigid body inertia tensor.
$\boldsymbol{P}$	Generalized forces.
$\boldsymbol{p}$	Generalized momentum.
$\boldsymbol{q}$	Generalized coordinates.
$\boldsymbol{V}$	Interaction port velocity.
$\boldsymbol{X}$	Interaction port coordinates.

Subscripts:

a	Actuator.
c	Controlled system.
e	Environment.
i	Interface.
m	Manipulator.
t	Coupled system.

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Introduction

THE WORK presented in this paper is part of an effort to develop a unified approach to the design and implementation of controllers for systems that may interact dynamically with their environment. One of the perceived problems is robustness: preserving stability and acceptable performance in the face of changes. One of the most profound changes occurs when a manipulator contacts an object. Before contact, the controller interacts with the hardware of the manipulator; after contact, the controller also interacts dynamically with the environment. In this sense, the physical system interacting with the controller may change dramatically as a result of contact. Consequently, the performance of the controller must change, and that change may be drastic. Even if the controlled system is stable when not in contact, when it contacts the environment that stability may be jeopardized. This phenomenon was reported by Whitney in 1977 [20] and subsequently by numerous others. It was identified recently [16] as one of the major challenges of robotics.

The problem of preserving controller stability in the face of changes has received considerable attention, especially in the last decade, but the changes are usually considered to be small in some sense—small changes in system parameters or the presence of unmodeled dynamics. Unmodeled dynamics are commonly assumed to be some subtle aspect of system behavior which only becomes important above the frequency range of normal operation.

The problems generated by dynamic interaction between manipulator and environment are at least an order of magnitude more severe, because the changes cannot be assumed to be small. Consider two people shaking hands: As soon as hands are clasped each person's control system interacts dynamically with the musculo-skeletal systems of both people. Thus the number of degrees of freedom of the physical system coupled to each biological controller approximately doubles when hands are clasped; a dramatic change. The changes cannot be assumed to occur at high frequency; both of the interacting subsystems—both people—have approximately the same frequency range of operation.

This paper will examine a simple control strategy which preserves controller stability in the face of such dramatic changes in the dynamics of the environment coupled to the manipulator.

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Impedance Control

Most of the prior art in control system design has been dominated by the problem or regulation or tracking. It is typically assumed that the objective of the controller is to minimize the error between a reference command vector and a selected output vector, subject to the limitations of the physical hardware. The influence of the environment on the control system is usually added as an afterthought and treated as a disturbance which will tend to thwart this objective. These disturbances are usually assumed to be generated by an independent process and it is typically assumed that the response to disturbances should be minimized, again subject to the limitations of the physical hardware.

When this approach is applied to dynamically interacting systems, it is quite misleading. First, because of the dynamic interaction, the "disturbances" due to the environment may no longer be described as independent of the state of the system, and this invalidates most of the usual analysis. Secondly, the response of the system to the environment becomes at least as important as its response to a reference input, if not more so. The typical regulator design procedures for linear systems focus on shaping the transfer function matrix relating an output vector to a command vector. If the system is to perform dynamically interactive tasks, it is equally important to shape the transfer function dictating the response of the system to its environmental inputs. Indeed, this may become the dominant concern in the design of the controller.

The approach presented in this paper is based on the recognition that a controller does more than merely regulate a system's output. The true action of a controller on a physical system is to modify its behavior. If we describe a physical system by a set of constitutive equations, the action of a controller is to change the apparent "constitutive behavior" of the system. In terms of the system's response to its environment, the controller acts to change the system's dynamic impedance[1] and the approach presented here has been termed impedance control [7]–[9].

Physical Equivalence

One of the ideas underlying this approach to controlling impedance is the conjecture of *physical equivalence* [9]. Any controlled system will consist of "hardware" components (e.g., sensors, actuators, and structures) coupled to controlling "software" (e.g., a brain or a computer). In general, the controller is a dynamic system, operating on a set of measurement inputs from the sensors and reference inputs or "high-level" instructions, to produce a set of output commands to the actuators. Aside from the limitations of computability, the dynamic behavior of the controller appears to be subject to no fundamental restrictions.

In general, the hardware components also comprise a dynamic system, but, unlike the controller, the dynamic behavior of the hardware is clearly subject to a number of important restrictions arising from fundamental physical laws. However, the controller acts on the environment through the hardware. The conjecture of physical equivalence is that, as seen from the environment, the dynamic behavior of the combination of hardware and software is restricted in the same way as any physical system.

One implication of this conjecture is that no controller need be considered unless it results in a behavior of the controlled system which can be described as equivalent to some physical system. In other words, the equations describing the controlled system should be subject to the same restrictions as the equations used to describe physical systems. Well-developed formalisms exist for describing physical systems (e.g., [3], [17], [18]). If true, the postulate of physical equivalence implies that the same techniques can be used to describe control systems. Furthermore, the concept of physical equivalence provides a powerful and intuitive way of thinking about control action in physical terms, and permits a unified approach to the design of both the controller and the physical hardware.

What distinguishes the differential equations used to model a physical system from any other general system of differential equations? No complete answer is attempted here, but the following sections consider some of the features of physical system models and their implications for system stability.

Energetic Structure of the Environmental Dynamics

If a manipulator (biological or artificial) is to interact dynamically with its environment then it is important to understand the dynamic behavior of the environment. We first define the class of environments to be considered in this paper. In the vast majority of cases, the environment a manipulator grasps consists of an inertial object, possibly kinematically constrained. That inertial object may in turn interact with other dynamic entities. Arbitrarily complex environments of this class can be described in the following form using Lagrange's equations:

$$L(q_e, \dot{q}_e) = E_k^*(q_e, \dot{q}_e) - E_p(q_e) \tag{1}$$

$$d[\partial L/\partial \dot{q}_e)]/dt - \partial L/\partial q_e = -D_e(q_e, \dot{q}_e) + P_e(t). \tag{2}$$

We will further restrict out attention to environments which are 1) passive and 2) stable in isolation. To analyze stability, it is convenient to express the dynamic equations in generalized Hamiltonian form. It is a common misconception that the Hamiltonian form can be used only for conservative systems, but this is not the case. A Hamiltonian may be formed by defining generalized momentum as the velocity gradient of kinetic co-energy and applying a Legendre transformation.

$$p_e \triangleq \partial L/\partial \dot{q}_e \tag{3}$$

$$E_k(p_e, q_e) = p_e^t \dot{q}_e - E_k^*(q_e, \dot{q}_e) \tag{4}$$

$$H_e(p_e, q_e) = E_k(p_e, q_e) + E_p(q_e) = p_e^t \dot{q}_e - L(q_e, \dot{q}_e). \tag{5}$$

The momentum gradient of the Hamiltonian yields differential equations for the displacement

$$\dot{q}_e = \partial H_e/\partial p_e. \tag{6}$$

[1] The term impedance is used here in the general sense of a dynamic relation between two variables whose inner product is power, e.g., as a generalization of the common electrical engineering term meaning a dynamic or frequency-dependent resistance.

Substituting into the Lagrangian form and rearranging yields the Hamiltonian form of the momentum equations

$$\dot{\boldsymbol{p}}_e = -\partial H_e/\partial \boldsymbol{q}_e - \boldsymbol{D}_e(\boldsymbol{p}_e, \boldsymbol{q}_e) + \boldsymbol{P}_e(t). \tag{7}$$

For notational convenience in subsequent analysis, denote the gradient of the Hamiltonian with respect to generalized displacements as $\boldsymbol{H}_{\mathrm{eq}}$ and with respect to generalized momenta as $\boldsymbol{H}_{\mathrm{ep}}$.

$$\boldsymbol{H}_{\mathrm{eq}} \triangleq \partial H_e/\partial \boldsymbol{q}_e \qquad \boldsymbol{H}_{\mathrm{ep}} \triangleq \partial H_e/\partial \boldsymbol{p}_e \tag{8}$$

$$\dot{\boldsymbol{q}}_e = \boldsymbol{H}_{\mathrm{ep}}(\boldsymbol{p}_e, \boldsymbol{q}_e) \tag{9}$$

$$\dot{\boldsymbol{p}}_e = -\boldsymbol{H}_{\mathrm{eq}}(\boldsymbol{p}_e, \boldsymbol{q}_e) - \boldsymbol{D}_e(\boldsymbol{p}_e, \boldsymbol{q}_e) + \boldsymbol{P}_e(t). \tag{10}$$

An advantage of the Hamiltonian formulation is that for this class of systems the Hamiltonian is identical to the total system energy and may be used to establish passivity and stability. If the environment is passive, its total energy has a lower bound, but that alone is not sufficient to establish stability in isolation—a passive system may be unstable [22]. To establish stability we examine equilibrium conditions.

If an equilibrium configuration exists the generalized velocity is zero, and from (5) and (9), the system is at a minimum of the kinetic energy with respect to the generalized momentum. In general, the kinetic energy is a positive-definite, nondecreasing function of the momentum, thus at equilibrium $\boldsymbol{p}_e = \boldsymbol{0}$.

We define this class of systems to be isolated whenever $\boldsymbol{P}_e(t) = \boldsymbol{0}$. At equilibrium in the absence of external forces the nonconservative internal forces vanish, $\boldsymbol{D}_e(\boldsymbol{0}, \boldsymbol{q}_e) = \boldsymbol{0}$, thus from (5) and (10), the equilibrium configuration is at a minimum of the potential energy with respect to the generalized displacement. We will consider only those environments for which potential energy is a positive-definite, nondecreasing function of the generalized displacements, thus there is an unique equilibrium configuration at $\boldsymbol{q}_e = \boldsymbol{0}$.

For systems of this class, the Hamiltonian may be used as a Lyapunov function for the analysis of stability [6]. The rate of change of the Hamiltonian is

$$\begin{aligned} dH_e/dt &= \boldsymbol{H}^t_{\mathrm{eq}}\dot{\boldsymbol{q}}_e + \boldsymbol{H}^t_{\mathrm{ep}}\dot{\boldsymbol{p}}_e \\ &= \boldsymbol{H}^t_{\mathrm{eq}}\boldsymbol{H}_{\mathrm{ep}} - \boldsymbol{H}^t_{\mathrm{ep}}\boldsymbol{H}_{\mathrm{eq}} - \boldsymbol{H}^t_{\mathrm{ep}}\boldsymbol{D}_e + \boldsymbol{H}^t_{\mathrm{ep}}\boldsymbol{P}_e \end{aligned} \tag{11}$$

$$dH_e/dt = -\dot{\boldsymbol{q}}^t_e\boldsymbol{D}_e + \dot{\boldsymbol{q}}^t_e\boldsymbol{P}_e. \tag{12}$$

If the system is isolated,

$$dH_e/dt = -\dot{\boldsymbol{q}}^t_e\boldsymbol{D}_e(\boldsymbol{p}_e, \boldsymbol{q}_e). \tag{13}$$

A sufficient condition for global asymptotic stability in isolation is

$$\dot{\boldsymbol{q}}^t_e\boldsymbol{D}_e > 0 \qquad \text{for all } \boldsymbol{p}_e \neq \boldsymbol{0} \tag{14}$$

that is, the unique equilibrium configuration of the environment is globally asymptotically stable if the net power absorbed by the internal nonconservative (generalized) forces are a positive-definite function of the generalized momenta.

Note that the describing equations for the environment are highly structured because of the energetic nature of physical systems. We need only make a few mildly restrictive assumptions to arrive at sufficient conditions for isolated stability. Summarizing those assumptions: 1) the system dynamics are described by (9) and (10), 2) the total system energy is a positive-definite, nondecreasing function of the generalized momenta and displacements, and 3) the *net* power absorbed by the nonconservative (generalized) forces internal to the environment is a positive-definite function of the generalized momenta. Of course, not all environments behave this way; however, the class of physical objects which is subsumed under this description is extremely large [4], [17].

Restrictions on Physical System Interactions

Another important feature of physical systems is that, unlike dynamic systems in general, the ways in which they may interact are restricted. Dynamic interactions between physical systems may be described (essentially be generalizing Kirchhoff's current and voltage laws) as *isenergic*, that is, involving an instantaneous exchange of energy without loss, storage, or generation of energy. Instantaneous energetic interaction or power flow between a physical system and its environment may always be described as a product of two variables, an effort (generalized voltage or force) and a flow (generalized current or velocity) [3], [17].

Note that in general, dynamic systems are not subject to this restriction. For example, two general linear systems may be coupled in cascade. The two systems may be represented by the following transfer functions:

$$y_1 = G_1(s)u_1 \tag{15}$$

$$y_2 = G_2(s)u_2. \tag{16}$$

Their cascade combination is

$$y_3 = G_3(s) \qquad u_3 = G_2(s)G_1(s)u_3. \tag{17}$$

The cascade coupling equations are

$$y_3 = y_2 \qquad u_2 = y_1 \qquad u_1 = u_3. \tag{18}$$

However, if (15) and (16) represent the dynamic responses of physical systems to their environments (i.e., impedances or admittances), then the products u_1y_1, u_2y_2, and u_3y_3 represent the power into[2] systems 1, 2, and 3, respectively. The isenergic nature of physical system interactions requires that the net power into the coupled system must be the sum of the power into the component systems

$$u_3y_3 \equiv u_1y_1 + u_2y_2. \tag{19}$$

Clearly, the cascade combination does not guarantee that this requirement is satisfied. Interactions between physical systems are more restricted than interactions between dynamic systems in general.

When a manipulator grasps an object, coupling between manipulator and environment takes place at a set of points on the object. These points define an *interaction port*. The

[2] The convention that power is positive into a system has been assumed.

position of the interaction port is a function of the generalized coordinates.

$$X_e = L_e(q_e). \quad (20)$$

The velocity of the interaction port V_e is related to the generalized velocity $\dot{q}_e$ through the Jacobian of the kinematic transformation from generalized coordinates to interaction port coordinates. Because the transformation is isenergic, the generalized input force P_e is related to the interaction port force F_e through the transposed Jacobian

$$V_e = J_e(q_e)\dot{q}_e \quad (21)$$

$$P_e = J_e(q_e)^t F_e. \quad (22)$$

The equations show that this class of environments accepts input forces and produces output motions in response. Note that the vector of generalized coordinates q_e need not have the same number of components as the vector of interaction port coordinates and hence the Jacobian need not be square. Therefore, it is not always possible to reformulate the equations in the dual form with velocity as the input and force as the output; this system is best described as a generalized mechanical admittance[3]. Once again, not all environments behave this way, but the class of physical objects which fit this description is extremely large.

Dynamic Behavior of the Manipulator

The idea of physical equivalence is that the dynamic behavior of the controlled manipulator is subject to the same restrictions as any physical system. Dynamic interaction between two physical systems imposes a constraint on the forms of their input/output relations. If one system is described as a (generalized) admittance, accepting effort (e.g., force) input and producing flow (e.g., motion) output, the other should be described as a (generalized) impedance, accepting flow (e.g., motion) input and producing effort (e.g., force) output. If we choose to model the environment as above, then to be physically compatible, the manipulator should behave as a generalized impedance, accepting motion inputs, and producing force outputs in response.

Imposing appropriate dynamic behavior is the objective of impedance control. Some questions immediately arise: First, can it be done? The feasibility of imposing a desired impedance on a manipulator has been demonstrated and discussed in detail elsewhere [8], [9], [11], [13]-[15], [21]. Secondly, is it worth doing? As detailed elsewhere [1], [9] impedance control provides a unified framework for coordinating free motions, obstacle avoidance, kinematically constrained motions, and motions involving dynamic interaction.

The paper will consider another aspect of impedance control: the preservation of stability in the face of large changes in the dynamic environment to which a manipulator is coupled. Large changes occur when the manipulator makes the transition from unconstrained motion (infinite environmental admittance) to contact with an object of unspecified dynamic complexity (any member of the class of systems described by (9), (10), (20)-(22)).

First we will consider the desired dynamic behavior. A class of *simple impedances* will be defined and it will be shown that if the manipulator had the behavior of this general class of impedances then a sufficient condition for the manipulator and the environment to be stable in isolation from one another would also be sufficient to guarantee that dynamic interaction between manipulator and environment would not cause instability [10]. Then we will consider one approach to achieving this behavior and examine the conditions under which it also has the coupled stability property.

[3] Note that although the terms impedance and admittance are commonly restricted to linear systems, they are applied here to nonlinear systems, hence the terms *generalized* impedance and *generalized* admittance. In general, they are causal dynamic operators which map an input time function $u(t)$ onto an output time function $y(t)$ such that the present value of the output $y(t)$ may depend on the entire past history of the input $u(t-\tau)$ for $0 < \tau < \infty$ [17].

Simple Impedances

One simple (but versatile) class of impedances has the following behavior:

$$F_z = K(X_z - X_0) + B(V_z) \quad (23)$$

where X_0 is a vector of equilibrium positions of the port of interaction (e.g., the manipulator end-effector) when it is isolated from its environment. In the following it will be assumed to be a constant, corresponding to the maintenance of a fixed posture.

To facilitate stability analysis we express this impedance in Hamiltonian form. The function relating force to displacement from the equilibrium posture may be nonlinear, but if it corresponds to a generalized elastic behavior, physical systems theory requires that a potential function can be defined, which can be used as the Hamiltonian for this impedance.

$$H(q_z) \triangleq \int K(q_z)\, dq_z \quad (24)$$

where

$$q_z \triangleq X_z - X_0. \quad (25)$$

Rewriting (23) in Hamiltonian form

$$\dot{p}_z = H_{zq}(q_z) + B(V_z) \quad (26)$$

$$\dot{q}_z = V_z(t) \quad (27)$$

$$F_z = \dot{p}_z. \quad (28)$$

The rate of change of the Hamiltonian is

$$dH_z/dt = H_{zq}^t V_z. \quad (29)$$

In the absence of imposed motions, V_z is identically zero and this system is isolated. The rate of change of the Hamiltonian is then zero, but no statement can be made about the system's asymptotic stability. However, the impedance described by (23) is to be the target behavior for a manipulator and one of the assumptions underlying impedance control [9] is that an impedance-controlled manipulator should be at least capable of stably positioning an arbitrarily small unconstrained mass (i.e., a rigid body). To determine the restrictions this places on the impedance, we will next consider the stability of this impedance coupled to an unconstrained rigid body.

The Hamiltonian for a rigid body may be equated with its kinetic energy

$$H_e(p_e)=E_k(p_e)=1/2p_e^tM^{-1}p_e. \quad (30)$$

Because the inertia tensor is positive-definite, the Hamiltonian is positive-definite and nondecreasing. The equations of motion may be written as follows:

$$\dot{q}_e=H_{ep}(p_e) \quad (31)$$

$$\dot{p}_e=F_e(t) \quad (32)$$

$$V_e=\dot{q}_e. \quad (33)$$

The rate of change the Hamiltonian of this system is

$$dH_e/dt=H_{ep}^tF_e. \quad (34)$$

Thus an unconstrained rigid body also has the property that when the force F_e is zero and the system is isolated, its energy is nonincreasing but no statement can be made about its asymptotic stability.

Assume the rigid body and the impedance are coupled so that their interaction ports share a common velocity. Isenergic coupling requires the net power generated to be zero.

$$F_e^tV_e+F_z^tV_z=0. \quad (35)$$

Thus the forces F_e and F_z have opposite signs. The Hamiltonian for the coupled system is its total energy which is the sum of the energies of the component systems.

$$H_t(p_e, q_z)=H_e(p_e)+H_z(q_z). \quad (36)$$

The equations for the coupled system are as follows:

$$\dot{p}_e=-H_{tq}(q_z)-B(H_{tp}(p_e)) \quad (37)$$

$$\dot{q}_z=H_{tp}(p_e). \quad (38)$$

This system is at equilibrium when $p_e = 0$. If the potential energy of the impedance is a positive-definite, nondecreasing function of the displacements then the system has a unique equilibrium point at the origin of the state space ($p_e = 0$, $q_z = 0$). The rate of change of the Hamiltonian is

$$dH_t/dt=H_{tq}^t\dot{q}_z-H_{tp}^t\dot{p}_e$$
$$=H_{tq}^tH_{tp}-H_{tp}^tH_{tq}-H_{tp}^tB=-\dot{q}_z^tB. \quad (39)$$

A sufficient condition for global asymptotic stability of that equilibrium point is

$$\dot{q}_z^tB>0 \qquad \text{for } p_e\neq 0. \quad (40)$$

An impedance of the form of (23) which satisfies the conditions that 1) the potential energy of the impedance is a positive-definite, nondecreasing function of the displacements q_z and 2) the net power dissipated by the nonconservative forces is a positive-definite function of the generalized momenta p_e will be referred to as a *simple impedance*.

Coupled Stability of Simple Impedances

Now consider the stability of the system formed when a simple impedance interacts dynamically with the more general environment described by (9), (10), (20)–(22). Once again, because of the isenergic nature of the interaction, the coupled system Hamiltonian is the sum of the Hamiltonians of the components.

$$H_t(p_e, q_e)=E_k(p_e, q_e)+E_p(q_e)+H_z(L_e(q_e)-X_0). \quad (41)$$

The equilibrium configuration for the environment is at the minimum of $E_p(q_e)$. The equilibrium configuration for the simple impedance is at X_0, the minimum of $H_z(q_z)$. We will assume that the two systems are at equilibrium and that the positions of their interaction ports coincide before they are coupled. Consequently, the minima of H_z and E_p coincide and the coupled system Hamiltonian H_t is a positive-definite, nondecreasing function of the momenta p_e and displacements q_e. The rate of change of the coupled system Hamiltonian is

$$dH_t/dt=H_{zq}^tJ_eH_{ep}+H_{eq}^tH_{ep}-H_{ep}^tH_{eq}-H_{ep}^tD_e$$
$$-H_{ep}^tJ_e^tH_{zq}-H_{ep}^tJ_e^tB \quad (42)$$

$$dH_t/dt=-\dot{q}_e^tD_e-\dot{q}_z^tB. \quad (43)$$

Thus the sufficient conditions for assymptotic stability of each of the two individual systems (14) and (40) are also sufficient to guarantee that the rate of change of the Hamiltonian of the coupled system is never positive. In physical terms, because the isenergic coupling does not generate energy, there is no mechanism through which the total system energy can grow, and the nonconservative terms ensure it will decrease. Furthermore, the coupled system has a unique equilibrium state and therefore the coupled system is globally asymptotically stable.

Summarizing briefly, this discussion has shown that a simple impedance described by (23) where $K(q_z)$ is the gradient of a positive-definite, nondecreasing function and $V_z^tB > 0$ for $V_z \neq 0$ has strong stability robustness properties. Dynamic interaction with *any* object which is a generalized admittance described by (9), (10), (20)–(22), where $H_e(p_e, q_e)$ is a positive-definite, nondecreasing function of p_e and q_e and $\dot{q}_e^tD_e > 0$ for all $p_e \neq 0$ cannot cause instability.

If the behavior of this simple impedance can be imposed on a manipulator, then if the manipulator is stable in isolation it will remain stable when it grasps any of a large class of objects of arbitrary dynamic complexity, provided that object was also stable in isolation.

Implementing a Simple Impedance

We will next consider the implementation of a simple impedance on a class of manipulators. There are, of course, many ways to implement a given desired impedance; some have been discussed elsewhere [9]. For simplicity, we will restrict our attention to manipulators which, in the absence of a controller, are neutrally stable at each configuration in the workspace. Any mechanism which is dynamically balanced or statically balanced has this property. Robots of this type can exhibit the problems of contact instability [11]. We will further assume that the manipulator may be adequately modeled as a rigid-body kinematic mechanism driven by controllable-force actuators. This is an idealizing assumption common in the robotic literature, and the recent work on direct-drive robot designs [2] has resulted in practical robots which are well-described by these assumptions. Such a manipulator is usually described using a Lagrangian formulation similar to (1) and (2), but to simplify the stability analysis, we will use a generalized Hamiltonian formulation.

The Hamiltonian for the manipulator without a controller is the kinetic energy of the mechanism.

$$H_m \triangleq p_m^t I(q_m)^{-1} p_m. \tag{44}$$

The generalized input forces to this mechanism are the actuator efforts and the forces from the environment. Defining an interaction port for the manipulator, its position is a function of the generalized coordinates.

$$X_m = L_m(q_m). \tag{45}$$

Given the kinematic transformations, the relation between velocities and forces may be derived and the input/output relation at the interaction port is as follows:

$$\dot{q}_m = H_{mp} \tag{46}$$

$$\dot{p}_m = -H_{mq} - D_m + P_a + J_m^t F_m \tag{47}$$

$$V_m = J_m \dot{q}_m. \tag{48}$$

We wish to impose the behavior of the target impedance of (23). Suppose that impedance were coupled to the manipulator interaction port, i.e., $X_z = X_m$. Given the kinematic transformations and (35), the target impedance may be transformed into the generalized coordinates of the manipulator.

$$P_a = -J_m(q_m)^t K(L_m(q_m) - X_0) - J_m(q_m)^t B(J_m(q_m)\dot{q}_m). \tag{49}$$

If we assume that measurements of the position and velocity of the manipulator generalized coordinates are available, then one simple way to implement the target impedance is to regard (49) as the specification of a nonlinear feedback control law [12]. Note in passing that this approach requires no inversion of the kinematic equations or of the Jacobian and thus is computable for all configurations of the manipulator.

This controller coupled to the manipulator may be described as an equivalent physical system in generalized Hamiltonian form. The Hamiltonian for the controlled system H_c is the sum of the Hamiltonians for the uncontrolled manipulator and the impedance.

$$H_c(p_m, q_m) = H_m(p_m, q_m) + H_z(L_m(q_m) - X_0). \tag{50}$$

Thus we arrive at the following representation:

$$\dot{q}_m = H_{cp} \tag{51}$$

$$\dot{p}_m = -H_{cq} - D_m - J_m^t B + J_m^t F_m \tag{52}$$

$$V_m = J_m \dot{q}_m. \tag{53}$$

Note that the structure of the equations for the controlled system is the same as that for the uncontrolled physical system. In this sense the controlled system is equivalent to a physical system (and subject to the same constraints). This is an example of physical equivalence, the starting assumption behind impedance control.

To assess stability we examine the rate of change of the Hamiltonian H_c.

$$\begin{aligned} dH_c/dt = {} & H_{cq}^t H_{cp} - H_{cp}^t H_{cq} - H_{cp}^t D_m \\ & - H_{cp}^t J_m^t B + H_{cp}^t J_m^t F_m \end{aligned} \tag{54}$$

$$dH_c/dt = -\dot{q}_m^t D_m - V_m^t B + V_m^t F_m. \tag{55}$$

At first glance it might appear that the properties of the simple impedance are sufficient to guarantee asymptotic stability of the controlled manipulator in isolation, but this is not necessarily the case. A feature of this implementation of the simple impedance controller is that if the mechanism is capable of joint motions which do not affect the motion of the interaction port (motions in the null-space of the Jacobian) the impedance imposed by the controller is unaffected by those motions. This is advantageous insofar as any desired strategy for using those "extra" degrees of freedom can be applied in conjunction with the controller. On the other hand, the converse is also true: this controller will have no effect on null-space motions. The simple impedance only guarantees asymptotic stability of the interaction port and not of the configuration of the manipulator.

There are several ways to overcome this problem. One is to augment this implementation of the simple impedance with a joint-space controller which stabilizes the manipulator configuration; for example, the position and velocity feedback controllers discussed by Takegaki and Arimoto [19]. However, in the interest of simplicity, in this paper we will assume that the manipulator and the interaction port have the same number of degrees of freedom so that the manipulator cannot make null-space motions.

Even given this assumption, there exist equilibrium configurations other than $q_m = \mathbf{0}$ for which the rate of change of the Hamiltonian vanishes identically. For example, a planar two-segment mechanism with no limits on joint motion can reach any end-point position not on the boundaries of its workspace at two joint configurations (the "left-hand" and "right-hand" solutions of the inverse kinematic equations). As a result, this implementation of the simple impedance does not provide global asymptotic stability.

However, it does provide local asymptotic stability because there exists a region centered on the equilibrium configuration $q_m = \mathbf{0}$ in which the Hamiltonian H_c is a positive-definite, nondecreasing function of the displacements q_m. Because of the inertial properties of the mechanism, the Hamiltonian is a positive-definite, nondecreasing function of the momenta p_m. When the system is isolated $F_m = \mathbf{0}$ and from the properties of the simple impedance, $V^t B \geq 0$ for all p_m. A sufficient condition for isolated stability of the impedance-controlled manipulator is

$$\dot{q}_m^t D_m > 0, \qquad \text{for } p_m \neq \mathbf{0}. \tag{56}$$

This result is similar to that presented by Takegaki and Arimoto [19] which established the stability properties of general position and velocity feedback controllers. Those authors also point out that if the manipulator is coupled to an ideal kinematic constraint, the effect is to reduce the number of degrees of freedom of the coupled system, but the Hamiltonian defined above still remains locally positive-definite. However, in general, dynamic interaction with an environment will *add* to the degrees of freedom of the coupled system (rather than reduce them) and couple the environmental dynamics to the manipulator and its controller, thereby placing stability in jeopardy. The next section will show that this impedance-controlled manipulator has the stability robustness property of the target impedance.

Coupled Stability of the Controlled Manipulator

If the manipulator and environment are coupled at a set of points of common velocity, some of the generalized coordinates of the two systems will become interdependent. In that case, a new set of generalized coordinates q_t for the coupled system may be defined in terms of the old.

$$q_t = q_t(q_m, q_e). \tag{57}$$

Because of this constraint equation, the generalized momenta of the two systems become interdependent, as do the generalized velocities and forces. The effect of the coupling is to confine the system trajectories to a subspace of the state space defined by the momenta and displacements of the manipulator, p_m and q_m, and the environment, p_e and q_e. Nevertheless, the total energy for the coupled system is still the sum of the energies of the two systems in isolation, and the analysis of stability may proceed as before.

$$H_t = E_k(p_e, q_e) + E_p(q_e) + H_m(p_m, q_m) + H_z(L_m(q_m) - X_0). \tag{58}$$

The equilibrium configuration for the environment is at the minimum of $E_p(q_e)$. The equilibrium configuration for the simple impedance is at X_0, the minimum of $H_z(q_z)$. Assuming that the two systems are at equilibrium and that the positions of their interaction ports coincide before they are coupled, the minima of H_z and E_p coincide and there is a region centered on the equilibrium configuration in which the coupled system Hamiltonian H_t is a positive-definite, nondecreasing function of the momenta p_e and p_m, and displacements q_e and q_m.

Using (12), (22), and (55), the rate of change of the coupled system Hamiltonian is as follows:

$$dH_t/dt = -\dot{q}_e^t D_e + \dot{q}_e^t J_e^t F_e - \dot{q}_m^t (D_m + J_m^t B) + \dot{q}_m^t J_m^t F_m. \tag{59}$$

Using (21) and (48) we may identify the terms representing the power generated by the coupling.

$$dH_t/dt = -\dot{q}_e^t D_e - \dot{q}_m^t (D_m + J_m^t B) + V_e^t F_e + V_m^t F_m. \tag{60}$$

Because the coupling cannot generate power, the last two terms sum to zero.

$$dH_t/dt = -\dot{q}_e^t D_e - \dot{q}_m^t D_m - V_m^t B. \tag{61}$$

Thus the sufficient conditions for assymptotic stability of each of the two individual systems (14), (40), and (56) are also sufficient to guarantee that the rate of change of the Hamiltonian of the coupled system is never positive. Because the coupled system Hamiltonian is locally positive-definite, the coupled system is locally asymptotically stable. Dynamic interaction with a large class of objects which are stable in isolation cannot cause instability.

Insensitivity to Kinematic Errors

How sensitive is this result to the inevitable departure of a real manipulator or a real control algorithm from the idealizing assumptions made by using (46)–(48)? This is a topic of current investigation [5] which will not be discussed in depth here, but robustness to certain classes of errors will be demonstrated. One interesting and useful result is that the coupled stability property is completely insensitive to errors in the kinematic equations.

Suppose (49) is implemented as a control algorithm using kinematic equations which differ from the correct equations.

$$P_a = -{}'J(q_m)^t K({}'L(q_m) - X_0) - {}'J(q_m)^t B({}'J(q_m)\dot{q}_m). \tag{62}$$

The erroneous kinematic equations map the coordinates q_m onto a point $'X$ where

$$'X = {}'L(q_m). \tag{63}$$

Define the potential function

$$'H_z(q_m) \triangleq H_z({}'L(q_m) - X_0). \tag{64}$$

It is a nonnegative function of the displacements q_m with a local minimum at $'X = X_0$ Define the candidate Lyapunov function

$$'H_c(p_m, q_m) \triangleq H_m(p_m, q_m) + H_z({}'L_m(q_m) - X_0). \tag{65}$$

Its rate of change is

$$\begin{aligned} d'H_c/dt = {} & H_{zq}^t[\partial' L/\partial q_m] H_{mp} + H_{mq}^t H_{mp} \\ & - H_{mp}^t H_{mq} - H_{mp}^t D_m - H_{mp}^t{}' J^t H_{zq} \\ & - H_{mp}^t{}' J^t B + J_m^t F_m. \end{aligned} \tag{66}$$

If the (erroneous) Jacobian $'J(\cdot)$ is the correct derivative of the (erroneous) kinematic equation $'L(\cdot)$ then $\partial L/\partial q_m = {}'J$

$$d'X/dt = {}'V = {}'J(q_m)\dot{q}_m. \tag{67}$$

If the manipulator is isolated $F_m(t) \equiv 0$

$$d'H_c/dt = -\dot{q}_m^t D_m - {}'V^t B. \tag{68}$$

Equation (56) (a property of the manipulator) and (40) (a property of the target impedance) guarantee that these nonconservative forces are a positive-definite function of the momenta, and that is sufficient to establish isolated local asymptotic stability of the controlled manipulator.

We can then proceed as above (57)–(61) to analyze the stability of the controlled manipulator coupled to the environment. The Hamiltonian of the coupled system is

$$'H_t = E_k(p_e, q_e) + E_p(q_e) + H_m(p_m, q_m) + H_z('L(q_m) - X_0). \quad (69)$$

Its rate of change is

$$d'H_t/dt = -\dot{q}_e^t D_e - \dot{q}_m^t D_m - 'V^t B. \quad (70)$$

Thus conditions sufficient to guarantee local asymptotic stability of the controlled manipulator in isolation are also sufficient to guarantee that dynamic interaction with a large class of objects which are stable in isolation cannot cause instability.

An interesting point is that this result does not require any assumption of small errors. Differences between the true kinematic equations and those implemented in the controller could arise due to kinematic calibration errors, in which case we might expect the differences to be small. But they could also arise if the manipulator contacted the environment at a point other than the expected interaction port (e.g., when the forearm touches down before the hand does). In that case, the differences need not be small by any measure. However, the coupled stability property remains intact: if the manipulator is (locally) stable in isolation, it will remain (locally) stable when coupled to an arbitrarily complex environment of the class defined above.

Insensitivity to Interface Dynamics

Another important way a real manipulator may depart from the idealizing assumptions made above is that the interface between the manipulator and environment (e.g., the hand, the fingers) may exhibit dynamic behavior which this simple implementation does not take into account. We will next show that the coupled stability property is insensitive to a broad class of unmodeled interface dynamics.

The simple implementation described above which treats (49) as a nonlinear feedback control law is not completely successful at achieving the desired impedance. It adds the desired impedance to the manipulator but does nothing to change the manipulator's fundamental inertial behavior. (A more sophisticated controller would be required to do that, e.g., [11], [14], [21].) The dynamics of the controlled manipulator is composed of the imposed impedance coupled to the intrinsic admittance of the manipulator.

To the extent that the environment is an admittance (our starting assumption), a compatible model of the interface behavior is that of an impedance. This is physically reasonable: The soft fingertip pads of the human hand are predominantly visco-elastic; the wrist force sensor used in many robots is basically an instrumented spring. In both cases, the dynamics of the interface may be well described as a generalized mechanical impedance.

Now consider the stability of the coupled system composed of the manipulator, a dynamic environment of an arbitrary number of degrees of freedom, and this interface. Any force exerted on the environment is also exerted on the interface and is equal and opposite to the force on the manipulator

$$F_i = F_e = -F_m. \quad (71)$$

The velocity associated with the interface is the difference between the velocity of the manipulator interaction port and the velocity of the environment interaction port

$$V_i = V_m - V_e. \quad (72)$$

Consequently, as required by physical systems theory, the net power generated by the coupling is zero

$$F_i^t V_i + F_e^t V_e + F_m^t V_m = 0. \quad (73)$$

From this fact we can proceed as above to show that if 1) the interface dynamics are stable in isolation, 2) the environment is stable in isolation, and 3) the manipulator is stable in isolation, then the total energy of the coupled system is the sum of the energies of the components and never increases. Therefore, dynamic interactions between these three systems cannot cause instability.

Just as the environment dynamics may be of arbitrary order, this result could be derived for interface dynamics of arbitrary order, but for brevity we will show it for the following case in which the interface is a simple visco-elastic system. Denoting the displacement of the interface by q_i, where

$$q_i = X_m - X_e \quad (74)$$

we will assume the interface is described by the following equations:

$$F_i = K_i(q_i) + B_i(V_i). \quad (75)$$

For stability analysis we will represent this impedance in Hamiltonian form. The Hamiltonian for the interface is the potential function defined by the elastic behavior

$$H_i(q_i) \triangleq \int K_i(q_i)\, dq_i \quad (76)$$

$$\dot{p}_i = H_{iq}(q_i) + B_i(V_i) \quad (77)$$

$$\dot{q}_i = V_i(t) \quad (78)$$

$$F_i = \dot{p}_i. \quad (79)$$

To ensure that this interface is stable when the manipulator grasps an arbitrarily small rigid body, we will assume that $H_i(q_i)$ is a positive-definite nondecreasing function of q_i and that $V_i^t B_i > 0$ for $V_i \neq \mathbf{0}$.

When the manipulator, interface, and environment are coupled, the generalized coordinates of the total system are q_m and q_e, the generalized coordinates of the interface being determined by (74). The Hamiltonian for the coupled system H_t may be obtained by adding the individual Hamiltonians

$$H_t(p_e, q_e, p_m, q_m) = H_c(p_m, q_m) + H_e(p_e, q_e) + H_i(L_m(q_m) - L_e(q_e)). \quad (80)$$

Assuming that the systems are at equilibrium and that the positions of their interaction ports coincide before they are coupled, there is a region centered on the equilibrium configuration in which the coupled system Hamiltonian H_t is a positive-definite, nondecreasing function of the momenta p_e and p_m, and displacements q_e and q_m. The rate of change of the coupled system Hamiltonian is as follows:

$$dH_t/dt = dH_c/dt + dH_e/dt + H_{iq}^t V_i. \quad (81)$$

Using (12), (21), (22), (55), and (77)

$$dH_t/dt = -\dot{q}_e^t D_e + V_e^t F_e - \dot{q}_m^t D_m - V_m^t B + V_m^t F_m - V_i^t B_i + V_i^t F_i. \quad (82)$$

But the power generated by the coupling is zero (73), therefore

$$dH_t/dt = -\dot{q}_e^t D_e - \dot{q}_m^t D_m - V_m^t B - V_i^t B_i. \quad (83)$$

Thus the properties of the environment, the manipulator, the simple impedance, and the interface are sufficient to guarantee that the Hamiltonian for the coupled system is 1) locally positive-definite and 2) never increases, thus the coupled system is locally asymptotically stable. In physical terms, because the interface dynamics add degrees of freedom to the system but cannot supply energy without bound, dynamic interactions between the controlled manipulator, the interface, and the environment cannot cause instability.

Summary

The approach discussed in this paper and elsewhere—impedance control—is founded on physical systems theory. One important aspect of the dynamic equations of a physical system is their structure. This paper has shown that imposing appropriate structure on the dynamic behavior of a manipulator can result in superior stability robustness properties. It must be stressed that in general the stability of a dynamic system is jeopardized when it is coupled to a stable dynamic environment. In contrast, in this paper it was shown that if the manipulator has the behavior of a *simple impedance* then the stability of the manipulator is preserved when it is coupled to a large class of stable environments.

If the manipulator is isolated or the environment is an ideal kinematic constraint, the results presented here and in [10] reduce to those of Takegaki and Arimoto [19]. However, this analysis goes considerably further; no such restrictive assumptions about the environment are necessary. In fact, to prove the results requires almost no information about the environment. The key assumption is that the environment is stable in isolation. No restriction need be placed on its complexity or linearity. The environment may be of arbitrary order; it may even be a continuous, distributed system (as real environments are). The environment may contain nondifferentiable nonlinearities (as real environments do). For this reason, the robustness property is extremely general. For example, it is straightforward to show [5] that if the manipulator is coupled to a nonstationary support of arbitrary dynamic complexity, then, if the support is stable in isolation, the stability of the manipulator is not jeopardized by dynamic interaction with the support.

Robustness to dynamic interaction is not an exclusive property of the simple impedance nor of the simple nonlinear position and velocity feedback control law considered in this paper. The arguments presented here can readily be extended to more general forms of the target impedance and more sophisticated controllers can be found to implement the target impedance without losing the basic result. The only restriction is that the controlled system have a Hamiltonian representation with a positive-definite energy function and positive-definite nonconservative internal forces.

The reason for this strong stability robustness property is because although the manipulator and controller are clearly an active system, the behavior of the controlled manipulator as seen from the environment masquerades as a passive and dissipative system. Note that imposing passive and dissipative behavior is not essential to impedance control, but it does offer significant benefits. For example, any "non-ideal" aspect of a particular implementation of impedance control (e.g., the failure of the simple controller considered in this paper to impose the target impedance under all conditions) which preserves this apparently passive and dissipative behavior will also preserve the stability robustness property. This is the reason why the algorithm considered in this paper is *completely* insensitive to kinematic errors.

However, this restriction to apparently passive and dissipative behavior is a limitation. For example, it was assumed throughout that X_0 is constant. If it is assumed to vary with time, then the stability of the manipulator is no longer guaranteed. This does not mean that the manipulator is necessarily unstable, but the theoretical result no longer holds. Similarly, it the environment contains active power sources (as many real environments will) the manipulator stability is no longer guaranteed. Kazerooni *et al.* [13] have shown that if the environment contains active sources which do not depend on the state variables of the manipulator, then the stability robustness of the simple impedance is preserved. However, that result was derived from a local, linear analysis which assumes that the environment is either linear or at best differentiable. In contrast, the analysis presented here and in [10] requires no such assumptions.

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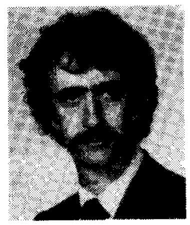

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